Supplemental Material for: Yoram Halevy, Dotan Persitz, Lanny Zrill. 2018. "Parametric Recoverability of Preferences." Journal of Political Economy 126(4). DOI: 10.1086/697741.

# Parametric Recoverability of Preferences: Online Appendix

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## 1 Proofs

### 1.1 Fact 1.

**Let $\mathbf{v}' \leq \mathbf{v}$. Then: $R^0_{D,\mathbf{v}'} \subseteq R^0_{D,\mathbf{v}}$, $P^0_{D,\mathbf{v}'} \subseteq P^0_{D,\mathbf{v}}$ and $R_{D,\mathbf{v}'} \subseteq R_{D,\mathbf{v}}$.**

*Proof.* For example, the proof of $R^0_{D,\mathbf{v}'} \subseteq R^0_{D,\mathbf{v}}$ is as follows: If $x = x^i$ the statement holds by Definition 1.1. Otherwise, if $x^i R^0_{D,\mathbf{v}'} x$ then $v'^i p^i x^i \geq p^i x$ . $\mathbf{v}' \leq \mathbf{v}$ implies that for every observation $i$, $v'^i \leq v^i$ . Therefore, $v^i p^i x^i \geq p^i x$, meaning $x^i R^0_{D,\mathbf{v}} x$. □

### 1.2 Fact 2.

**Every $D$ satisfies $GARP_{\mathbf{0}}$.**

*Proof.* For every pair of observed bundles $x^i$ and $x^j$, $x^j P^0_{D,\mathbf{0}} x^i$ is false since for every bundle $x$, $p^j x \geq 0 = 0 \times p^j x^j$ ($P^0_{D,\mathbf{0}}$ is the empty relation). □

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### 1.3 Fact 3.

**Let $\mathbf{v}, \mathbf{v}' \in [0,1]^n$ and $\mathbf{v} \geq \mathbf{v}'$. If $D$ satisfies $GARP_{\mathbf{v}}$ then $D$ satisfies $GARP_{\mathbf{v}'}$.**

*Proof.* By Fact 1, for every pair of observed bundles $x^i$ and $x^j$, $x^i R_{D,\mathbf{v}'} x^j$ implies $x^i R_{D,\mathbf{v}} x^j$. By Definition 2, since $D$ satisfies $GARP_{\mathbf{v}}$ for every pair of observed bundles $x^i$ and $x^j$, $x^i R_{D,\mathbf{v}'} x^j$ implies not $x^j P^0_{D,\mathbf{v}} x^i$. By Fact 1, for every pair of observed bundles $x^i$ and $x^j$, $x^i R_{D,\mathbf{v}'} x^j$ implies not $x^j P^0_{D,\mathbf{v}'} x^i$. Therefore, $D$ satisfies $GARP_{\mathbf{v}'}$. □

### 1.4 Theorem 1.

*Notation.* Let $\succeq$ be a binary relation. Then, $\succ$ is defined as $x \succ y$ if and only if $x \succeq y$ and not $y \succeq x$, while $\sim$ is defined as $x \sim y$ if and only if $x \succeq y$ and $y \succeq x$. Denote by $X/\sim$ the set of all equivalence classes on $X$ induced by $\sim$. Also, denote by $\succeq^\star$ the transitive closure of $\succeq$ and by $\succeq^c$ the relation where $x \succeq^c y$ if and only if $y \succeq x$.

**Definition 1.** Let $\mathbf{v} \in [0,1]^n$. A transitive and reflexive binary relation $\succeq$ $\mathbf{v}$-*rationalizes-by-relation* $D$, if $R^0_{D,\mathbf{v}} \subseteq \succeq$ and $P^0_{D,\mathbf{v}} \subseteq \succ$.

*Notation.* Let $x \in \Re^K_+$ and $\delta > 0$. $B_\delta(x) = \left\{ y \in \Re^K_+ : \|y - x\| < \delta \right\}$.

**Definition 2.** A utility function $u : \Re^K_+ \to \Re$ is

1. *Locally non-satiated* if $\forall x \in \Re^K_+$ and $\forall \epsilon > 0$, $\exists y \in B_\epsilon(x)$ such that $u(x) < u(y)$.

2. *Continuous* if $\forall x \in \Re^K_+$ and $\forall \epsilon > 0$ there exists $\delta > 0$ such that $y \in B_\delta(x)$ implies $u(y) \in B_\epsilon(u(x))$.

3. *Acceptable* if $\forall x \in \Re^K_+$, $u(0) \leq u(x)$.[1]

[1] For every $D = \left\{ (p^i, x^i)_{i=1}^n \right\}$ and for every $\mathbf{v} \in [0,1]^n$, $\forall i \in 1, \ldots, n : x^i R^0_{D,\mathbf{v}} 0$ (where 0 is the zero bundle). Therefore, a necessary condition for a binary relation $\succeq$ to $\mathbf{v}$-*rationalize-by-relation* $D$ is that for every observed bundle $x \in \Re^K_+$, $x \succeq 0$. Similarly, for a utility function $u(x)$ to $\mathbf{v}$-*rationalize* $D$ it must be that for every observed bundle $x \in \Re^K_+$, $u(x) \geq u(0)$.

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4. *Monotone* if $\forall x, y \in \Re_+^K$, $x \leq y$ implies $u(x) \leq u(y)$.

5. *Concave* if $\forall x, y \in \Re_+^K$ and $0 \leq \alpha \leq 1$: $\alpha u(x) + (1-\alpha) u(y) \leq u(\alpha x + (1-\alpha) y)$.

**Lemma 1.** *Let $\succeq$ be transitive and reflexive binary relation on a set $X$. Then, there exists a complete, transitive and reflexive binary relation $\succeq'$ on $X$ such that $\succeq \subseteq \succeq'$ and $\succ \subseteq \succ'$.*

*Proof.* Construct the mapping $\Pi : X \to X/\sim$ where each element of $X$ is mapped into its equivalence class (the Canonical Projection Map). Consider the relation $\bar{\succeq}$ on $X/\sim$ where $x \succeq y$ implies $\Pi(x) \bar{\succeq} \Pi(y)$. $\bar{\succeq}$ is reflexive and transitive since $\succeq$ is reflexive and transitive. Also, $\bar{\succeq}$ is antisymmetric since if $x \sim y$ then $\Pi(x) = \Pi(y)$. By Szpilrajn (1930)'s Extension Theorem, there is a complete, transitive, reflexive and antisymmetric binary relation, $\bar{\succeq}'$, such that $\bar{\succeq} \subseteq \bar{\succeq}'$. Consider now the relation $\succeq'$ on $X$ where $\Pi(x) \bar{\succeq}' \Pi(y)$ implies $x \succeq' y$. $\succeq'$ is complete, reflexive and transitive since $\bar{\succeq}'$ is complete, reflexive and transitive. Also, suppose $x \succeq y$, then, by the first construction, $\Pi(x) \bar{\succeq} \Pi(y)$, by the Extension Theorem $\Pi(x) \bar{\succeq}' \Pi(y)$ and by the second construction $x \succeq' y$. Therefore $\succeq \subseteq \succeq'$. Similarly, $\succ \subseteq \succ'$. □

**Lemma 2.** *Let $R$ and $P$ be two arbitrary binary relations on $X$. The following statements are equivalent:*

*1. There exists a transitive and reflexive binary relation $\succeq$ on $X$ such that $R \subseteq \succeq$ and $P \subseteq \succ$.*

*2. There exists a complete, transitive and reflexive binary relation $\succeq'$ on $X$ such that $R \subseteq \succeq'$ and $P \subseteq \succ'$.*

*3. $(R \cup P)^\star \cap P^c = \emptyset$.*

*Proof.* By Lemma 1, the first two statements are equivalent. Next, suppose *(1)* holds. Then, $(R \cup P) \subseteq (\succeq \cup \succ)$ and therefore $(R \cup P)^\star \subseteq (\succeq \cup \succ)^\star$. Also, $P^c \subseteq \succ^c$. Therefore, $(R \cup P)^\star \cap P^c \subseteq (\succeq \cup \succ)^\star \cap \succ^c$. Since $\succ \subseteq \succeq$ and since $\succeq$ is transitive we get $(R \cup P)^\star \cap P^c \subseteq \succeq \cap \succ^c$. But, $\succeq \cap \succ^c = \emptyset$ and hence $(R \cup P)^\star \cap P^c = \emptyset$.

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Last, suppose *(3)* holds. We construct a transitive and reflexive binary relation $\succeq$ on $X$ such that $R \subseteq \succeq$ and $P \subseteq \succ$. Let $\succeq$ be such that $x \succeq y$ if and only if $x(R \cup P)^\star y$ or $x = y$. $\succeq$ is reflexive by definition and transitive since $(R \cup P)^\star$ is transitive. Moreover, since $R \subseteq (R \cup P)^\star$ and $P \subseteq (R \cup P)^\star$ then $R \subseteq \succeq$ and $P \subseteq \succeq$. It is left to show that $P \subseteq \succ$. Suppose $xPy$. Since $P \subseteq \succeq$ then $x \succeq y$. Moreover, since $xPy$ then $yP^c x$ and since $(R \cup P)^\star \cap P^c = \emptyset$ we get that it cannot be that $y(R \cup P)^\star x$. In particular, it cannot be that $yPx$ and therefore $x \neq y$. Thus, by the definition of $\succeq$, it cannot be that $y \succeq x$. Therefore, $x \succ y$ and we conclude that $P \subseteq \succ$. □

**Lemma 3.** *Let* $\mathbf{v} \in [0,1]^n$ *and let* $D = \left\{(p^i, x^i)_{i=1}^n\right\}$ *be a finite data set of choices from budget sets. The following statements are equivalent:*

*1. There exists a transitive and reflexive binary relation* $\succeq$ *on* $\Re_+^K$ *such that* $\succeq$ $\mathbf{v}$*-relation-rationalizes* $D$*.*

*2. There exists a complete, transitive and reflexive binary relation* $\succeq'$ *on* $\Re_+^K$ *such that* $\succeq'$ $\mathbf{v}$*-relation-rationalizes* $D$*.*

*3.* $D$ *satisfies* $GARP_{\mathbf{v}}$*.*

*Proof.* By Lemma 2 and Definition 1 if $X = \Re_+^K$, $R = R_{D,\mathbf{v}}^0$ and $P = P_{D,\mathbf{v}}^0$ then the first two statements are equivalent and both are also equivalent to $(R_{D,\mathbf{v}}^0 \cup P_{D,\mathbf{v}}^0)^\star \cap {P_{D,\mathbf{v}}^0}^c = \emptyset$. But, $(R_{D,\mathbf{v}}^0 \cup P_{D,\mathbf{v}}^0)^\star \cap {P_{D,\mathbf{v}}^0}^c = \emptyset$ holds if and only if for every pair of bundles $x$ and $y$, $xR_{D,\mathbf{v}}y$ implies not $yP_{D,\mathbf{v}}^0 x$. If $x$ is an unobserved bundle or $y$ is an unobserved bundle then by Definition 1 in the main text, $xR_{D,\mathbf{v}}y$ implies not $yP_{D,\mathbf{v}}^0 x$. Therefore, $(R_{D,\mathbf{v}}^0 \cup P_{D,\mathbf{v}}^0)^\star \cap {P_{D,\mathbf{v}}^0}^c = \emptyset$ holds if and only if for every pair of observed bundles $x$ and $y$, $xR_{D,\mathbf{v}}y$ implies not $yP_{D,\mathbf{v}}^0 x$. Hence, by Definition 2 in the main text, $(R_{D,\mathbf{v}}^0 \cup P_{D,\mathbf{v}}^0)^\star \cap {P_{D,\mathbf{v}}^0}^c = \emptyset$ holds if and only if $D$ satisfies $GARP_{\mathbf{v}}$. □

**Lemma 4.** *Let* $D = \left\{(p^i, x^i)_{i=1}^n\right\}$ *be a finite data set and let* $\left\{\left(z_i : \Re_+^K \to \Re\right)_{i=1}^n\right\}$ *be a family of real functions. Define the following two binary relations on* $\left\{(x^i)_{i=1}^n\right\}$*:* $x^i R x^j \Leftrightarrow z_i(x^j) \leq 0$ *and* $x^i P x^j \Leftrightarrow z_i(x^j) < 0$*. If there exists a transitive and reflexive binary relation* $\bar{\succeq}$ *on* $\left\{(x^i)_{i=1}^n\right\}$ *such that* $R \subseteq \bar{\succeq}$ *and* $P \subset \bar{\succ}$ *then there exists a function* $f(x) = \min_{i \in \{1,\ldots,n\}} f_i + \lambda_i z_i(x)$ *such that* $\lambda_i > 0$ *and* $f(x^i) \geq f_i$*.*

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*Proof.* By Lemma 2, there exists a complete, transitive and reflexive binary relation $\succeq$ on $\left\{(x^i)_{i=1}^n\right\}$ such that $R \subseteq \succeq$ and $P \subset \succ$. Since $\succeq$ is complete and transitive and $\left\{(x^i)_{i=1}^n\right\}$ is finite we can partition the observed bundles and rank them according to $\succeq$. Let $I = \{1, \ldots, n\}$. Then $E_1 = \{i \in I | \nexists y \in \{(x^i)_{i \in I}\}, y \succ x^i\}$ is the set of indices of those observed bundles that are not dominated by any other observed bundle according to $\succeq$. Similarly, from the remaining observed bundles, $E_2 = \{i \in I/E_1 | \nexists y \in \{(x^i)_{i \in I/E_1}\}, y \succ x^i\}$, is the set of indices of those observed bundles that are not dominated according to $\succeq$ by any other observed bundle, and so $E_3 = \{i \in I/(E_1 \cup E_2) | \nexists y \in \{(x^i)_{i \in I/(E_1 \cup E_2)}\}, y \succ x^i\}$, etc. Denote the number of classes by $l$. Transitivity guarantees that there are no empty classes while completeness assures that for every $k \in 1, \ldots, l$ and for every pair of observed bundles $x, y \in E_k$ it must be that $x \sim y$.

