Extinction efficiency of obliquely and randomly oriented infinite cylinders

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The extinction efficiency is calculated for a volume containing randomly oriented long circular cylinders. The results are compared with the extinction efficiency of spheres with the same refractive index and are shown to be almost identical. Special attention is given to a corrected definition of the extinction efficiency for obliquely oriented cylinders with respect to the incident radiation direction. The corrected definition presented here is justified by use of the physical concept of the extinction efficiency as used in the case of other scatterers such as spheres. Calculations are presented for a polarized incident light (normal to the incident plane) when discussing the definition of the extinction efficiency. For the randomly oriented particles the incident light is assumed to be unpolarized.

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Lind an Greenberg[1] used the following determinations for the extinction of infinite cylinders obliquely oriented to the incident light:

$$Q_{1\text{ext}} = \frac{2}{ka}\,\text{Re}\Big(b_{0,1} + 2\sum_{n=1}^{\infty} b_{n,1}\Big),$$
$$Q_{2\text{ext}} = \frac{2}{ka}\,\text{Re}\Big(a_{0,2} + 2\sum_{n=1}^{\infty} a_{n,2}\Big), \qquad (1a)$$

where $k = 2\pi/\lambda$, λ is the incident radiation wavelength, a is the radius of the cylinder, $a_{i,2}$ and $b_{i,1}$ are coefficients (the exact definition of which can be found in Refs. 1 and 2) dependent on ka; m, the relative refractive index of the cylinder; and ϕ, the angle between the incident direction (a plane wave) and the normal to the oblique cylinder in the plane containing the incident direction and the cylinder (the incident plane).

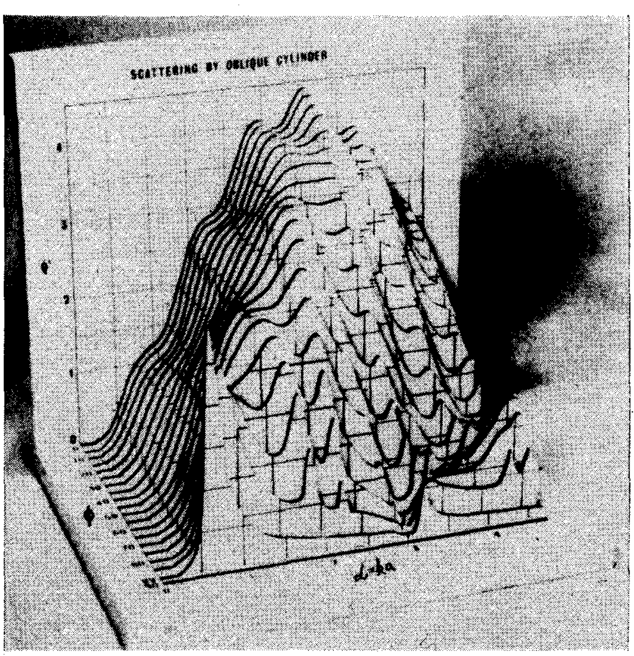

FIG. 1. Extinction efficiencies for an obliquely oriented infinite cylinder. The index of refraction is $m = 1.6$. Variation of extinction with the size parameter $\alpha = ka$ is plotted to the right. Variation with angle of incidence ϕ is plotted back ($\phi = 90°$) to front ($\phi = 89°$). $\phi = 0°, 5°, 85°$, and $89°$ after Ref. 1, Fig. 2. The incident light is polarized normal to the incident plane.

Equations (1a) are also given by Kerker[2] in his Eqs. (6.1.45) and (6.1.48). Here we suggest that Eqs. (1a) do not correspond to the usual concept of the extinction efficiency.

