Radiation pressure on randomly oriented infinite cylinders

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Expressions are developed for the radiation pressure on infinite dielectric cylinders caused by an oblique incidence as a function of the size parameter $\alpha = 2\pi a/\lambda$. It is shown that for nonabsorbing cylinders the radiation pressure is always perpendicular to the axis of the cylinder and thus not along the direction of the incident radiation except for the case of normal incidence. This result applies also for other small nonspherical particles. Consequently, the radiation pressure on a randomly oriented nonspinning group of small nonspherical particles causes the particles to spread away from the direction of propagation of the incident radiation. It is suggested that this conclusion should be taken into account when discussing the effect of the radiation pressure on small particles in space, as compared with other forces such as the dynamic pressure on the solar wind.

I. Introduction

The radiation pressure caused by scattering spherical particles has been discussed and calculated for size parameters ranging from the Rayleigh-scattering domain ($\alpha = 2\pi a/\lambda \ll 1$) through the Mie-scattering domain ($\alpha \tilde{>} 1$) and to the geometrical optics region $\alpha \gg 1$ (see, for example, Refs. 1–3).

The main effort has been concentrated in the calculation of the asymmetry factor (A.F.) of spherical scatterers produced by the stronger forward scattering and defined as

$$\text{A.F.} = \frac{\iint I(\theta) \cos\theta d\theta d\phi \sin\theta}{\iint I(\theta) d\theta d\phi \sin\theta}, \tag{1}$$

where $I(\theta)$ is the differential scattering intensity at a given direction relative to the direction of the incidence $(-1 \leqslant \cos\theta \leqslant 1)$.

Although the value of the A.F. for a spherical scatterer may range from -1 (predominant backscattering) to $+1$ (predominant forward scattering), depending on the size parameter and the refractive index, the direction of the radiation pressure always coincides with the direction of the propagation of light (the incident radiation).

This is not normally the case for nonspherical scatterers: Let us consider a tilted nonspherical particle, which is for simplicity small enough to satisfy the Rayleigh approximation. The induced dipole is dependent on the polarizability tensor, which is not a constant multiple by the unit matrix (since we deal with a nonspherical particle) even along the main axis of the particle. If we take, for example, a linear diatomic molecule, the two polarizability coefficients to be considered are $\alpha_\parallel$—along the axis of the molecule—and $\alpha_\perp$—normal to the axis.[4] The induced dipole $\mathbf{P}$ can be expressed by the following two equations:

$$P_\parallel = \alpha_\parallel E_i \sin\Phi,$$

$$P_\perp = \alpha_\perp E_i \cos\Phi,$$

if the incident light is polarized in the plane containing the incident direction and the diatomic molecule, where E_i is the incident electric field and Φ is the angle between the direction of the incidence and the axis of the molecule.

Since for a nonspherical particle $\alpha_\parallel \neq \alpha_\perp$, $\mathbf{P}$ is not along $\mathbf{E}_i$ as it is for a sphere. On the other hand, since the scattered radiation is symmetric about $\mathbf{P}$, it is not symmetric about $\mathbf{E}_i$, thus leading to a component of radiation pressure perpendicular to the direction of the incident radiation.

As will be shown, a nonabsorbing infinite cylinder, which is scattering radiation, will always produce radiation pressure normal to its axis. The radiation pressure on cylinders was discussed by Thilo[5] only for normal incidence, and therefore this fact has not been emphasized.

It follows that randomly oriented small ($\alpha \tilde{\gtrless} 1$) nonspinning cylinders being radiated by an infinite plane wave will spread away relative to the direction of propagation.

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II. Radiation Pressure Expressions

The scattering per unit length by an infinite cylinder is characterized by a cone (of unit thickness) with an apical opening angle of $180 - 2\Phi$ (where Φ is the incident angle, see Fig. 6.3 in Ref. 3, p. 264), the envelope of which represents all possible directions of the scattered light. Let $I_0(\Phi)$ be the incident irradiance falling on an infinite cylinder with an angle Φ to its normal.

Here we shall deal only with incident radiation linearly polarized in the incident plane (the plane containing the incident direction and the cylinder axis). This incident light will be scattered in two polarizations I_1 and I_2, where I_1 is polarized in the plane containing the scattered light direction and the cylinder axis, and I_2 is polarized perpendicular to this plane:

$$I_1(\theta) = \frac{2I_0}{\pi r k} \left| \sum_{n=-\infty}^{\infty} b_n \exp(in\theta) \right|^2, \tag{2}$$

$$I_2(\theta) = \frac{2I_0}{\pi r k} \left| \sum_{n=-\infty}^{\infty} a_n \exp(in\theta) \right|^2. \tag{3}$$

For the determination of r, k, b_η, and a_η,[6] θ is the scattering angle defined in the plane normal to the cylinder axis.