The following procedure uses this partition and the functions $\left\{\left(z_i : \Re_+^K \to \Re\right)_{i=1}^n\right\}$ to construct a mapping $(f_i, \lambda_i) : \left\{(x^i)_{i=1}^n\right\} \to \Re^2$ such that $\lambda_i > 0$ and $f(x^i) \geq f_i$ where $f(x) = \min_{i \in I} \{f_i + \lambda_i z_i(x)\}$:

1. For every $i \in E_1$, set $f_i = 1$ and $\lambda_i = 1$. Also, set $k = 1$. If $l = 1$ the procedure terminates, otherwise continue.
2. Set $k := k + 1$.
3. Denote $B_k = \cup_{m=1}^{k-1} E_m$.
4. Calculate $\alpha_k = \min_{i \in B_k} \min_{j \in E_k} \min \left\{ f_i + \lambda_i z_i(x^j), f_i \right\}$.
5. Choose some $f < \alpha_k$ and set $f_j = f$ for every $j \in E_k$.
6. Calculate $\beta_k = \max_{i \in B_k} \max_{j \in E_k} \frac{f_i - f_j}{z_j(x^i)}$.
7. Choose some $\lambda > \beta_k$ and set $\lambda_j = \lambda$ for every $j \in E_k$.
8. If $k < l$ return to step 2, otherwise the procedure terminates.

Stage 1 guarantees that for every $i \in E_1$, $\lambda_i = 1$ and $f_i = 1$. Suppose $i \in E_1$, $l \geq 2$ and $k \in \{2, \ldots, l\}$. Then $i \in B_k$ and for every $j \in E_k$, $x^i \succ x^j$ (since $\succeq$ is complete). Steps 4 and 5 guarantee that $x^i \succ x^j$ implies that $f_i > f_j$ or $f_i - f_j > 0$. In addition, $x^i \succ x^j$ implies that $z_j(x^i) > 0$ (otherwise $x^j R x^i$ and therefore $x^j \succeq x^i$). Therefore, steps 6 and 7 guarantee that for every observation $i \in I$, $\lambda_i > 0$. It is left to show that for every observation $i \in I$, $f(x^i) \geq f_i$. That is, $\min_{j \in I} [f_j + \lambda_j z_j(x^i)] \geq f_i$ or, equivalently, for every pair of observations $i, j \in I$, $f_j + \lambda_j z_j(x^i) \geq f_i$. First, if $x^j \succ x^i$ steps 4 and 5 guarantee that $f_j + \lambda_j z_j(x^i) > f_i$. If $x^j \sim x^i$ then $z_j(x^i) \geq 0$ (otherwise $x^j P x^i$ and therefore $x^j \succ x^i$) and in addition by step 5, $f_j = f_i$. Since for every $j \in I$, $\lambda_j > 0$ we get that $x^j \sim x^i$ implies $f_j + \lambda_j z_j(x^i) \geq f_i$. Last, if $x^i \succ x^j$ then $z_j(x^i) > 0$ and $f_i - f_j > 0$ and steps 6 and 7 guarantee that $\lambda_j > \frac{f_i - f_j}{z_j(x^i)}$. Therefore, $x^i \succ x^j$ implies $f_j + \lambda_j z_j(x^i) > f_i$. Thus, for every observation $i \in I$, $f(x^i) \geq f_i$. If $l = 1$ then for every pair of observations $i, j \in I$, we have $x^i \sim x^j$. Therefore, for every pair of observations $i, j \in I$,$z_j(x^i) \geq 0$. In addition, for every $i \in I$, $\lambda_i = 1$ and $f_i = 1$. Hence, for every $i \in I$, $f(x^i) \geq f_i$. $\square$

**Lemma 5.** *If $u$ is a locally non satiated utility function that $\mathbf{v}$-rationalizes $D = \left\{(p^i, x^i)_{i=1}^n\right\}$, then $x^i P^0_{D,\mathbf{v}} x$ implies $u\left(x^i\right) > u\left(x\right)$.*

*Proof.* If $x^i P^0_{D,\mathbf{v}} x$ then $x^i R^0_{D,\mathbf{v}} x$. Since $u\left(\cdot\right)$ $\mathbf{v}$-rationalizes $D$, $x^i R^0_{D,\mathbf{v}} x$ implies $u\left(x^i\right) \geq u\left(x\right)$. Suppose that $u\left(x^i\right) = u\left(x\right)$. Since $v^i p^i x^i > p^i x$, $\exists \epsilon > 0$ such that $\forall y \in B_\epsilon\left(x\right) : v^i p^i x^i > p^i y$. By local non-satiation $\exists y' \in B_\epsilon\left(x\right)$ such that $u\left(y'\right) > u\left(x\right) = u\left(x^i\right)$. Thus, $y'$ is a bundle such that $v^i p^i x^i > p^i y'$ and $u\left(y'\right) > u\left(x^i\right)$, in contradiction to $u\left(\cdot\right)$ $\mathbf{v}$-rationalizing $D$. Therefore, $u\left(x^i\right) > u\left(x\right)$. $\square$

We proceed to the proof of Theorem 1,

*Proof.* First, suppose there exists a locally non-satiated utility function $u(\cdot)$ that $\mathbf{v}$-rationalizes $D$. If, in negation, $D$ does not satisfy $GARP_\mathbf{v}$ then, by Definition 2 in the main text, there are two observed bundles $x^i, x^j$ such that $x^i R_{D,\mathbf{v}} x^j$ and $x^j P^0_{D,\mathbf{v}} x^i$. By Definition 1.3 in the main text, $x^i R_{D,\mathbf{v}} x^j$ implies that there exists a sequence of observed bundles $\left(x^k, \ldots, x^m\right)$ such that $x^i R^0_{D,\mathbf{v}} x^k, \ldots, x^m R^0_{D,\mathbf{v}} x^j$. Therefore, by Definition 3 in the main text, $x^i R_{D,\mathbf{v}} x^j$ implies $u(x^i) \geq u(x^k) \geq \cdots \geq u(x^m) \geq u(x^j)$, meaning $x^i R_{D,\mathbf{v}} x^j$ implies $u(x^i) \geq u(x^j)$ . However, by Lemma 5, since $u\left(\cdot\right)$ is a locally non-satiated utility function that $\mathbf{v}$-rationalizes $D$, $x^j P^0_{D,\mathbf{v}} x^i$ implies $u\left(x^j\right) > u\left(x^i\right)$. Contradiction. Therefore, $D$ satisfies $GARP_\mathbf{v}$.

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Since the third statement implies the first statement, it is left to be shown that if $D$ satisfies $GARP_{\mathbf{v}}$ then there exists a continuous, concave, acceptable and monotone utility function that $\mathbf{v}$-rationalizes $D$.

By Lemma 3 and by Definition 1, we have to show that for every data set $D$ and adjustments vector $\mathbf{v}$, if $\succeq$ is a transitive and reflexive binary relation on $\Re^K_+$ such that $R^0_{D,\mathbf{v}} \subseteq \succeq$ and $P^0_{D,\mathbf{v}} \subseteq \succ$ then there exists a continuous, concave, acceptable and monotone utility function that $\mathbf{v}$-rationalizes $D$.

Define $z_i(x) = \frac{1}{v_i} p^i x - p^i x^i$ if $x \neq x^i$ and zero otherwise. Then, $x^i R^0_{D,\mathbf{v}} x \Leftrightarrow z_i(x) \leq 0$ and $x^i P^0_{D,\mathbf{v}} x \Leftrightarrow z_i(x) < 0$. Thus, by Lemma 4, there exists a function $f(x) = \min_{i \in \{1,\ldots,n\}} f_i + \lambda_i z_i(x)$ such that $\lambda_i > 0$ and $f(x^i) \geq f_i$.

Next we show that $f(\cdot)$ $\mathbf{v}$-rationalizes $D$. Suppose $x^i R^0_{D,\mathbf{v}} x$. By the definition of $f$ we get $f(x) \leq f_i + \lambda_i z_i(x)$. Since, $\lambda_i > 0$ and since $x^i R^0_{D,\mathbf{v}} x$ we get $\lambda_i z_i(x) \leq 0$ and therefore $f(x) \leq f_i$. However, $f(x^i) \geq f_i$. Therefore, $x^i R^0_{D,\mathbf{v}} x$ implies $f(x^i) \geq f(x)$, that is $f(\cdot)$ $\mathbf{v}$-rationalizes $D$.

The functions $z_i$ are discontinuous at $x^i$ when $v_i < 1$. Therefore, $f$ is continuous everywhere except maybe at the observed bundles. We use $f$ to construct a continuous utility function $\hat{f}$ that $\mathbf{v}$-rationalizes $D$. Let $\hat{z}_i(x) = \lim_{y \to x} z_i(y)$ then $\hat{z}_i(x) \geq z_i(x)$ for $x = x^i$ and $\hat{z}_i(x) = z_i(x)$ otherwise. Construct $\hat{f}(x) = \min_{i \in \{1,\ldots,n\}} f_i + \lambda_i \hat{z}_i(x)$ where $f_i$ and $\lambda_i$ are the same as in $f$ and therefore $\lambda_i > 0$ and $f(x^i) \geq f_i$. Note that $\hat{z}_j(x^i) \geq z_j(x^i) = 0$ for all $j \in \{1,\ldots,n\}$ implies $\hat{f}(x^i) \geq f(x^i) \geq f_i$. If $x \neq x^i$ then $z_i(x) \leq 0$ implies $\hat{z}_i(x) \leq 0$ and therefore $\hat{f}(x) \leq f_i$. Hence, for every bundle $x \neq x^i$ such that $z_i(x) \leq 0$ we get $\hat{f}(x) \leq \hat{f}(x^i)$. Thus, for every bundle $x$ such that $x^i R^0_{D,\mathbf{v}} x$ we get $\hat{f}(x) \leq \hat{f}(x^i)$, that is $\hat{f}$ $\mathbf{v}$-rationalizes $D$. Obviously, $\hat{z}_i(x)$ is continuous and therefore for every observation $i \in I$, $f_i + \lambda_i \hat{z}_i(x)$ is continuous. Since the minimum of any finite number of continuous functions is continuous we get that $\hat{f}(x) = \min_{i \in \{1,\ldots,n\}} f_i + \lambda_i \hat{z}_i(x)$ is continuous.

For every $i \in I$, since $\hat{z}_i(x)$ is linear with positive slope, the zero bundle, $x = 0$, minimizes $f_i + \lambda_i \hat{z}_i(x)$. Therefore, $\hat{f}(0) = \min_{x \in \Re^K_+} \hat{f}(x)$. Hence, $\hat{f}$ satisfies acceptability. Also, since $\hat{z}_i(x)$ is increasing monotonically, for every observation $i \in I$, $f_i + \lambda_i \hat{z}_i(x)$ is increasing monotonically and therefore $\hat{f}$ is monotonic. $\hat{z}_i(x)$ is linear and therefore for every observation $i \in I$, $f_i + \lambda_i \hat{z}_i(x)$

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is linear. Since the minimum of a set of linear functions is concave, $\hat{f}$ is concave. □

## 1.5 Fact 4.

**$I_V(D,f)$, $I_A(D)$ and $I_{HM}(D,f)$ always exist.**

*Proof.* The aggregator function $f(\cdot)$ is bounded. In addition, by Fact 2, the sets $\{\mathbf{v} \in [0,1]^n : D\ satisfies\ GARP_{\mathbf{v}}\}$, $\{\mathbf{v} \in \mathcal{I} : D\ satisfies\ GARP_{\mathbf{v}}\}$ and $\{\mathbf{v} \in \{0,1\}^n : D\ satisfies\ GARP_{\mathbf{v}}\}$ are non-empty. Hence, $I_V(D,f)$, $I_A(D)$ and $I_{HM}(D,f)$ always exist. □

## 1.6 Proposition 1.

**Let $D = \left\{(p^i,x^i)_{i=1}^n\right\}$, $u \in \mathcal{U}^c$ and $\mathbf{v} \in [0,1]^n$. $u(\cdot)$ $\mathbf{v}$-*rationalizes* $D$ if and only if $\mathbf{v} \leqq \mathbf{v}^\star(D,u)$.**

*Proof.* First, let us show that if $u(\cdot)$ $\mathbf{v}$-rationalizes $D$ then $\mathbf{v} \leqq \mathbf{v}^\star(D,u)$. Suppose that $\mathbf{v}$ is such that $u(\cdot)$ $\mathbf{v}$-rationalizes $D$ and for observation $i$, $v^i > v^{\star i}(D,u)$. By Definition 3 in the main text, $u(x^i) \geq u(x)$ for all $x$ such that $v^i p^i x^i \geq p^i x$. By Definition 8 in the main text and since $v^i > v^{\star i}(D,u)$ we get that $v^i p^i x^i > m(x^i,p^i,u) = p^i x^\star$ where $x^\star \in argmin_{\left\{y \in \Re^K_+ : u(y) \geq u(x^i)\right\}} p^i y$. It follows that $\exists \epsilon > 0$ such that $\forall y \in B_\epsilon(x^\star) : v^i p^i x^i > p^i y$. By local non-satiation $\exists y' \in B_\epsilon(x^\star)$ such that $u(y') > u(x^\star) \geq u(x^i)$. Thus, $y'$ is a bundle such that $v^i p^i x^i > p^i y'$ and $u(y') > u(x^i)$ contradicting that $u(\cdot)$ $\mathbf{v}$-rationalizes $D$.

