Cross sections for extinction of tilted infinite circular cylinders

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Received 18 November 1978.
0003-6935/79/162728-01$00.50/0.

The far-field scattering of a tilted infinite cylinder is given by[1]

$$\left.\begin{aligned} I_{11} &= 2(k_0\pi r)^{-1}|T_{11}(\theta)|^2 = 2(k_0\pi r)^{-1}\left|b_{0\mathrm{I}} + 2\sum_{n=1}^{\infty} b_{n\mathrm{I}}\cos(n\theta)\right|^2 \\ &\vdots \\ I_{22} &= 2(k_0\pi r)^{-1}|T_{22}(\theta)|^2 = 2(k_0\pi r)^{-1}\left|a_{0\mathrm{II}} + 2\sum_{n=1}^{\infty} a_{n\mathrm{II}}\cos(n\theta)\right|^2 \end{aligned}\right\}. \tag{1}$$

According to the optical theorem the extinction cross section can be calculated by analyzing the result of the interference between the scattered wave in the forward direction ($\theta = 0$) and the incident wave.

In Case I[1] only the scattered wave $T_{11}(\theta = 0)$ interferes with the incidence, and therefore

$$C_{1\,\mathrm{ext}} = C_{11\,\mathrm{sca}} + C_{12\,\mathrm{sca}} + C_{1\,\mathrm{abs}} = \frac{2\lambda}{\pi}\,\mathrm{Re}\left(b_{0\mathrm{I}} + 2\sum_{n=1}^{\infty} b_{n\mathrm{I}}\right), \tag{2}$$

where

$$\left.\begin{aligned} C_{11\,\mathrm{sca}} &= 2\pi r\int_0 I_{11}d\theta = 2\lambda/\pi\left(|b_{0\mathrm{I}}|^2 + 2\sum_{n=1}^{\infty}|b_{n\mathrm{I}}|^2\right) \\ C_{12\,\mathrm{sca}} &= 2\lambda/\pi\left(2\sum_{n=1}^{\infty}|b_{n\mathrm{II}}|^2\right) \end{aligned}\right\}. \tag{3}$$

Similarly, for Case II[1]

$$C_{2\,\mathrm{ext}} = C_{22\,\mathrm{sca}} + C_{21\,\mathrm{sca}} + C_{2\,\mathrm{abs}} = \frac{2\lambda}{\pi}\,\mathrm{Re}\left(a_{0\mathrm{II}} + 2\sum_{n=1}^{\infty} a_{n\mathrm{II}}\right).$$

It follows that Kerker's equations (6.1.46) and (6.1.47) should be deleted, and the quantities $C_{11\,\mathrm{ext}}$ and $C_{22\,\mathrm{ext}}$ should be replaced by $C_{1\,\mathrm{ext}}$ and $C_{2\,\mathrm{ext}}$.

Note that the forwardscattering ($\theta = 0$) wave as well as the backscattering wave from tilted infinite cylinders is not depolarized, an effect which is well known in the case of spheres. But in contradiction to the spheres, the following relations exist for nonabsorbing cylinders[2] (real refractive index):

$$\left.\begin{aligned} \mathrm{Re}(b_{n\mathrm{I}}) &= |b_{n\mathrm{I}}|^2 + |b_{n\mathrm{II}}|^2 \\ \mathrm{Re}(a_{n\mathrm{II}}) &= |a_{n\mathrm{II}}|^2 + |a_{n\mathrm{I}}|^2 \end{aligned}\right\}. \tag{4}$$

From these expressions and since[1] $|a_{n\mathrm{I}}|^2 = |b_{n\mathrm{II}}|^2$, it can be shown that

$$|b_{n\mathrm{I}} - 1/2|^2 = |a_{n\mathrm{II}} - 1/2|^2, \tag{5}$$

which equals 1/4 (as in the case of spheres[3]) merely for normal incidence $\theta = 0$. Otherwise ($\theta \neq 0$; $n \geq 1$):

$$|b_{n\mathrm{I}} - 1/2|^2 = |a_{n\mathrm{II}} - 1/2|^2 = 1/4 - |b_{n\mathrm{II}}|^2 = 1/4 - |a_{n\mathrm{I}}|^2. \tag{6}$$

References

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2. C. Lind and J. M. Greenberg, J. Appl. Phys. 37, 3195 (1966).
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Thermal blooming in axial pipe flow

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Received 22 March 1979.
0003-6935/79/162728-03$00.50/0.

Thermal blooming in the beam path between a laser cavity and a telescope is a major source of optical distortion inside a high-power laser system. Two methods have been proposed to convect laser-produced heating out of the beam path in order to reduce the blooming—axial flow and cross flow.[1] In this Letter we report three new results that are important in the design of beam paths for high-power lasers. First, we show that turbulent axial flow has a large mass flow advantage over cross flow for beam paths with L/D ratios greater than about 10 (L = path length, D = diam of beam tube). Second, we show that in axial flow an obscured or annular beam causes much smaller distortions than an unobscured beam. Third, for the practical regime $20 < L/D < 100$, the temperature solutions for L/D large, used previously,[1] overpredict the size of optical path differences (OPDs) and incorrectly predict the shape of the OPDs.

Our analysis starts with the steady-state heat equation for the mean temperature T in the beam path,

$$\mathbf{v}\cdot\nabla T - \epsilon_\theta\nabla^2 T = \dot{Q}/(\rho c_p), \tag{1}$$

where $\mathbf{v}$ is the mean velocity in the beam path (axial or cross

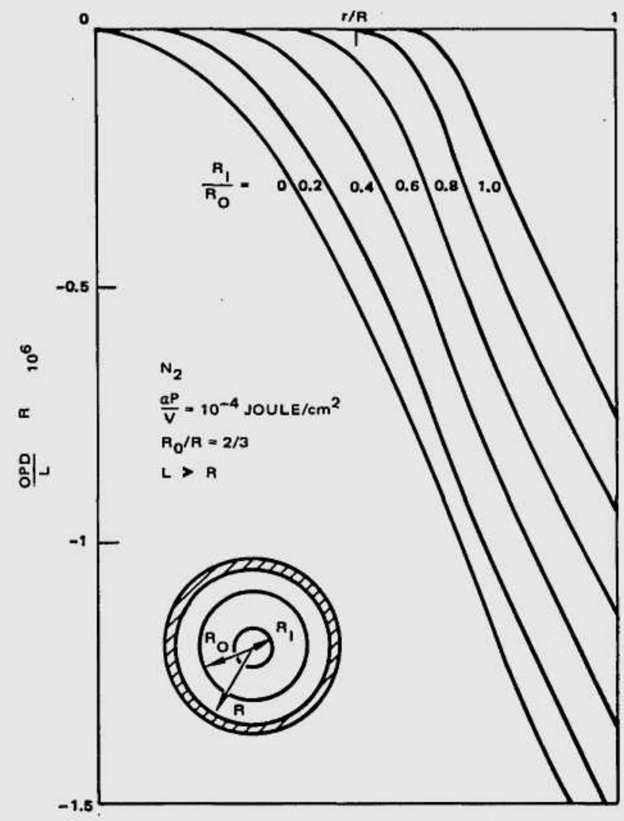

Fig. 1. The effect of obscuration ratio (R_i/R_0) on beam-path OPD as a function of radial position in a tube.