The expressions for the scattering irradiances by an infinite cylinder were first derived for a normal incidence (see, for example, Van de Hulst[3]) and calculated for a unit length of the cylinder. Therefore the extinction efficiency was defined there as the total scattering absorption divided by the geometrical cross section 2a. A tilted cylinder of unit length has a geometrical cross section normal to the incident direction of 2a cos ϕ. Hence, the extinction efficiencies should be defined as

$$Q_{1\text{ext}} = \frac{2}{ka\cos\phi}\,\text{Re}\Big(b_{0,1} + 2\sum_{n=1}^{\infty} b_{n,1}\Big),$$
$$Q_{2\text{ext}} = \frac{2}{ka\cos\phi}\,\text{Re}\Big(a_{0,2} + 2\sum_{n=1}^{\infty} a_{n,2}\Big), \qquad (1b)$$

For very large particles ($a \gg \lambda$) Babinet's principle[4] helps explain the asymptotic value of $Q = 2$ for any scatterer independent of its shape or refractive index as long as Q is defined relative to the geometrical cross section of the scatterer nor-

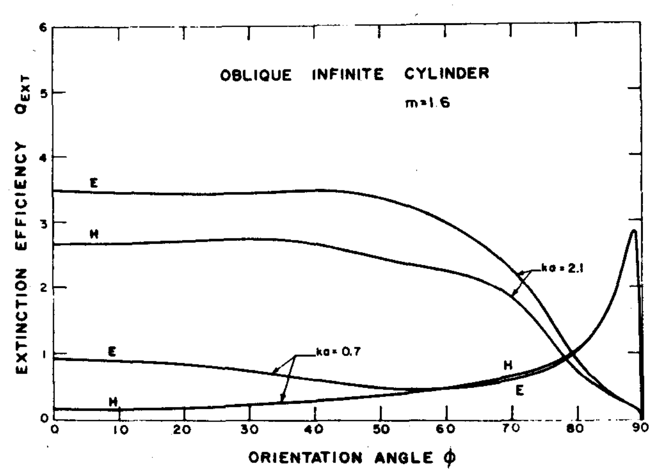

FIG. 2. Orientation dependence of extinction efficiencies for $\alpha = ka = 0.7$ and 2.1–after Ref. 1, Fig. 10. (E corresponds to the extinction of an incident light polarized normal to the incident plane and H is for an incident light normal to E).

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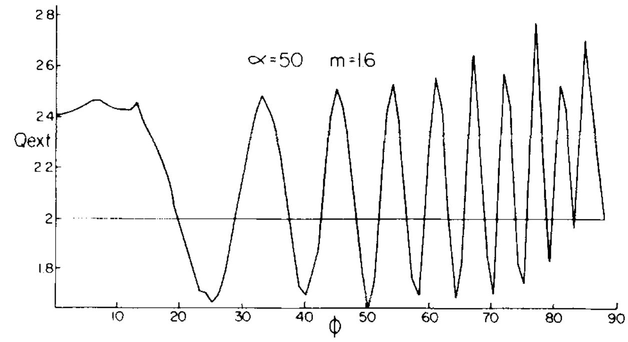

FIG. 3. Corrected orientation dependence of extinction efficiencies for tilted infinite circular cylinders for $\alpha = ka = 50$, $(Q_{1ext} \equiv Q^E)$.

mal to the incidence. Lind and Greenberg suggested that for a large $(a \tilde{>} \lambda)$ cylinder Q decreases with an increased value of ϕ (see Fig. 1) until "For $\phi = 90°$ end-on incidence Q_{1ext} and Q_{2ext} are zero for an infintie cylinder. Thus, ignoring the fluctuating part, the extinction efficiency which is proportional to the cross section per unit length, decreases inversely with the length (of finite cylinders) for sufficiently long cylinders" (Ref. 1, p. 3197). This correct but obvious conclusion is mentioned by the authors of Ref. 1 after being misled by the use of Eqs. (1a) which suggest that for $\phi \to 90°$, $Q \to 0$.

Physically, it is expected that the cylinder length should merely have a minor effect on the extinciton efficiency of any part of the cylinder which is not close to its edges. Consequently, we suggest that the concept of efficiency per unit length of a tilted cylinder used previously [1,2] has no direct physical meaning is in contradiction to the usual concept of efficiency, and if not treated carefully can lead to an erroneous conclusion.