Next, let us show that if $\mathbf{v} \leqq \mathbf{v}^\star(D,u)$ then $u(\cdot)$ $\mathbf{v}$-rationalizes $D$. We begin by establishing that $u(\cdot)$ $\mathbf{v}^\star(D,u)$-rationalizes $D$. Suppose, in negation, that for some observation $(p^i,x^i) \in D$ there exists a bundle $x$ such that $x^i R^0_{D,\mathbf{v}^\star(D,u)} x$ and $u(x^i) < u(x)$. If $x = 0$ then we get a contradiction by acceptability. If $x \neq 0$ then by Definition 1.1 in the main text, $v^{\star i}(D,u) p^i x^i \geq p^i x$. By Definition 8 in the main text, $m(x^i,p^i,u) \geq p^i x$. By continuity of $u(\cdot)$ there exists $\gamma > 0$ such that $u(x^i) < u((1-\gamma)x)$. However, since $p^i(1-\gamma)x < m(x^i, p^i, u)$, we reach a contradiction to Definition 8 in the main text.

Finally, since $u(\cdot)$ $\mathbf{v}^\star(D,u)$-rationalizes $D$, for every observation $(p^i, x^i) \in D$, $v^{\star i}(D,u)p^i x^i \geq p^i x$ implies $u(x^i) \geq u(x)$. Since $\mathbf{v} \leqq \mathbf{v}^\star(D,u)$, for every observation $(p^i, x^i) \in D$, $v^{\star i}(D,u)p^i x^i \geq v^i p^i x^i$. Therefore, for every observation $(p^i, x^i) \in D$, $v^i p^i x^i \geq p^i x$ implies $u(x^i) \geq u(x)$. Hence, $u(\cdot)$ $\mathbf{v}$-rationalizes $D$. □

## 1.7 Proposition 2.

**Let $D = \{(p^i, x^i)_{i=1}^n\}$, $u \in \mathcal{U}^c$ and $\mathbf{b} \in \{0,1\}^n$. $u(\cdot)$ $\mathbf{b}$-*rationalizes* $D$ if and only if $\mathbf{b} \leqq \mathbf{b}^\star(D,u)$.**

*Proof.* First, let us show that if $u(\cdot)$ $\mathbf{b}$-rationalizes $D$ then $\mathbf{b} \leqq \mathbf{b}^\star(D,u)$. Suppose, in negation, that $\mathbf{b}$ is such that $u(\cdot)$ $\mathbf{b}$-rationalizes $D$ and for observation $i$, $b^i = 1$ while $b^{\star i}(D,u) = 0$. By Definition 10 in the main text, $b^{\star i}(D,u) = 0$ implies that there exists $y \in \Re_+^K$ such that $p^i x^i \geq p^i y$ and $u(y) > u(x^i)$. Thus, $x^i R_{D,\mathbf{b}}^0 x$ does not imply $u(x^i) \geq u(x)$, contradicting that $u(\cdot)$ $\mathbf{b}$-rationalizes $D$.

Next, let us show that if $\mathbf{b} \leqq \mathbf{b}^\star(D,u)$ then $u(\cdot)$ $\mathbf{b}$-rationalizes $D$. Since, $\mathbf{b} \leqq \mathbf{b}^\star(D,u)$, for every observation $(p^i, x^i) \in D$, $b^i = 1$ implies $b^{\star i}(D,u) = 1$. By Definition 10 in the main text, this means that $b^i p^i x^i \geq p^i x$ implies $u(x^i) \geq u(x)$. Otherwise, if $b^i = 0$ by the acceptability of $u(\cdot)$, $b^i p^i x^i \geq p^i x$ implies $u(x^i) \geq u(x)$. Therefore, $b^i p^i x^i \geq p^i x$ implies $u(x^i) \geq u(x)$ and by Definition 1.1 in the main text $x^i R_{D,\mathbf{b}}^0 x$ implies $u(x^i) \geq u(x)$. Hence, by Definition 3 in the main text, $u(\cdot)$ $\mathbf{b}$-rationalizes $D$. □

## 1.8 Fact 5.

**For every $\mathcal{U}' \subseteq \mathcal{U}$ : $I_M(D,f,\mathcal{U}) \leq I_M(D,f,\mathcal{U}')$ and $I_B(D,f,\mathcal{U}) \leq I_B(D,f,\mathcal{U}')$.**

*Proof.* $\mathcal{U}' \subseteq \mathcal{U}$ implies $\inf_{u\in\mathcal{U}'} f(\mathbf{v}^\star(D,u)) \geq \inf_{u\in\mathcal{U}} f(\mathbf{v}^\star(D,u))$ and therefore $I_M(D,f,\mathcal{U}) \leq I_M(D,f,\mathcal{U}')$ and similarly for the Binary Incompatibility Index.

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$\square$

## 1.9 Theorem 2.

*Proof.* We begin with the proof of part 1. First, we show that $I_V(D,f) \leq I_M(D,f,\mathcal{U}^c)$. If $I_V(D,f) = 0$ then by definitions 4 and 9 in the main text we get $I_V(D,f) \leq I_M(D,f,\mathcal{U}^c)$. Otherwise, if $I_V(D,f) > 0$, suppose that $I_V(D,f) > I_M(D,f,\mathcal{U}^c)$. Then, there exists $u \in \mathcal{U}^c$ such that $f(\mathbf{v}^\star(D,u)) < I_V(D,f)$. By Proposition 1, $u(\cdot)$ $\mathbf{v}^\star(D,u)$-rationalizes $D$. By Theorem 1 $D$ satisfies $GARP_{\mathbf{v}^\star(D,u)}$. However, since $D$ satisfies $GARP_{\mathbf{v}^\star(D,u)}$ and $f(\mathbf{v}^\star(D,u)) < I_V(D,f)$, $I_V(D,f)$ cannot be the infimum of $f(\cdot)$ on the set of all $\mathbf{v} \in [0,1]^n$ such that $D$ satisfies $GARP_\mathbf{v}$. Contradiction.

For the converse direction note that by Theorem 1, $D$ satisfies $GARP_\mathbf{v}$ if and only if there exists $u \in \mathcal{U}^c$ that $\mathbf{v}$-rationalizes $D$. By Proposition 1, $\mathbf{v} \leqq \mathbf{v}^\star(D,u)$. Since $f(\cdot)$ is weakly decreasing $f(\mathbf{v}^\star(D,u)) \leq f(\mathbf{v})$. Therefore, by Definition 9, $D$ satisfies $GARP_\mathbf{v}$ implies that $I_M(D,f,\mathcal{U}^c) \leq f(\mathbf{v})$. Since $I_V(D,f) = \inf_{\mathbf{v}\in[0,1]^n : D \text{ satisfies } GARP_\mathbf{v}} f(\mathbf{v})$ we have $I_V(D,f) \geq I_M(D,f,\mathcal{U}^c)$. Hence, $I_V(D,f) = I_M(D,f,\mathcal{U}^c)$.

To prove part 2 we first show that $I_{HM}(D,f) \leq I_B(D,f,\mathcal{U}^c)$. If $I_{HM}(D,f) = 0$ by definitions 4 and 10 in the main text we get $I_{HM}(D,f) \leq I_B(D,f,\mathcal{U}^c)$. Otherwise, if $I_{HM}(D,f) > 0$ suppose that $I_{HM}(D,f) > I_B(D,f,\mathcal{U}^c)$. Then, there exists $u \in \mathcal{U}^c$ such that $f(\mathbf{b}^\star(D,u)) < I_{HM}(D,f)$. By Proposition 2 $u(\cdot)$ $\mathbf{b}^\star(D,u)$-rationalizes $D$. By Theorem 1, $D$ satisfies $GARP_{\mathbf{b}^\star(D,u)}$. However, since $D$ satisfies $GARP_{\mathbf{b}^\star(D,u)}$ and $f(\mathbf{b}^\star(D,u)) < I_{HM}(D,f)$, $I_{HM}(D,f)$ cannot be the infimum of $f(\cdot)$ on the set of all $\mathbf{v} \in \{0,1\}^n$ such that $D$ satisfies $GARP_\mathbf{v}$. Contradiction.

Second, by Theorem 1, $D$ satisfies $GARP_\mathbf{b}$ if and only if there exists $u \in \mathcal{U}^c$ that $\mathbf{b}$-rationalizes $D$. By Proposition 2, $\mathbf{b} \leqq \mathbf{b}^\star(D,u)$. Since $f(\cdot)$ is weakly decreasing $f(\mathbf{b}^\star(D,u)) \leq f(\mathbf{b})$. Therefore, by Definition 10 un the main text, $D$ satisfies $GARP_\mathbf{b}$ implies that $I_B(D,f,\mathcal{U}^c) \leq f(\mathbf{b})$. Since $I_{HM}(D,f) = \inf_{\mathbf{v}\in\{0,1\}^n : D \text{ satisfies } GARP_\mathbf{v}} f(\mathbf{v})$ we have $I_{HM}(D,f) \geq I_B(D,f,\mathcal{U}^c)$. Hence, $I_{HM}(D,f) = I_B(D,f,\mathcal{U}^c)$.

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We conclude with the proof of part 3. By part 1, since $f(\mathbf{v}) = 1 - \min_{i\in\{1,\ldots,n\}} v_i$ is continuous and weakly decreasing then for every finite data set $D$, $I_V(D, f) = I_M(D, f, \mathcal{U}^c)$. By Definition 6, since $\mathcal{I} \subset [0, 1]^n$ then if $f(\mathbf{v}) = 1 - \min_{i\in\{1,\ldots,n\}} v_i$ we get $I_V(D, f) \leq I_A(D)$. Suppose that $I_V(D, f) < I_A(D)$, then there exists $\hat{\mathbf{v}} \in [0, 1]^n$ such that $D$ satisfies $GARP_{\hat{\mathbf{v}}}$ and $f(\hat{\mathbf{v}}) < I_A(D)$. By Fact 3, for every $\mathbf{v} \in [0, 1]^n$ such that $D$ satisfies $GARP_{\mathbf{v}}$ there exists $\mathbf{v}' \in \mathcal{I}$ such that $D$ satisfies $GARP_{\mathbf{v}'}$ where $v' = \min_{i\in\{1,\ldots,n\}} v_i$. Hence, there exists $\hat{\mathbf{v}}' \in \mathcal{I}$ such that $D$ satisfies $GARP_{\hat{\mathbf{v}}'}$ and $f(\hat{\mathbf{v}}) = f(\hat{\mathbf{v}}') < I_A(D)$. Contradiction. $\square$

# 2 Inconsistency Indices

This appendix provides detailed information regarding inconsistency indices mentioned or related to this work. Section 2.1 describes the theoretical and practical computational issues concerning the indices analyzed in Theorem 2. Three important alternative inconsistency indices based on revealed preferences are discussed in Section 2.2. A fourth alternative, which is not based on revealed preferences, is discussed in Section 2.3.[2]

## 2.1 Computation

Theorem 2 relates three inconsistency indices to loss functions used in the recovery of parametric preferences. Since an inconsistency index is constant (given a data set), its value is inconsequential to the selection of the best approximating function within a parametric family. However, the value of the index is necessary in order to determine the decomposition of the loss between the subject's inconsistency and the researcher's inaccuracy in her choice of functional form. Therefore, a practical consideration in the choice of a loss function is the computability of the corresponding inconsistency index.

[2]We do not discuss indices based on the number of violations of the revealed preference axioms (see Swofford and Whitney (1987); Famulari (1995) and Harbaugh et al. (2001)) or indices based on the distance of the observed Slutsky matrix from the set of rational Slutsky matrices (see Jerison and Jerison, 1993; Aguiar and Serrano, 2017, 2015).

