Comments on "A One-Level Mesoscale Model for Diagnosing Surface Winds in Mountainous and Coastal Regions"

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ABSTRACT

Mass and Dempsey (1985) have reformulated the equations for the one-level sigma-system model over complex terrain and suggested they have removed the large hydrostatic part of the horizontal pressure-gradient force (PGF). It is shown that although the coefficient of $g\nabla_\sigma Z_s$ was greatly reduced (compared to 1), the complete elimination of the hydrostatic PGF cannot be done unless the surface temperature is redefined as the sum of a perturbation temperature and a basic rest-state temperature. The alternative formula in which the hydrostatic PGF terms were totally removed is presented.

1. Introduction

Mass and Dempsey (1985; henceforth MD) describe a one-level sigma-coordinate model for diagnosing surface winds over complex terrain. The model was successfully applied by Alpert and Getenio (1988) and Alpert et al. (1988). Unlike earlier similar models, e.g., Danard (1977), they eliminated explicit reference to the surface pressure and expressed surface pressure gradients in terms of surface temperature and its temporal and spatial derivatives.

One of the important advantages of the reformulated model as claimed by MD (p. 1212) is the removal of the hydrostatic part of the horizontal pressure-gradient force (PGF) term that dominates in sigma coordinates over sloping terrain. [For methods to overcome this problem in the sigma system, see Kurihara (1968) and Phillips (1973).] In particular, MD suggest a specific way for the calculation of the PGF by first applying the Leibnitz rule for differentiating the surface pressure, p_s, given by [MD, Eq. (5)].

$$\ln p_s = \ln p_R + \left(\frac{g}{R}\right)\int_{Z_s}^{Z_R} 1\Big/T dZ, \qquad (1)$$

to obtain [MD, Eq. (12)]

$$\nabla_\sigma \ln p_s = \left(\frac{g}{R}\right)\left[T_R^{-1}\nabla_\sigma Z_R - T_s^{-1}\nabla_\sigma Z_s - \int_{Z_s}^{Z_R} T^{-2}(Z)\nabla_\sigma T(Z)dZ\right]. \qquad (2)$$

The notation follows MD exactly.

Now, (2) is substituted into the PGF term along the $\sigma = 1$ surface to obtain

$$g\nabla_\sigma Z_s + RT_s\nabla_\sigma \ln p_s = g\left[(T_s/T_R)\nabla_\sigma Z_R - T_s\int_{Z_s}^{Z_R} T^{-2}(Z)\nabla_\sigma T(Z)dZ\right]. \qquad (3)$$

During the last manipulation the large hydrostatic term $g\nabla_\sigma Z_s$ was canceled and MD therefore suggest that the hydrostatic variation has been removed. The explicit integration of (3) leads to [MD, Eq. (14)],

$$g\nabla_\sigma Z_s + RT_s\nabla_\sigma \ln p_s = g\Big\{\left(e_1 - \frac{H}{T_H}\right)\nabla_\sigma T_s - [e_1 + (e_2/(\gamma T_R))(T_H - T_R)]\nabla\sigma T_R + (\gamma e_1 - e_2 + 1)\nabla_\sigma Z_s + (e_2 - \gamma e_1)\nabla_\sigma Z_R\Big\}, \qquad (4)$$

where

$$e_1 = (T_s\gamma_2^{-1})[T_H^{-1} - (H\gamma_2)^{-1}\ln(T_s/T_H)]$$

$$e_2 = T_s/T_H.$$

One will notice that the third part in the $\nabla_\sigma Z_s$ term (with the coefficient of 1) contains exactly the supposedly canceled term $g\nabla_\sigma Z_s$. Actually, we will show that although the second part in that term, i.e., $-ge_2\nabla_\sigma Z_s$, largely counterbalances the large hydrostatic term $g\nabla_\sigma Z_s$, it cannot be completely removed by this method since the hydrostatic part of the $\nabla_\sigma T_s$ term also contributes to the cancellation of $g\nabla_\sigma Z_s$.