An interesting example of the above conclusion is presented in Figs. 2 and 3. Figure 2 presents the extinction efficiency of oblique infinite cylinders of size parameters $ka = 0.7$ and $ka = 2.1$ as calculated by Lind and Greenberg. For the cylinder of $ka = 2.1$ the dependence of Q on the orientation angle essentially represents fluctuations around the function $y = A \cos\phi$ (where y corresponds to the Q coordinate and A is a constant). The larger the cylinder is, the closer its curve is expected to approximate $y = A \cos\phi$ [Eqs. (1a)]. We calculated such a dependence for $\alpha = ka = 50$ $(m = 1.6)$ and by use of the corrected Eqs. (1b) we found that Q fluctuates for all orientation angles $(0° < \phi < 89°)$ around $Q = 2$ $(1.7 \lesssim Q \lesssim 2.5)$–see Fig. 3. This result is in agreement with the physically expected efficiency value as predicted by Babiner's principle for very large particles.

An important result of the correct use of Eqs. (1b) is apparent in analyzing the curve for $ka = 0.7$ in Fig. 2: The fact that the extinction efficiency increases with the orientation angle ϕ means that for a given length of cylinder, the cylinder can scatter more light when tilted (say $\phi = 89°$) compared to the total scattering even when it is normal to the direction of the incident light. [Note that this resonance ef-

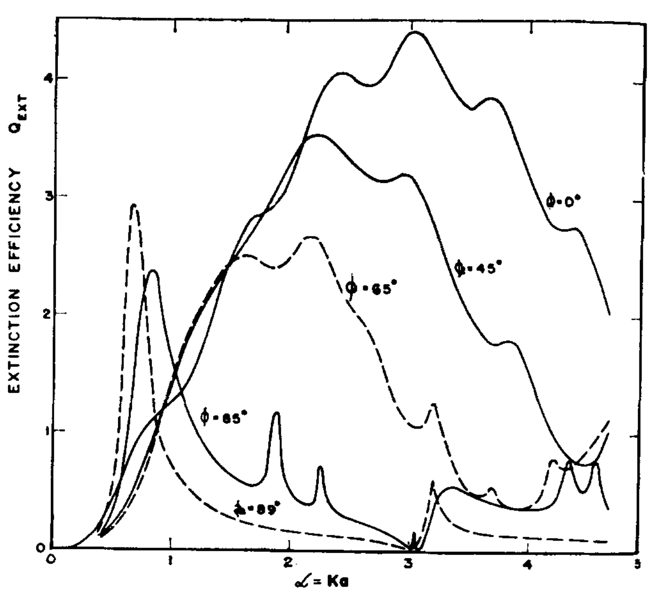

FIG. 4. Selected curves from Fig. 1 showing details of the extinction efficiencies as a function of ka, after Ref. 1, Fig. 8. (Note that in Ref. 1 the orientation angle 25° should be corrected to 65°).

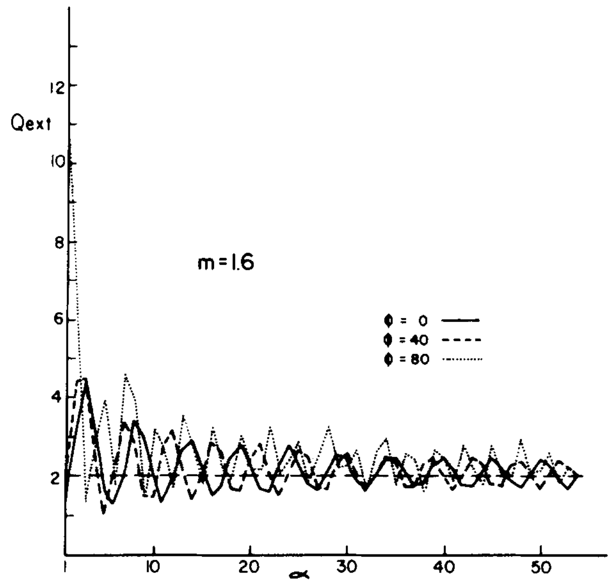

FIG. 5. Corrected Q_{1ext} values versus $\alpha = ka = 1(1)50$ for three orientation angles $\phi = 0°$, 40°, and 80° $(m = 1.6$ as in Fig. 4).