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#### 2.1.1 Afriat's Inconsistency Index

Theorem 3 in Afriat (1973) suggests an NP-Hard algorithm to calculate Afriat's inconsistency index. Based on a similar idea, Smeulders et al. (2014) provide a polynomial time algorithm to calculate this index. Houtman and Maks (1987) describe an efficient binary search routine that approximates Afriat's inconsistency index with an arbitrary accuracy in polynomial time.

In the supplemented code package we follow Houtman and Maks (1987). Let $GL$ denote a lower bound on the index (initialized to zero) and let $GU$ denote an upper bound on the index (initialized to one). At each iteration we cut the difference between the bounds by half, by testing the data for $GARP_{\frac{GU+GL}{2}}$ and updating the upper bound in case of a failure and the lower bound otherwise. $l$ iterations guarantee an accuracy of approximately $\log_{10} 2^l \approx 0.3l$ significant decimal digits (we implement $l = 30$). Finally, we report $GL$.

#### 2.1.2 Varian's Inconsistency Index

The problem of finding the exact value of Varian's Inconsistency Index is equivalent to solving the *minimum cost feedback arc set problem.*[3] Karp (1972) shows that this problem is NP-Hard and therefore finding the exact value of Varian's Inconsistency Index is also NP-Hard (as mentioned in Varian (1990)).[4] Moreover, Smeulders et al. (2014) show that no polynomial time algorithm can achieve a constant factor approximation (a ratio of $o\left(n^{1-\delta}\right)$). Tsur (1989), Varian (1993) and Alcantud et al. (2010) suggest approximation algorithms that overestimate the actual Varian's Inconsistency Index.

Our calculation of Varian's Inconsistency Index in the supplemented code package attempts to take advantage of the moderate size of the analyzed datasets (at most 50 observations per subject). Denote the number of GARP violations by $m$ and the set of all GARP violations by $M = \{h_1, \ldots, h_m\}$ (each element is an ordered sequence of observations). For every violation $h_i$, denote

[3] Given a directed and weighted graph, find the "cheapest" subset of arcs such that its removal turns the graph into an acyclic graph.

[4] Smeulders et al. (2014) show a similar result for the generalized mean aggregator function.

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the set of budget line adjustments that can potentially prevent it by $H_i$ (each element is an ordered pair of an observation and an adjustment percentage).

If $\sum_{i=1}^{m}|H_i| < K_1$ then we take a "brute force" approach (we implement $K_1 = 26$). For each subset of $\cup_{i=1}^{m} H_i$, we construct the corresponding adjustment vector $\mathbf{v}$ and check whether $GARP_{\mathbf{v}}$ is satisfied. We report three versions of Varian's Inconsistency Index, each minimizing a different aggregator function - the Minimum aggregator $(1 - \min_{i\in\{1,\dots,n\}} v_i)$, the MEAN aggregator $(\frac{1}{n}\sum_{i=1}^{n}(1-v^i))$ and the SSQ aggregator $(\sqrt{\frac{1}{n}\sum_{i=1}^{n}(1-v^i)^2})$.

Otherwise, we take advantage of the small commodity space ($K = 2$). Rose (1958) shows that in this case WARP is satisfied if and only if SARP is satisfied. Denote the set of WARP violations by $W$ (each element, $w_i$, is an unordered pair of observations). If $|W| \leq K_2$ we take a similar approach, on budget adjustments that can prevent the WARP violations (we implement $K_2 = 12$). For each of $\cup_{i=1}^{|W|} w_i$, we construct the corresponding adjustment vector $\mathbf{v}$ and check whether $GARP_{\mathbf{v}}$ is satisfied. We report the minimum of the three aggregators mentioned above. We observe that resolving WARP violations provides a very good approximation to the actual Varian's Inconsistency Index.

Finally, if $\sum_{i=1}^{m}|H_i| \geq K_1$ and $|W| > K_2$ we implement Algorithm 3 of Alcantud et al. (2010). This algorithm initializes the vector of adjustments, $\mathbf{v}$, to $\mathbf{1}$. Then, a loop is implemented that ends only when the data satisfies $GARP_{\mathbf{v}}$. Inside the loop, the matrix $A$ is maintained where the cell in the $i^{th}$ row and the $j^{th}$ column contains $\frac{p_j x_i}{v_j p_j x_j}$ if $x_i R_{\mathbf{v},D} x_j$ and $x_j P^0_{\mathbf{v},D} x_i$ and zero otherwise. In each iteration, the maximal element of $A$ is picked and substituted into the corresponding element in the vector of adjustments. We report the three aggregators mentioned above operated on the resulted vector of adjustments $\mathbf{v}$.

For the data collected in the first part of our experiment, where each subject made 22 choices from linear budget lines, we are able to calculate the Varian Inconsistency Index exactly for 91.6% (186 out of 203) of the subjects. We fail to calculate a reliable index for only 3 subjects (we provide good approximation for 14 subjects). Since Choi et al. (2007) collected 50 observations per subject,

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the success rate of our algorithm is somewhat lower. We are able to calculate the index exactly for 72.3% (34 out of 47) of the subjects and to provide good approximation for 4 other subjects. We fail to calculate a reliable result for 9 subjects.

### 2.1.3 Houtman-Maks Inconsistency Index

Boodaghians and Vetta (2015) show that there exists a polynomial time algorithm to calculate the Houtman-Maks Inconsistency Index for the two commodities case ($K = 2$).[5] In addition, they follow Houtman and Maks (1985) and Smeulders et al. (2014) to show that for three commodities or more, calculating the Houtman-Maks Inconsistency Index is NP-Hard. Smeulders et al. (2014) show that no polynomial time algorithm can achieve a constant factor approximation (a ratio of $o\left(n^{1-\delta}\right)$) for this Index (see also the discussion in Boodaghians and Vetta (2015) following Lemma 2.1).

Our calculation of the Houtman-Maks Inconsistency Index in the supplemented code package begins with an exhaustive search approach.[6] Given a dataset $D$ of size $n$, denote by $D_m$ the set of all subsets of $D$ of size $m < n$. Also, denote $M = \min_{m\in\{1,\ldots,n-1\}} m \quad s.t. \quad |\cup_{l=m}^{n-1} D_l| < K_3$. The algorithm first goes over every element in $D_{n-1}$, then over every element in $D_{n-2}$, etc. The algorithm terminates either after an adjusted dataset that satisfies $GARP$

[5] Rose (1958) shows that in the two commodity case ($K = 2$) WARP is satisfied if and only if SARP is satisfied. Let $G$ be an undirected graph where each node is a chosen bundle and two nodes are linked if they constitute a pair that violates WARP. Boodaghians and Vetta (2015) use Rose (1958) to prove that in the two commodity case calculating the Houtman-Maks Inconsistency Index is equivalent to finding the minimal vertex cover of $G$ (the smallest set of nodes $S$ such that every edge in $G$ has an endpoint in $S$). Next, a graph is perfect if the chromatic number (the smallest number of colors needed to color all nodes where no two adjacent vertices share the same color) of every induced subgraph equals the size of the largest clique (a set of fully connected nodes) of that subgraph. Boodaghians and Vetta (2015) show that $G$ is perfect and recall that finding the minimal vertex cover of a perfect graph is solvable in polynomial time. Hence, they conclude that the calculation of the Houtman-Maks Inconsistency Index in two commodities case is also solvable in polynomial time.

[6] Algorithm 1 in Gross and Kaiser (1996) is a different, more efficient algorithm, for an exact calculation of the Houtman-Maks Inconsistency Index.

is found, or after every element in $D_M$ was checked (we implement $K_3 = 10^8$).[7] If the algorithm terminated without finding a subset that satisfies GARP, we use a modified complementary package[8] where the Houtman-Maks Inconsistency Index problem for the case of two goods is represented as an integer linear program which is solved by an approximation algorithm provided by Matlab. This solution is an upper bound since the removals suggested by the linear program might not be minimal.[9]

For the data collected in the first part of our experiment, where each subject made 22 choices from a linear budget line, we are able to calculate the Houtman-Maks Inconsistency Index for all subjects. For the data collected by Choi et al. (2007) (50 observations per subject) we failed to calculate the exact index for 7 subjects (14.9%).

## 2.2 Alternative Indices Based on Revealed Preferences

The Money Pump Index (MPI, Echenique et al., 2011) and the Minimum Cost Index (MCI, Dean and Martin, 2015) are recently proposed alternatives to the Varian, Afriat and Houtman-Maks Inconsistency Indices. In this section we describe and discuss these indices and their relation to those characterized by Theorem 2. In addition, we discuss an additional possible inconsistency index, and highlight the challenges in its application.

[7] For example, if the data set includes 50 observations then all subsets of size 46 or more are tested while if the data set is of size 22 then all subsets are checked (in fact for every dataset of size 23 or less, all subsets will be examined).

[8] Downloaded from Daniel Martin's personal website on November $5^{th}$ 2011. The modifications are mainly due to the simplifications enabled by the result of Rose (1958) for the case of two commodities. The second algorithm in Heufer and Hjertstrand (2015) is closely related to Martin's implementation.

[9] Another, more efficient approximation is implemented by the algorithm suggested by Algorithm 2 in Gross and Kaiser (1996) and Algorithm 1 in Heufer and Hjertstrand (2015).

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### 2.2.1 Money Pump Index

The premise of the MPI is that every violation of GARP corresponds to a cycle of observed bundles.[10] Each cycle can be interpreted as a sequence of trades, resulting in a sure loss of money, that the DM will accept. The MPI of a cycle is the monetary loss, relative to the total income in the cycle, incurred by one sequence of these trades. The MPI of a data set is an aggregation of these losses.[11] The MPI is the only inconsistency index mentioned in the current study that does not minimize any loss function, but rather calculates some measure of severity for each GARP violation.[12] In addition, the MPI takes into account every link in a cycle, rather than focusing only on the weakest link as the other indices analyzed here.

### 2.2.2 Minimum Cost Index

The MCI is based on the fact that SARP is satisfied if and only if the direct revealed preference relation is acyclic. Dean and Martin (2015) suggest to remove direct revealed preference relations between observed bundles until $R^0_{D,\mathbf{1}}$ becomes acyclic. They calculate the cost of removing the ordered pair $(x^{k_i}, x^{k_{i+1}})$ from $R^0_{D,\mathbf{1}}$ by $\frac{p^{k_i}x^{k_i}-p^{k_i}x^{k_{i+1}}}{\sum_{k\in 1,\ldots,n} p^k x^k}$, and propose the MCI as the minimal cost of removals that make $R^0_{D,\mathbf{1}}$ acyclic. The MCI does not take into account that a budget line adjustment required to remove one relation may also remove additional relations. In comparison, such inter-dependencies between cycles are accounted for by the Varian Inconsistency Index.

### 2.2.3 MCI and MPI vs. other Indices

Echenique et al. (2011, Section III.B) and Dean and Martin (2015, Section 2.1) provide thorough discussions on the relative merits of the MPI and the MCI,

[10] A sequence of observed bundles $x^{k_1}, x^{k_2}, \ldots, x^{k_l}$ in dataset $D$ is a cycle of length $l$ if $x^{k_1} R^0_{D,\mathbf{1}} x^{k_2},\ldots,x^{k_{l-1}} R^0_{D,\mathbf{1}} x^{k_l}$ and $x^{k_l} P^0_{D,\mathbf{1}} x^{k_1}$.

[11] Echenique et al. (2011) suggest the mean and the median aggregators, while Smeulders et al. (2013) recommend, due to computational complexity concerns, the minimum or the maximum aggregators.

[12] Counting the violations of the revealed preference axioms is a similar approach in this respect, see Swofford and Whitney (1987); Famulari (1995); Harbaugh et al. (2001).

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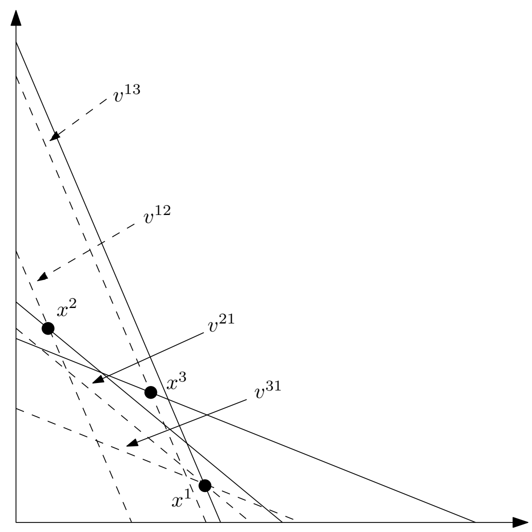


Figure 2.1: Comparing Inconsistency Indices.

respectively. Here, we provide an example that highlights a property common to both indices. Note that the MPI is defined over cycles of observations and the MCI over pairs of observations, while the Varian Inconsistency Index is defined observation-by-observation. As a consequence, the latter internalizes the effect of a single adjustment on all cycles or pairs (in which this observation is involved), while the former two do not. The most important implication of this property, in the context of parametric recovery of preferences, is that it is not clear that there exist corresponding measures of incompatibility that can be decomposed into these inconsistency indices (MPI or MCI) and misspecification measures, in the spirit of Theorem 2.