For a nonabsorbing cylinder the total intensity of the light affected by the presence of the cylinder is given by

$$C_{\text{ext}} = C_{\text{scat}} = \text{const} \times r \times 2 \int_0^\pi [I_1(\theta) + I_2(\theta)] d\theta \equiv I_T. \tag{4}$$

I_T represents the intensity lost by the incident beam, and therefore the momentum lost is expressed by I_T/c, where c is the speed of light. This loss of momentum by the incident beam can be separated into two components. We chose one component to be in the direction of the cylinder axis so that

$$I_{\text{axis}}/c = I_T \sin\phi/c \equiv \mathbf{R.P.}_{\text{I}}, \tag{5}$$

and the second component to be in the incident plane normal to the cylinder axis:

$$I_{\text{normal}}/c = I_T \cos\phi/c \equiv \mathbf{R.P.}_{\text{II}}. \tag{6}$$

Since no light is absorbed, the loss of momentum can be compensated in part by the scattered light. This is exactly true for the first component $\mathbf{R.P.}_{\text{I}}$.

The momentum gained by the light scattered at a direction θ along the cylinder axis is

$$\frac{1}{c} [I_1(\theta) + I_2(\theta)] \sin\phi,$$

and the total gain,

$$\mathbf{C}_{\text{I}} = \frac{1}{c} \times \text{const} \times r \times 2 \int_0^\pi [I_1(\theta) + I_2(\theta)] \sin\phi d\theta = \sin\phi \frac{I_T}{c} = \mathbf{R.P.}_{\text{I}}. \tag{7}$$

We conclude that no radiation pressure occurs along the cylinder axis. For the second component the compensation is expressed by

$$\mathbf{C}_{\text{II}} = \frac{\cos\phi}{c} \times \text{const} \times r \times 2 \int_0^\pi [I_1(\theta) + I_2(\theta)] \cos\theta d\theta. \tag{8}$$

Note that $\mathbf{C}_{\text{II}}$ is exactly equal to $\mathbf{R.P.}_{\text{II}}$ only when

$$I_1(\theta) + I_2(\theta) = \begin{cases} I_1(0) + I_2(0) & \theta = 0°, \\ 0 & \theta \neq 0°, \end{cases}$$

and therefore the scattered light cannot compensate the momentum loss normal to the cylinder axis, and the conservation of momentum requires the scatterer to move normal to its axis in the incident plane.

III. Calculation of Radiation Pressure

The radiation pressure has been calculated for

$$\alpha = 0.01; 0.1(\Delta\alpha = 0.1)1(\Delta\alpha = 1)100,$$

and for

$$\phi = 0°(\Delta\phi = 10°)80°$$

by solving analytically Eq. (8) (making use of the following expressions:

$$\left.\begin{matrix} \int_0^\pi \cos(k\theta)\cos(l\theta)\cos\theta d\theta \\ \left[\int_0^\pi \sin(k\theta)\sin(l\theta)\cos\theta d\theta = \right] \end{matrix}\right. = \begin{cases} 0; & |l-k| \neq 1, \\ \pi/4; & |l-k| = 1; l+k \neq 1, \\ \pi/2; & l+k = 1, \end{cases}$$

l,k = integers):

$$\mathbf{C}_{\text{II}} = \frac{2\cos\phi A}{c} \left[2\sum_{n=1}^{\infty} \text{Re}(b_n^* b_{n-1}) + 2\sum_{n=2}^{\infty} \text{Re}(a_n^* a_{n-1}) \right], \tag{9}$$

where A is a constant.

The numerical results are summarized in Tables I and II for

$$\text{R.R.P.} = \frac{c}{I_T} (|\mathbf{R.P.}_{\text{II}}| - |\mathbf{C}_{\text{II}}|). \tag{10}$$

This last expression represents the relative radiation pressure (R.R.P.) in the direction normal to the cylinder axis for a unit loss of momentum by the incident radiation. Note that

$$\frac{I_T}{c} = \frac{\mathbf{R.P.}_{\text{II}}}{\cos\phi} = \frac{2A}{c} \left[|b_0|^2 + 2\sum_{n=1}^{\infty} (|b_n|^2 + |a_n|^2) \right], \tag{11}$$

and hence

$$\text{R.R.P.} = \cos\phi \left[1 - \frac{2\sum_{n=1}^{\infty} \text{Re}(b_n^* b_{n-1}) + 2\sum_{n=2}^{\infty} \text{Re}(a_n^* a_{n-1})}{|b_0|^2 + 2\sum_{n=1}^{\infty} (|a_n|^2 + |b_n|^2)} \right]. \tag{12}$$