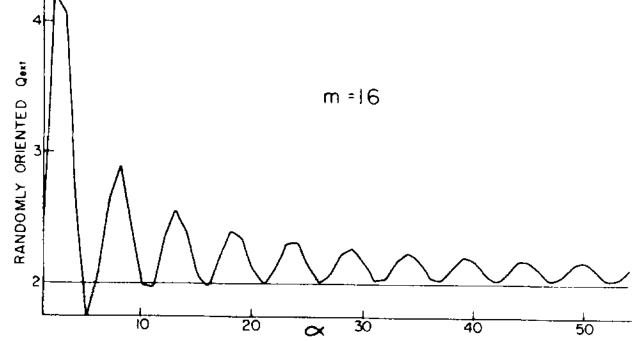

FIG. 6. Extinction efficiency for randomly oriented cylinders as a function of $\alpha = ka = 1(1)50$.

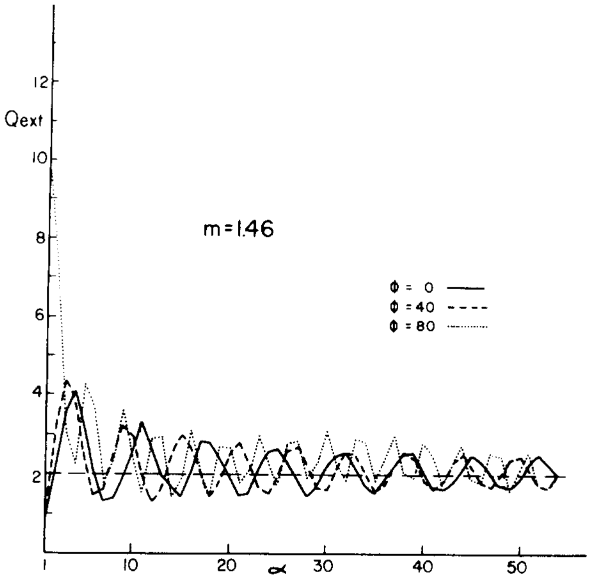

FIG. 7. Same as Fig. 5 for $m = 1.46$.

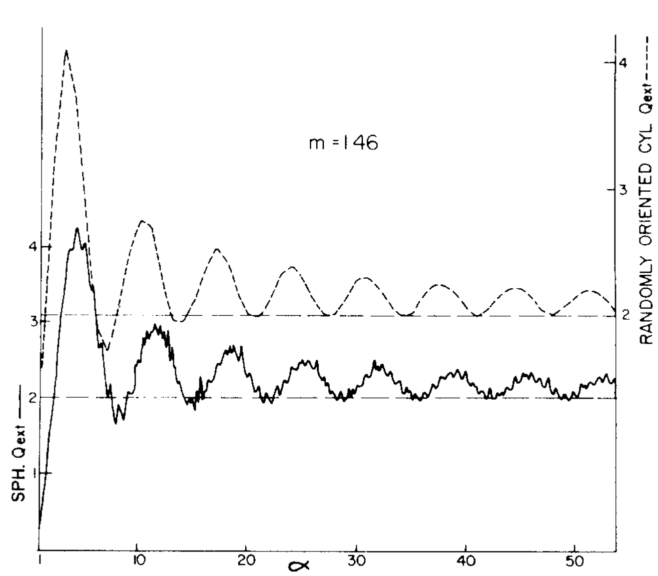

FIG. 8. Comparison between the extinction efficiency for randomly oriented cylinders (top) and spheres of same size parameters (bottom).

fect ($ka = 0.7$) is not generally ture.] This effect leads to the immediate conclusion that the real extinction efficiency per unit length normal to the incident beam is even higher for $\phi = 89°$ (relative to $\phi = 0°$) than the relative values suggested by Lind and Greenberg in Fig. 2. As a matter of fact, the absolute extinction efficiency for $ka = 0.7$ and $\phi = 89°$ calculated by us is as high as $Q_{\text{ext}} = 27$ and even higher values can be found in the resonance region[5] for $90° > \phi > 89°$ and $ka < 0.7$ (see Ref. 6).