Consider the data set demonstrated in Figure 2.1. This data set is of size 3, $D = \{(p^1, x^1), (p^2, x^2), (p^3, x^3)\}$ where $p^i x^i = 1$. The strict direct revealed preference relation $P^0_{D,\mathbf{1}}$ (and hence also $R^0_{D,\mathbf{1}}$) includes the ordered pairs $(x^1, x^2)$, $(x^2, x^1)$, $(x^1, x^3)$ and $(x^3, x^1)$ and therefore the data set is inconsistent with GARP. A budget set adjustment $v^{ij} p^i x^i$, where $v^{ij}$ is such that $v^{ij} p^i x^i = p^i x^j$, is the dashed line denoted by $v^{ij}$.

We first attend to the Varian Inconsistency Index. There are three possible minimal adjustment vectors $\mathbf{v}$ such that $GARP_{\mathbf{v}}$ is satisfied: $\mathbf{v}_A = (v^{12}, 1, 1)$, $\mathbf{v}_B = (v^{13}, v^{21}, 1)$ and $\mathbf{v}_C = (1, v^{21}, v^{31})$. Note that in $\mathbf{v}_A$, where the budget line of Observation 1 is adjusted to $x^2$, both cycles $((x^1, x^2, x^1)$ and $(x^1, x^3, x^1))$, are broken at once. Therefore $I_V(D, f) = min\{f(\mathbf{v}_A), f(\mathbf{v}_B), f(\mathbf{v}_C)\}$ and if $f$ is the MEAN aggregator of $\mathbf{1} - \mathbf{v}$, then $I_V(D, f) = min\{\frac{1-v^{12}}{3}, \frac{2-v^{21}-\max\{v^{13},v^{31}\}}{3}\}$. Alternatively, if we use the minimum aggregator $(f(\mathbf{v}) = 1 - \min_{i\in\{1,\dots,n\}} v_i)$ we get that $I_V(D, f) = 1 - \max\{v^{12}, \min\{v^{13}, v^{21}\}, \min\{v^{21}, v^{31}\}\}$. By Theorem 2.3, $I_A(D) = 1 - \max\{v^{12}, \min\{v^{13}, v^{21}\}, \min\{v^{21}, v^{31}\}\}$, as well. There are two minimal adjustment vectors for the Houtman-Maks Inconsistency Index: $\mathbf{v}_{A'} = (0, 1, 1)$ and $\mathbf{v}_{C'} = (1, 0, 0)$. Therefore, $I_{HM}(D, f) = min\{f(\mathbf{v}_{A'}), f(\mathbf{v}_{C'})\}$. If $f$ is anonymous then $I_{HM}(D, f) = f(\mathbf{v}_{A'})$.

The MPI takes into account three cycles - $(x^1, x^2, x^1)$, $(x^1, x^3, x^1)$ and $(x^2, x^1, x^3, x^1, x^2)$. For each cycle it accounts for all the links. Therefore, the measure for $(x^1, x^2, x^1)$ is $\frac{2-v^{12}-v^{21}}{2}$, the measure for $(x^1, x^3, x^1)$ is $\frac{2-v^{13}-v^{31}}{2}$ and the measure for $(x^2, x^1, x^3, x^1, x^2)$ is $\frac{4-v^{12}-v^{21}-v^{13}-v^{31}}{4}$ and using the MEAN aggregator we get $MPI = \frac{4-v^{12}-v^{21}-v^{13}-v^{31}}{4} \geq I_V(D, f)$.

The MCI ignores the fact that adjusting the budget line of Observation 1 to $x^2$ resolves also the cycle that includes $x^1$ and $x^3$. Therefore, $MCI = \frac{2-\max\{v^{12},v^{21}\}-\max\{v^{13},v^{31}\}}{3} \geq I_V(D, f)$.

#### 2.2.4 Area-based Measures

A natural alternative to the incompatibility indices discussed in the current study is an Intersection Incompatibility Index, which is based on the area bounded between the upper contour set of the indifference curve passing through the chosen bundle and the set of feasible alternatives.

A related measure is introduced in the online appendix (Part D.3) of Apesteguia and Ballester (2015) in which they extend their Minimal Swaps

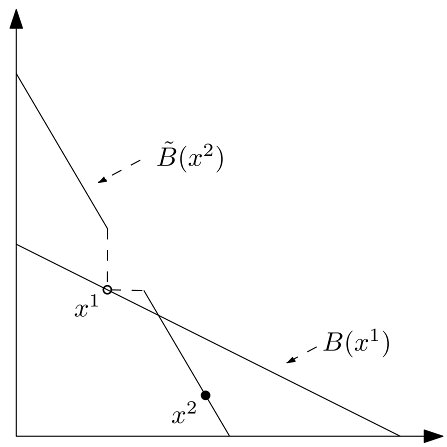


Figure 2.2: Modified budget sets

Index to the case of infinite number of alternatives.[13] Their proposal is based on the Lebesgue measure of the bounded area and the sum aggregator over observations. They define the Consumer Setting Swaps Index as the infimum of this sum over the set of all continuous, strictly monotone and quasi-concave utility functions.

In light of Theorem 2, one needs to have, in addition, a corresponding measure of inconsistency, so that when the set of utility functions is restricted, this index measures the inconsistency embedded in choices, while the remainder of the Intersection Incompatibility Index represents the misspecification implied by the chosen parametric family.

One option is to define an index of inconsistency based on the area of intersection between the revealed preferred set and the budget set corresponding to an observed choice. Define the *revealed preferred* set of a given bundle as *only* those bundles that are either revealed preferred or those that monotonically dominate a bundle that is revealed preferred to the given bundle. Hence, as

[13]For the case of finite number of alternatives, Apesteguia and Ballester (2015) define the Swaps Index of a given preference relation to be the minimal number of swaps required to reconcile the observations with the ranking induced by the given preference. The Minimal Swaps Index minimizes the Swaps Index over all possible rankings. Applying the current paper's terminology, the Swaps Index is an incompatibility measure. However, since Apesteguia and Ballester (2015) domain includes a finite number of alternatives and therefore a finite number of rankings, the Minimal Swaps Index becomes an inconsistency measure, in the spirit of Theorem 2.

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illustrated in Figure 2.2, violations of consistency are removed by modifying budget sets so as to eliminate the area of overlap between the budget set and those bundles which are revealed preferred. These violations can be measured and aggregated to construct the Area Inconsistency Index.[14]

Nevertheless, the Area Inconsistency Index is not ideal. First, currently, there does not exist an elegant theoretical analog to Theorem 1 with respect to the modified budget sets in Figure 2.2 as there does for the specific type of adjustments utilized in calculating the Varian and the Houtman-Maks Inconsistency Indices. Therefore a decomposition result may be difficult to achieve. Second, computing the inconsistency index suggested above would not be any easier than computing the Varian or Houtman-Maks Inconsistency Indices, problems which are NP-hard (see Section 2.1 above). Third, we conjecture that any recovery procedure related to the Area Inconsistency Index would be biased towards non-convex preferences due to the geometric characteristics of the suggested budget line adjustments. Finally, the Area Inconsistency Index lacks intuitive interpretation that the considered indices enjoy. All these are surmountable difficulties, that we think are worthwhile pursuing in future work.

## 2.3 Distance-based Indices

The common method for parametric recovery of individual preferences minimizes some loss function of the distance between observed and predicted bundles. Similar to the money metric and binary incompatibility measures, the result of this method can also be decomposed into an inconsistency and misspecification measures.

One example of such decomposition can be based on an inconsistency index suggested by Beatty and Crawford (2011). This index measures the Euclidean distance between the observed data set and the set of potential data sets

[14] Heufer (2008, 2009, Section 9.2.3) suggests, in the spirit of of Varian's (1982) non-parametric bounds, a similar inconsistency index with the additional external assumption of convexity of preferences. Apesteguia and Ballester (2015) provide a simple example in their online appendix in which they implement a measure that corresponds to Heufer's index, assuming its equivalence to their Consumer Setting Swaps Index.

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that satisfy GARP.[15] It can be shown that a generalization of the proposed index equals the infimum of the appropriate loss function calculated over all continuous and locally non-satiated utility functions. Therefore, the difference between the minimal loss calculated over a subset of utility functions and the proposed inconsistency index results in a natural measure of misspecification.

However, this method ignores the fact that making a choice from a menu reveals that the chosen alternative is preferred to *every* other feasible alternative, not only to the predicted one. In addition, this measure entails an additional assumption on the ranking of unchosen alternatives. It requires that the closer is a bundle to the choice, the higher it is ranked. Such ranking can be justified only by the auxiliary assumption that the choices were generated through a maximization of convex preferences, which is not part of revealed preference theory. Therefore, if choices were generated by a maximization of non-convex preferences then this additional assumption will lead to an erroneous ranking of unchosen alternatives, as demonstrated by the results of the experiment reported in sections 6 and 7 of the main text.

# 3 Decomposition: Graphical Example

Figure 3.1 demonstrates the decomposition graphically. Consider the data set: $D = \{(p^1, x^1), (p^2, x^2)\}$. The data set is inconsistent with GARP since $x^1 R_{D,\mathbf{1}} x^2$ and $x^2 P^0_{D,\mathbf{1}} x^1$. Note that the dashed line $v^2 p^2 x^2$, together with the original budget line from which $x^1$ was chosen, represent graphically the adjustments that lead $D$ to satisfy $GARP_{(1,v^2)}$. If $v^2 \geq v^1$, for any anonymous aggregator, the Varian Inconsistency Index is $I_V(D, f) = f((1, v^2))$ and the Houtman-Maks Inconsistency Index is $I_{HM}(D, f) = f((1, 0))$.

Now consider the monotonic and continuous function $u$. Since $\{(p^1, x^1)\}$ is rationalizable by this utility function, then $v^{\star 1}(D, u) = b^{\star 1}(D, u) = 1$. In

[15] Let $R_i = \{x \in \Re^K_+ : p^i x = p^i x^i\}$ be the set of bundles that cost $p^i x^i$ at prices $p^i$. Then, the set of all potential data sets given data set $D$ is $\left\{\left\{(p^i, x)^n_{i=1}\right\} : x \in R_i\right\}$. Beatty and Crawford (2011) propose $1 - \frac{d}{d^{max}}$ as an inconsistency index where $d$ is the Euclidean distance between the data set and the closest element in the set of potential data sets that satisfy GARP and it is normalized by $d^{max}$ to restrict the index to $[0, 1]$.

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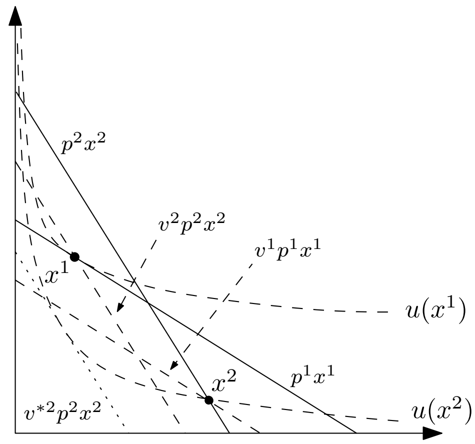


Figure 3.1: Decomposition

addition, $v^{\star 2}(D, u)$ is the minimal expenditure required to achieve utility level of $u(x^2)$ under prices $p^2$, which is represented graphically by the dotted line $v^{\star 2}p^2x^2$ while $b^{\star 2}(D, u) = 0$ since $u$ does not rationalize $\{(p^2, x^2)\}$.

Thus, $I_M(D, f, \{u\}) = f((1, v^{\star 2}(D, u)))$ and since $v^{\star 2}(D, u)$ is smaller than $v^2$, it implies that $I_M(D, f, \{u\})$ is weakly greater than $I_V(D, f)$. Since in this specific example, no other adjustments are required, the difference between the original budget line from which $x^2$ was chosen and the dashed line - $v^2p^2x^2$, represents graphically the inconsistency implied by $D$, while the difference between the dashed line and the dotted line - $v^{\star 2}p^2x^2$, represents the misspecification implied by $u$. Their sum is the goodness of fit measured by the money metric index. However, $I_B(D, f, \{u\}) = I_{HM}(D, f)$, meaning that no misspecification is implied by the binary incompatibility index since $u$ rationalizes $\{(p^1, x^1)\}$ which is the largest subset of $D$ that can be rationalized by any utility function as suggested by the Houtman-Maks Inconsistency Index.[16]

[16] If one considers an alternative utility function $u'$ such that $\{(p^1, x^1)\}$ is not rationalizable by $u'$ (but suppose $v^{\star 2}(D, u') = v^{\star 2}(D, u)$), this would not affect the inconsistency indices but would imply weakly higher loss indices than those measured for $u$ (e.g. $I_B(D, f, \{u'\}) = f(\mathbf{0})$).