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2. The complete removal of the large hydrostatic term

Let us redefine the surface temperature T_s as the sum of the hydrostatic surface temperature T_{s0} and a perturbation T_{s1}. Hence,

$$T_s = T_{s0} + T_{s1} \tag{5}$$

where

$$T_{s0} = T_{00} - \gamma Z_s.$$

The T_{00} and γ are the constant sea-level temperature and lapse-rate at the basic rest state, respectively. Hence, the surface temperature gradient along the σ-surface is

$$\nabla_\sigma T_s = -\nabla_\sigma T_{s0} + \nabla_\sigma T_{s1} = -\gamma \nabla_\sigma Z_s + \nabla_\sigma T_{s1}. \tag{6}$$

Now the *basic state PGF* can be calculated from (4) to be

$$(g\nabla_\sigma Z_s + RT_s \nabla_\sigma \ln p_s) = g\{(e_1 - H/T_H)(-\gamma \nabla_\sigma Z_s) + (\gamma e_1 - e_2 + 1)\nabla_\sigma Z_s\} = g\{(-H/T_H)(\nabla_\sigma T_{s0}) + (-e_2 + 1)(\nabla_\sigma Z_s)\}, \tag{7}$$

where $\nabla_\sigma Z_R = \nabla_\sigma T_R = 0$ is assumed for the basic rest state. As the basic state PGF must disappear, Eq. (7) indicates that the large hydrostatic term $g\nabla_\sigma Z_s$ is exactly canceled by

$$RT_s \nabla_\sigma \ln p_s = \{(-H/T_H)\nabla_\sigma T_{s0} - e_2 \nabla_\sigma Z_s\}g. \tag{8}$$

Equation (8) presents the terms in Eq. (4) which have not been removed along with $g\nabla_\sigma Z_s$ from the total PGF. In particular, the term $\gamma e_1 \nabla_\sigma Z_s$ is completely canceled by the basic-state component of $e_1 \nabla_\sigma T_s$ (i.e., $e_1 \nabla_\sigma T_{s0} = \gamma e_1 \nabla_\sigma Z_s$).

In order to thoroughly remove the hydrostatic variation from the PGF, Eq. (7) should be substracted from Eq. (4) to yield

$$g\nabla_\sigma Z_s + RT_s \nabla_\sigma \ln p_s = g\left\{\left(e_1 - \frac{H}{T_H}\right)\nabla_\sigma T_{s1} - [e_1 + (e_2/(\gamma T_R))(T_H - T_R)]\nabla_\sigma T_R + \frac{(\gamma - \gamma_2)H}{T_H}\nabla_\sigma Z_s + (e_2 - \gamma e_1)\nabla_\sigma Z_R\right\}. \tag{9}$$

In the basic state where $T_{s1} = 0$, $\nabla_\sigma T_R = \nabla_\sigma Z_R = 0$ and $\gamma = \gamma_2$, the PGF is identically zero. Equation (9) is thus suggested to replace Eq. (4) herein [in MD, Eq. (14)] in order to completely eliminate the hydrostatic PGF. Now, comparing MD's formula for the PGF, Eq. (4), with our formula, Eq. (9), one will notice that, in our formulation, the coefficient of $\nabla_s Z_s$ is identically zero when starting the simulations (i.e., $\gamma_2 = \gamma$) and becomes a small perturbation only when γ_2 differs from γ due to prognostic variations in the surface temperature, T_s. In addition, $\nabla_\sigma T_s$ in Eq. (4) is replaced by the gradient of a much smaller variable, T_{s1}, which does not include the large hydrostatic variation of surface temperature over sloping terrain, and which contributes nothing (except perhaps truncation errors) to the PGF over topography. Consequently, the numerical calculation of the $\nabla_\sigma Z_s$ and $\nabla_\sigma T_s$ terms from Eq. (4) will inevitably involve larger truncation errors than the $\nabla_\sigma Z_s$ and $\nabla_\sigma T_{s1}$ terms in Eq. (9). As mentioned earlier regarding the coefficient of $g\nabla_\sigma Z_s$, i.e., $(\gamma e_1 - e_2 + 1)$, although the term $-e_2$ approximately cancels $+1$, the remaining γe_1 term cannot be balanced in the MD formulation through the $\nabla_\sigma Z_s$ coefficient since as shown in Eqs. (7) and (8), the γe_1 term exactly cancels the basic-state component of $e_1 \nabla_\sigma T_s$. Hence, Eq. (9) thoroughly *eliminates* these large terms which exactly cancel each other, and therefore eliminates possible truncation errors. From these reasons, Eq. (9) is superior to Eq. (4)—the MD formulation for the PGF.

Another point to mention is that the "Leibnitz" derivation proposed by MD is unnecessary since the direct substitution of $\nabla_\sigma \ln p_s$ [their Eq. (10)] is sufficient to obtain exactly the same formula (14). Hence, their warning not to follow the latter way seems unjustified.

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