IV. Conclusions

From Tables I and II it can be seen that, for very thin nonabsorbing cylinders ($\alpha \ll 1$), the radiation pressure is maximized since the scattering has a symmetry as a function of θ. On the other hand for very large particles ($\alpha \gg 1$), most of the scattered radiation is in the forward direction, and therefore almost no radiation pressure is expected. The numerical values are generally smaller as the angle Φ increases. This is because the normal component of the incident radiation is also decreasing

Table I. $0.01 \leqslant \alpha \leqslant 1$ [a]

Φ \ α	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0[b]	1	0.997	0.990	0.978	0.961	0.940	0.912	0.878	0.834	0.779	0.712
10	0.962	0.960	0.953	0.942	0.926	0.905	0.879	0.846	0.804	0.753	0.689
20	0.852	0.850	0.845	0.835	0.822	0.809	0.782	0.753	0.719	0.676	0.625
30	0.681	0.680	0.676	0.669	0.659	0.647	0.631	0.611	0.587	0.558	0.524
40	0.494	0.493	0.491	0.486	0.481	0.473	0.464	0.453	0.441	0.426	0.410
50	0.372	0.371	0.370	0.368	0.366	0.362	0.358	0.353	0.347	0.340	0.333
60	—	0.335	0.336	0.336	0.336	0.335	0.334	0.332	0.330	0.327	0.322
70	—	0.286	0.287	0.287	0.288	0.289	0.289	0.290	0.291	0.291	0.291

[a] Relative radiation pressure (R.R.P.) values [Eq. (10)] for nonabsorbing infinite cylinders having a refractive index $m = 1.31 + oi$ (the average refractive index of ice in the visible) as a function of the size parameter $\alpha = 2\pi a/\lambda$ (a is the radius of the cylinder; λ is the incident wavelength) and the incident angle Φ (Φ is measured relative to the normal to the cylinder axis). The incident radiation is linearly polarized having its electric field in the incident plane. In Table I $\Phi = 0°(10°)70°$, and in Table II $\Phi = 0°(10°)80°$.

[b] Normal incidence.

Table II. $1 \leqslant \alpha \leqslant 100$ [a]

Φ \ α	1	2	3	4	5	10	20	30	50	70	100
0	0.712	0.252	0.136	0.100	0.092	0.239	0.140	0.133	0.124	0.095	0.048
10	0.690	0.242	0.133	0.100	0.093	0.242	0.147	0.158	0.120	0.122	0.067
20	0.625	0.218	0.126	0.101	0.096	0.281	0.183	0.161	0.135	0.117	0.052
30	0.525	0.192	0.124	0.103	0.098	0.335	0.162	0.139	0.084	0.077	0.061
40	0.411	0.188	0.130	0.104	0.096	0.224	0.101	0.083	0.103	0.098	0.068
50	0.333	0.201	0.126	0.098	0.108	0.107	0.074	0.116	0.070	0.085	0.006
60	0.322	0.207	0.122	0.108	0.141	0.067	0.152	0.063	0.070	0.071	0.072
70	0.291	0.221	0.109	0.115	0.187	0.120	0.081	0.072	0.109	0.076	0.108
80	0.169	0.181	0.069	0.079	0.189	0.094	0.090	0.028	0.059	0.048	0.088

[a] See Data in Table I.

vs Φ. It is interesting to note that the asymmetry factor for cylinders, which can be defined in a similar way to Eq. (1) (keeping in mind that the scattering is calculated per unit length),

$$\text{A.F. of cylinders} = \frac{\int I(\theta)\cos\theta d\theta}{\int I(\theta) d\theta} \tag{13}$$

can even reach negative values.

The general decreasing values as a function of α show fluctuations with a pronounced maximum near α = 6–10 (depending on Φ), following a strong minimum in the vicinity of α = 3–5. The resulting radiation pressure is always directed normal to the cylinder axis forming an angle $\Phi < 90°$ relative to the direction of the incident radiation.

Although the calculations are presented for infinite nonabsorbing cylindrical particles, the conclusions are applicable also for absorbing nonspherical particles: the resulting radiation pressure on a partially absorbing particle will also have a component along the axis of the cylinder, but combined with the normal component it will not usually sum along the incident direction. We therefore conclude that radiation pressure on small particles ($\alpha \simeq 1$) in space will not usually move the particles along the incident radiation, and for randomly oriented small nonspherical and nonspinning particles the radiation pressure effect will be to decrease the local number density.

Note that, if the particles are spinning randomly, the forces on individual particles keep reversing themselves so that the dispersion tendency would be only a second-order phenomenon. Only if the alignment is preferential a first-order dispersion phenomenon is expected.

The discussion here ignores the existence and changing of the particles' angular momentum caused by external fields other than the infinite plane wave of the radiation field.

Ariel Cohen was on leave from Hebrew University of Jerusalem when this work was completed at Drexel University, Philadelphia.

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