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# 4 Disappointment Aversion Preferences

Let $p = (p_1, x_1; ...p_n, x_n)$ be a lottery such that $x_1 \leq \cdots \leq x_n$. Assuming (for simplicity) that $ce\,(p) \notin supp\,(p)$, the support of $p$ can be partitioned into elation and disappointment sets: there exists a unique $j$ such that for all $i < j$ : $(x_i, 1) \prec p$ and for all $i \geq j$ : $(x_i, 1) \succ p$. Let $\alpha = \sum_{i=j}^{n} p_i$. Gul's elation/disappointment decomposition is then given by $r = (x_1, r_1; \cdots; x_{j-1}, r_{j-1})$, $q = (x_j, q_j; \cdots; x_n, q_n)$ such that $r_i = \frac{p_i}{1-\alpha}$ and $q_i = \frac{p_i}{\alpha}$. Note that $p = \alpha q + (1-\alpha)\, r$. Then:

$$u_{DA}\,(p) = \gamma\,(\alpha)\, E\,(v, q) + (1 - \gamma(\alpha))\, E\,(v, r)$$

and $\exists -1 < \beta < \infty$ such that

$$\gamma\,(\alpha) = \frac{\alpha}{1 + (1-\alpha)\,\beta}$$

where $v\,(\cdot)$ is a utility index and $E\,(v, \mu)$ is the expectation of the functional $v$ with respect to measure $\mu$. If $\beta = 0$ disappointment aversion reduces to expected utility, if $\beta > 0$ the DM is disappointment averse ($\gamma\,(\alpha) < \alpha$ for all $0 < \alpha < 1$), and if $\beta < 0$ the DM is elation seeking ($\gamma\,(\alpha) > \alpha$ for all $0 < \alpha < 1$). Gul (1991) shows that the DM is averse to mean preserving spreads if and only if $\beta \geq 0$ and $v$ is concave. That is, if $v$ is concave then, by Yaari (1969), preferences are convex if and only if the DM is weakly disappointment averse.

For binary lotteries: Let $(x_1, p; x_2, 1-p)$ be a lottery. The elation component is $x_2$ and the disappointment component is $x_1$ and $\alpha = 1 - p$ (in our case $\alpha = 0.5$). Therefore:

$$u_{DA}\,(x_1, p; x_2, 1-p) = \gamma\,(1-p)\, v\,(x_2) + (1 - \gamma\,(1-p))\, v\,(x_1)$$

and since $\gamma\,(0) = 0, \gamma\,(1) = 1$ and $\gamma\,(\cdot)$ is increasing, $\gamma\,(\cdot)$ can be viewed as a weighting function, and DA is a special case of Rank Dependent Utility (Quiggin, 1982).

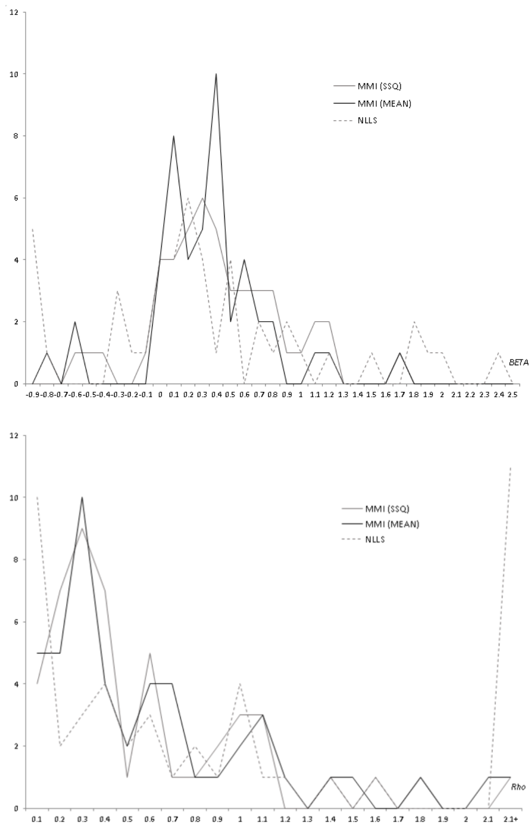


Figure 5.1: The distribution of the recovered $\beta$ (upper) and $\rho$ (lower) by MMI (SSQ), MMI (MEAN) and NLLS in Choi et al. (2007).

# 5 CRRA Parameters: Distributions

Figure 5.1 provides the distribution of the recovered parameters for the Disappointment Aversion functional form with the CRRA utility index by three recovery methods - NLLS, MMI (SSQ) and MMI (MEAN). Both distributions provide some evidence as to the extreme values recovered by NLLS.

Consider for example, the distribution of the disappointment aversion parameter (upper panel of Figure 5.1). The NLLS recovers $\beta < -0.9$ or $\beta > 1.3$ for 11 subjects, while the MMI methods recover such extreme values only once. Similar pattern can be easily observed in the lower panel for the CRRA parameter.

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# 6 The Experiment

## 6.1 Instructions

### Welcome

Welcome to the experiment. Please silence your cell phone and put it away for the duration of the experiment. Additionally, please avoid any discussions with other participants. At any time, if you have any questions please raise your hand and an experiment coordinator will approach you.

**Please note:** If you want to review the instructions at any point during the experiment, simply click on this window (the instructions window). To return to the experiment, please click on the experiment icon on the task bar.

### Study Procedures

This is an experiment in individual decision making. The study has two parts and the second part will begin immediately following completion of the first part. Before Part 1, the instructions will be read aloud by the experiment coordinator and you will be given an opportunity to practice. The practice time will allow you to familiarize yourself with the experimental interface and ask any questions you may have. We describe the parts of the experiment in reverse order, beginning with Part 2 now.

### Part 2

You will be presented with 9 independent decision problems that share a common form. In each round you will be given a choice between a pair of allocations of tokens between two accounts, labeled x and y. Each choice will involve choosing a point on a two-dimensional graph that represents the values in the two accounts. The x-account is represented by the x-axis and the y-account is represented by the y-axis.

For all rounds, in Option 1 the amount allocated to the x-account and y-account will differ, and in Option 2 the amount allocated to each account will be the same. For both options, the values allocated to each account will be displayed beside the point corresponding to each option on the graph, as well as, in the dialog box labeled "Options" on the right-hand side of the screen. Figure 6.1 illustrates some examples of types of choices you may face.

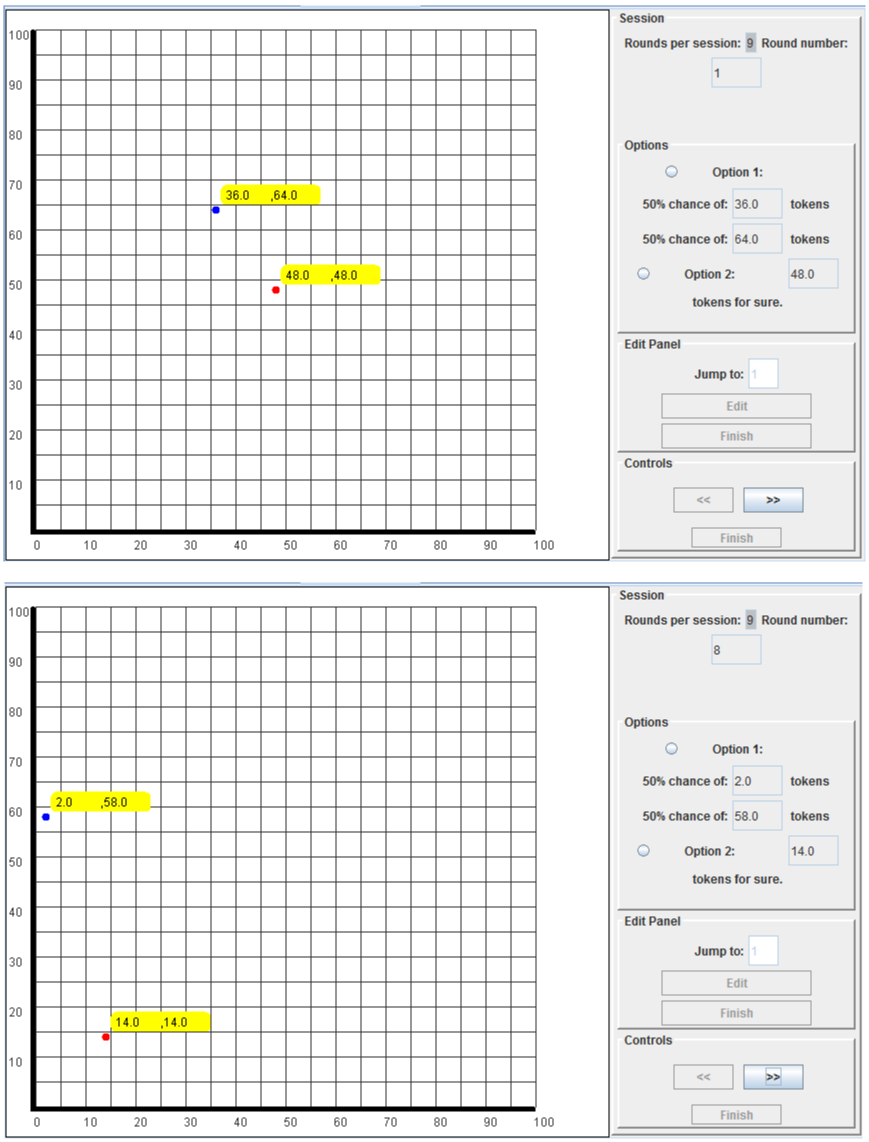


Figure 6.1: Pairwise Choices

For the round that is selected for payment, your payment is determined by the number of tokens allocated to each account. At the end of the experiment, you will toss a fair coin to randomly select one of the two accounts, x or y. For each participant, each account is equally likely to be chosen. That is, there is a 50% chance account x will be selected and a 50% chance account y will be chosen. You will only receive the amount of tokens you allocated to the account that was chosen. The round for which you will be paid will be selected randomly at the conclusion of the experiment and each round is equally likely to be chosen. Remember that tokens are valued at the following conversion rate: 2 tokens = $1.

**Please Note:** Only one round (from both parts combined) will be selected for payment and your payment will be determined only after completion of both parts.

Each round begins with the computer selecting a pair of allocations. For example, as illustrated in Figure 6.2, Option 1, if selected, implies a 50% chance of winning 32.0 tokens and a 50% chance of wining 58.0 tokens, where as Option 2, if selected, implies winning 43.0 tokens for sure.

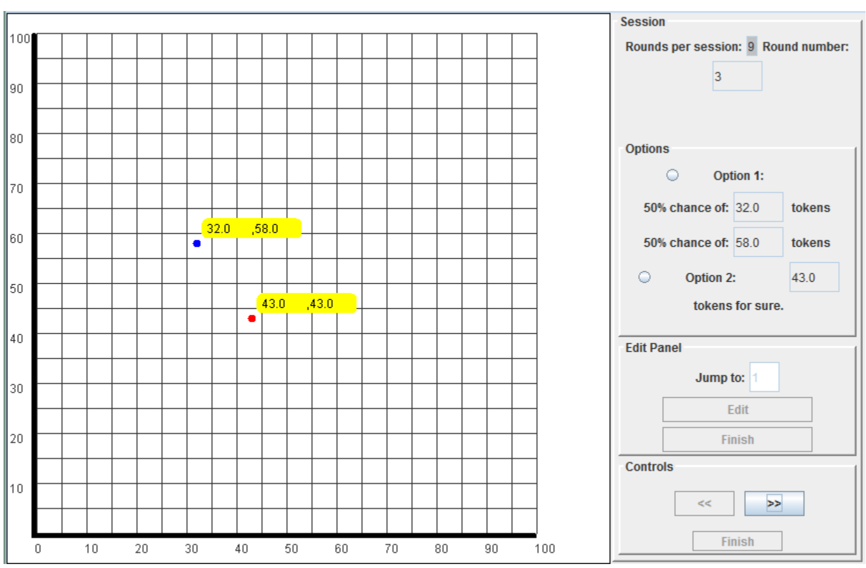


Figure 6.2: Pairwise Choices - Example

In some cases, the two options will be so close to each other that it will be difficult to distinguish between them graphically. In this case, you may refer to the "Options" box on the right-hand side of the screen where the values associated with each option are listed. Additionally, it may be difficult to select your preferred option by clicking on the graph itself, so instead you may use the radio buttons in the "Options" box to make you selection. Figure 6.3 provides an example of this situation.