Figures 4 and 5 represent selected curves for Q_{ext} as a function of the size parameter $\alpha = ka$ for given values of the orientation angle ϕ. Figure 4 is again taken from Refs. 1 and 2 and Fig. 5 shows our corrected curves. Figure 5 suggests that the Q_{ext} values oscillate around $Q = 2$ as a function of $\alpha = ka$ for the presented set of orientation angles.

In order to calculate the extinction efficiency of randomly oriented cylinders, values for Q were calculated for all orientation angles ϕ: 0°, 1°, and 89°. (Figure 5 presents the calculations for $\phi = 0°$, 40°, and 80°.) Since we assume that the cylinders are randomly oriented, the number of cylinders tilted in the direction ϕ (relative to the incident beam) is proportional to $\cos\phi$. Another assumption is that all cylinders have the same finite length which is much larger than their cross-sectional identical radius, hence their scattering properties can be approximated by the theory for infinite circular cylinders. It follows that the normalized expression for the extinction efficiency of our sample is given by

$$Q_{\text{ext}}(\alpha,m) = \pi/180 \sum_{i=0}^{89} Q_{\text{ext}}(\alpha,m,\phi = i) \cos i, \qquad (2)$$

where $Q_{\text{ext}} = \frac{1}{2}(Q_{1\text{ext}} + Q_{2\text{ext}})$ [Eqs. (1b)] and the incident light is assumed to be unpolarized. Q was calculated for the refractive index $m = 1.6$ and 1.46 for the range $\alpha = ka = 1(1)50$ and the results are presented in Figs. 6 and 7. It is interesting to note that whereas the individual curves for a given value of ϕ fluctuate below and above the value $Q = 2$ (versus α), the average value [Eq. (2)] is very similar to the behavior of the scattering by a sphere (Fig. 7) for which the fluctuations are above $Q = 2$. Also note that the number of maxima is the same for the randomly oriented cylinders versus the sphere (for the same refractive index m) as well as the corresponding maximum extinction value. Except for a small shift in the positions of the maxima and minima ($\Delta\alpha \approx 0.7$) which is detected for a polarized incidence normal or parallel to the incident plane, the curves are essentially identical. [The secondary fluctuations in Fig. 8 are due to the fact that Q was calculated for $\alpha = 1(0.1)50$]. A similar comparison has previously been suggested between spheres and cylinders which are all oriented normal to the incident beam (see, for example, the discussion in Ref. 2, p. 290).

A common approximation, used for example by atmospheric scientists dealing with light scattering, is to assume that the Mie scatterers can be represented by spherical particles. This is mainly based on the fact that in most cases the nonspherical particles are randomly oriented. Since the theory for nonspherical particles is not fully and generally developed, we presented here a theoretical examination of the above approximation for nonspherical particles having a high degree of symmetry. The results for such long cylindrical randomly oriented particles suggest that the behavior of the extinction efficiency versus the size parameter is similar to that of spheres (see Fig. 8).

[1]A.C. Lind and J.M. Greenberg, J. Appl. Phys. **37**, 3195 (1966).
[2]M. Kerker, *The Scattering of Light and Other Electromagnetic Radiation* (Academic, New York, 1969).
[3]H.C. Van de Hulst, *Light Scattering by Small Particles* (Wiley, New York, 1957).
[4]M. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, 1959).

[5]A detailed evaluation of the resonance region for which the usual physical concept of the extinction efficiency is considered requires a separate discussion and we are still in the process of analyzing the computational results.
[6]Extremely high values of extinction efficiencies ($Q \approx 750$) are also reported by M. Kerker [Appl. Opt. **17**, 3337 (1978)].