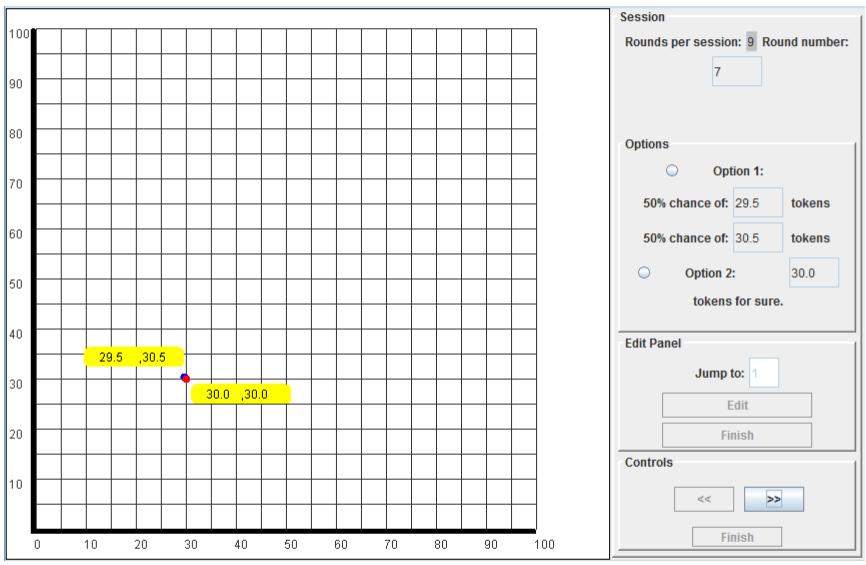


Figure 6.3: Pairwise Choices - Overlapping Points

In all rounds, you may select a particular allocation in either of two ways: 1) You may use the mouse to move the pointer on the computer screen to the option that you desire, and when you are ready to make your decision, simply left-click near that option, or 2) You may select your preferred option using the radio buttons on the right-hand side of the screen, and when you are ready to make your decision, simply left-click on the radio button that corresponds to your choice. In either case, a dialog box, illustrated in Figure 6.4, will ask you to confirm your decision by clicking "OK".

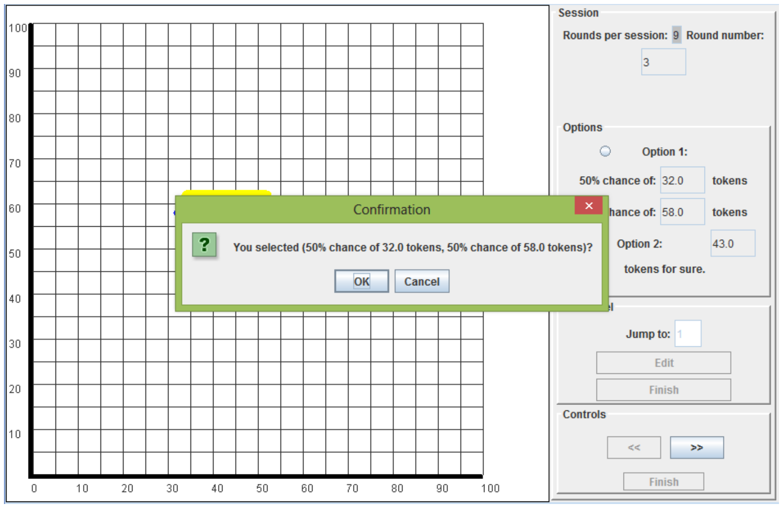


Figure 6.4: Pairwise Choices - Confirmation Screen

If you wish to revise your choice simply click "Cancel" instead. After you click "OK", your choice will be highlighted in green and the screen will darken, as illustrated in Figure 6.5, indicating that your choice is confirmed. You may proceed to the next round by clicking on the ">>" button located in the lower right-hand corner of the screen in the box labeled "Controls". Please note that you will be given an opportunity to review and edit your choices upon completion of Part 2 of the experiment.

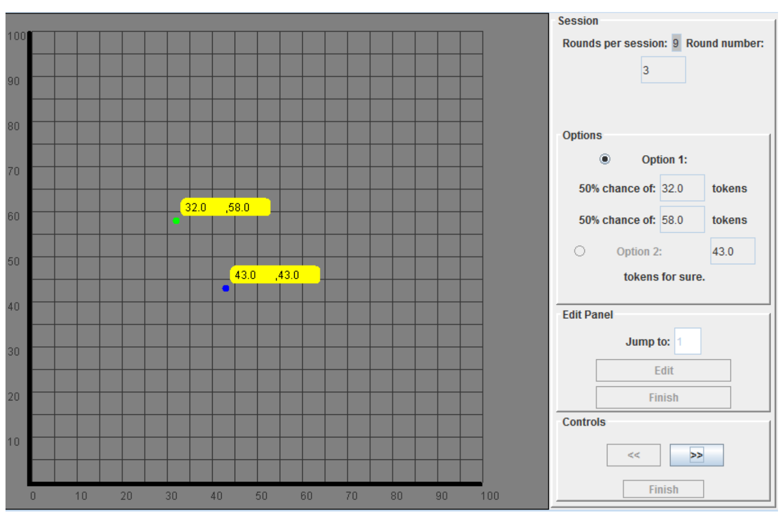


Figure 6.5: Pairwise Choices - Confirmed Choice

Next you will be asked to make an allocation in another independent decision problem. This process will be repeated until all 9 rounds are completed. At the end of the last round, you must click the “Finish” button, located in the lower right-hand corner of the screen in the box labeled “Controls”, and you will be given an opportunity to review your choices. You may use the navigation buttons to move between choices or the “Jump to” feature in the “Edit Panel” to navigate to a specific round. If you are content with your choices, you may exit the review by clicking on the “Finish” button. At this stage you may no longer go back to review and/or edit your choices. Instead, click “OK” to complete the experiment.

## Part 1

In Part 1, you will be presented with 22 independent decision problems that are very similar to those in Part 2. However, rather than selecting an allocation from among only two options, now you will have many options to choose from. In each round your available options will be illustrated by a straight line on the graph and you will make your choice by selecting a point on this line. As in Part 2, your payoff in the round that is selected for payment is determined by the number of tokens allocated to each account. Examples of different lines you may face are illustrated in Figure 6.6.

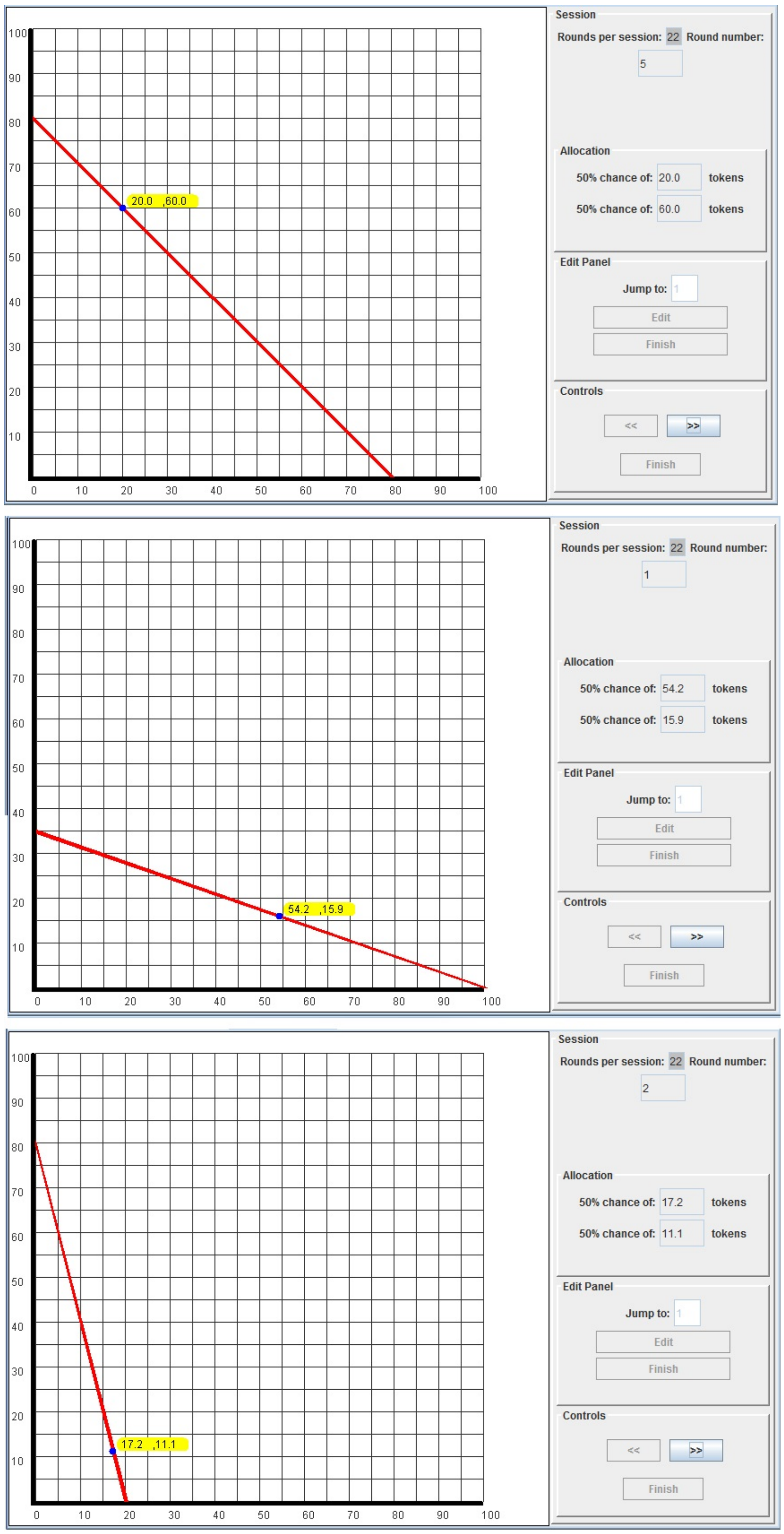


Figure 6.6: Budget Lines

Figure 6.7 illustrates the differences and similarities between the problems in Part 1 and Part 2. In Part 2, you are offered the choice between only two options, A and B. On the other hand, if we were to draw a straight line between these options and allow one to choose any point on this line, then this would increase the number of available choices. Notice, however, that the two original options are still available as well as many more. Hence, the problems in Part 1 are conceptually the same as in Part 2, but with many more possible allocations.

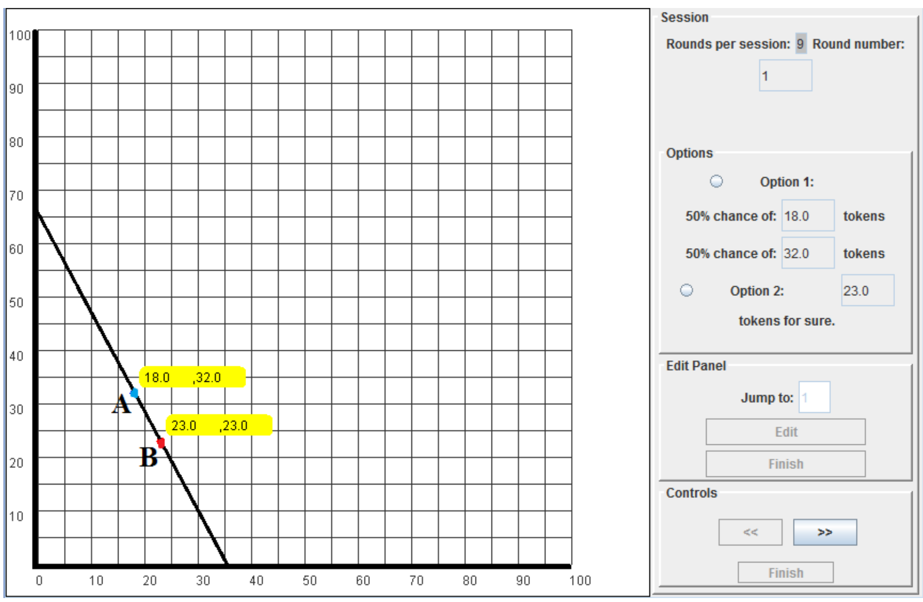


Figure 6.7: Budget Lines - Relationship to Pairwise Choice

The following two examples further illustrate the nature of the problem. If, in a particular round, you were to select an allocation where the amount in one of the accounts is zero, for example if you allocate all tokens to account x and $0 to account y (or vice versa), then in the event that this round is chosen for payment there is a 50% chance you will receive nothing at all, and a 50% chance you will receive the highest possible payment available in that round. In contrast, if you were to select an allocation where the amount in accounts x and y are equal, then in the event that this round is chosen for payment you will receive this amount regardless of which account is chosen by the coin toss.

Each round begins with the computer selecting a line. As in Part 2, the lines selected for you in different rounds are independent of each other. For example, as illustrated in Figure 6.8, choice A represents an allocation in which you allocate approximately 9.4 tokens in the x-account and 60.7 tokens in the y-account. Another possible allocation is choice B, in which you allocate 22.6 tokens in the x-account and 33.6 tokens in the y-account.

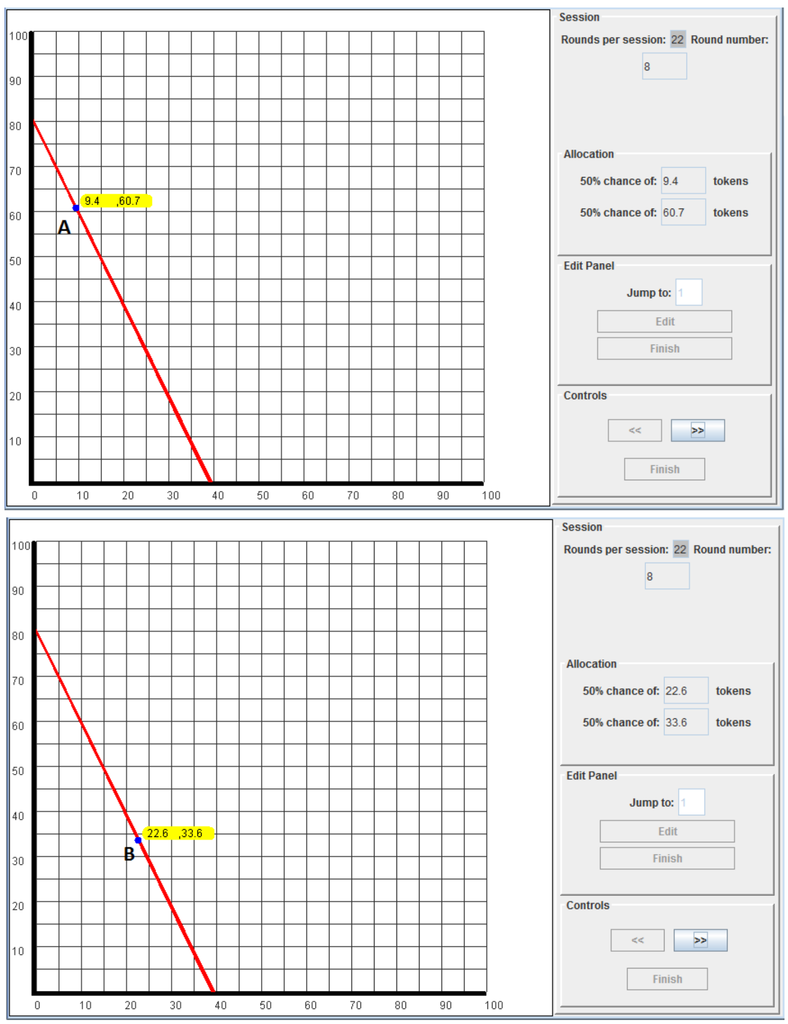


Figure 6.8: Budget Lines - Examples

To choose an allocation, use the mouse to move the pointer on the computer screen to the allocation that you desire. On the right hand side of the program dialog window you will be able to see the exact allocation where the pointer is located. Please note that, in each choice, you may only choose an allocation which lies on the line provided. Additionally, if you select an allocation that is close to the x-axis or the y-axis, you will be asked if you would like to select an allocation on the boundary or if you intended for your choice to be as originally selected. Similarly, if you select an allocation that is close to the middle, (roughly the same amounts in each account), you will be asked if you would like to select an allocation where the amounts in both accounts are exactly equal or if you intended for your choice to be as originally selected. The dialog boxes associated with these scenarios are illustrated in Figure 6.9.

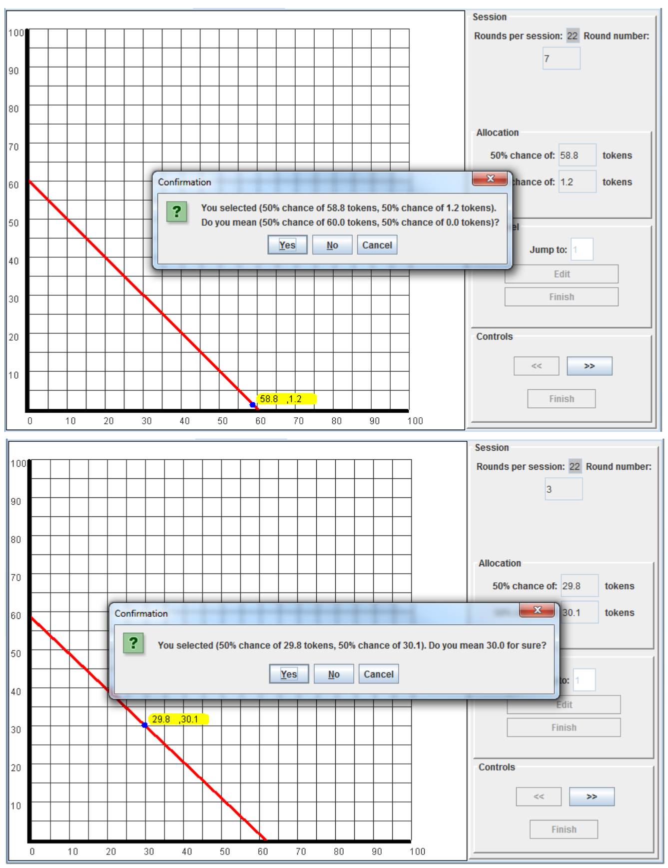


Figure 6.9: Budget Lines - Special Cases

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The controls to confirm your choices and navigate between rounds are identical to those described above for Part 2. Once you have finished with all 22 rounds, you will be given an opportunity to review your choices. You may conclude your review by clicking on the finish button in the "Edit Panel" at any time. Once complete, please click on the instructions window in order to move on to Part 2.

**Please remember that there are no "right" or "wrong" choices. Your preferences may be different from other participants, and as a result your choices can be different. Please note that as in all experiments in Economics, the procedures are described fully and all payments are real.**

## Compensation

After completing both parts of the experiment you will be informed of your payment via an on-screen dialog box. Payments are determined as follows:

The computer will randomly select one decision round from both parts (combined) to carry out. The round selected depends solely on chance and it is equally likely that any particular round will be chosen. The payment dialog box will inform you of which round was randomly chosen as well as your choice in that round. At this point please raise your hand and an experiment coordinator will provide you with a fair coin, e.g. a quarter. To determine your final payoff, please flip the coin. If it lands heads, you will be paid according to the amount of tokens in the x-account and if it lands tails, you will be paid according to the amount of tokens in the y-account. For both parts of the experiment, tokens are valued at the following conversion rate:

**2 tokens = $1**

You will receive your payment, along with the $10 show-up bonus, privately

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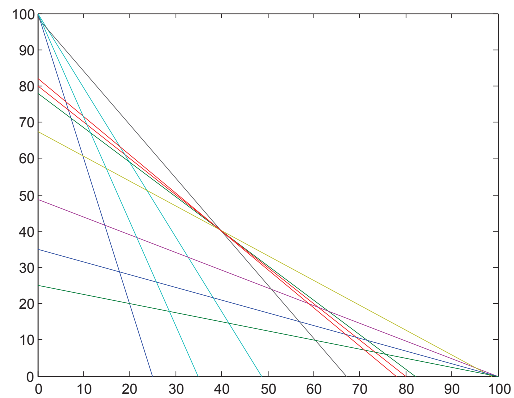


Figure 6.10: Subset of the budget lines shown in Part 1

before you leave the lab. You will be asked to sign a receipt acknowledging receipt of your payment, after which time you may leave.

## 6.2 Choice of Budget Lines

Section 6.2 of the main text describes the set of budget sets chosen for the first part of the experiment as a result of two considerations: sufficient power and first-order risk aversion/seeking identification. The 22 budget lines were divided into two subsets of 11 budget lines such that each subset was composed of the same price ratios, where the only difference was the wealth level. For each of the two subsets, 5 of the 11 price ratios had relatively moderate slopes, where as the other 6 were much steeper. Figure 6.10 shows the set of 11 budget lines for the higher wealth level.

To corroborate that this set of budget sets submits the subjects to a sufficiently powerful test of consistency, we conducted a power test (following Bronars (1987)) by constructing 1000 simulated data sets.[17] First, not a single simulated data set passed GARP while in the experimental data 44.4% were found to be consistent. Second, in the simulation, 1.3% (4.5%) of the data sets had Afriat Inconsistency Index below 0.05 (0.1) while in the experimental data 86% (93.7%) of the subjects exhibited this level of inconsistency.

[17]The results of the power test are available in a separate Excel file named "Halevy et al (2017) Part 1 - Power Test".

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Third, the Houtman-Maks Inconsistency Index (calculated exactly) suggests that 91.9% (57.9%) of the simulated data sets require at least 4 (6) observations to be discarded to satisfy GARP. However, in the experimental data, only 8.7% (0.05%) of the data sets require as many observations to be dropped to achieve consistency. Finally, while we were able to calculate the Varian Inconsistency Index exactly (or with very good approximation) for 98.6% of the experimental data sets, this was feasible for only 25.9% of the simulated data sets. In fact, even within this set, using the MEAN aggregator, while 57.1% of the simulated data sets exhibited Varian Inconsistency Index greater than 0.05, only 2.9% of the experimental data sets showed similar levels of inconsistency.

## 6.3 The Construction of the Pairwise Choices

In Section 6.3 of the main text we describe the basic logic behind the algorithm used to construct the pairwise choices for Part 2 of the experiment. Here, we provide a more detailed description of this algorithm.

Each pairwise choice is constructed using the following search algorithm. First, we fix an expected value for the risky portfolio. Then, we search over the line that connects all the portfolios with the same expected value until a risky portfolio, $x^R$, is found that satisfies certain stopping conditions. The starting point for the search as well as the stopping conditions are chosen to construct a sufficiently rich set of choices that are appropriate for addressing the research questions.

To investigate the nature of *local* risk attitudes across subjects we designated 6 out of the 9 pairwise choices to this task by beginning our search for $x^R$ at certainty and progressing along the equal expected value line in the direction of increasingly variable portfolios until the stopping rule is satisfied. In the case where both methods recover $\beta \geq 0$, the stopping rule requires that the difference in certainty equivalents exceeds one token. Hence, to construct these *low-variability portfolios* we search for the lowest variance portfolio among all those with the same expected value such that there is sufficient difference in certainty equivalents between recovered parameters. For sets of parameters

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where the difference between certainty equivalents does not exceed one token for *all* low-variability portfolios we reduce this threshold incrementally until a valid pairwise comparison is chosen. In all cases the safe portfolio, $x^S$, is chosen as the mid-point between the certainty equivalents of the risky portfolio, $x^R$ (see Section 6.3 of the main text).

For subjects where either or both methods recover $\beta < 0$, we use a different stopping rule. In these cases the search terminates as soon as a risky portfolio is found such that the certainty equivalent corresponding to one method exceeds the expected value of the portfolio and the certainty equivalent corresponding to the other method is less. Here we choose the safe portfolio as the expected value of the risky portfolio, i.e. $x^S = E[x^R]$.

The remaining 3 out of 9 pairwise choices are constructed such that the risky portfolio is close to, but not literally on, the axis. We refer to these pairwise choices as *high-variability portfolios*. We avoid offering corner choices as they can be difficult to rationalize with the CRRA functional form. We choose risky portfolios as close to the axis as possible by starting with a portfolio that includes a minimum payoff of two tokens and searching towards the certainty line. The stopping condition is that the difference in certainty equivalents is at least one token. High-variability portfolios are chosen in the same manner regardless of the recovered values for $\beta$.[18]

# 7 Part 1: Comparison to Choi et al. (2007)

This section compares the results of Part 1 of the experiment and the data collected by Choi et al. (2007). Table 1 summarizes the inconsistency indices and the parameters recovered for the Disappointment Aversion with CRRA utility index. We attribute the slight differences to the difference in instructions, interface, the number of rounds and to the variability and range of the price ratios.

[18]The six low-variability portfolios have expected values of 50, 45, 40, 35, 30, and 25 tokens, where as the three high-variability portfolios have expected values of 50, 40, and 30 tokens.

| | Choi et al. (2007) | | | | Part 1 of the Experiment | | | |
|---|---|---|---|---|---|---|---|---|
| **Consistent Subjects** | 12 (25.5%) | | | | 91 (44.8%) | | | |
| **Median (mean) Afriat Inconsistency Index*** | 0.045 (0.0881) | | | | 0.0126 (0.0374) | | | |
| **Median (mean) Houtman-Maks Inconsistency Index*** | 0.06 (0.079) | | | | 0.0909 (0.097) | | | |
| **Median (mean) Varian Inconsistency Index (SSQ)**** | 0.006 (0.007) | | | | 0.0027 (0.0084) | | | |
| | **MMI (SSQ)** | | **NLLS** | | **MMI (SSQ)** | | **NLLS** | |
| | $\beta$ | $\rho$ | $\beta$ | $\rho$ | $\beta$ | $\rho$ | $\beta$ | $\rho$ |
| **Complete Sample (Median)** | 0.3326 | 0.3559 | 0.171 | 0.5799 | 0.39 | 0.3764 | 0.3343 | 0.3674 |
| **Consistent Subjects Only (Median)** | 0.4121 | 0.7319 | 0.0058 | 1.277 | 0.4065 | 0.4137 | 0.3443 | 0.5597 |
| **# Subjects with Non-Convex Preferences** | 8 (17%) | | 15 (31.9%) | | 37 (18.2%) | | 45 (22.2%) | |
| **Subjects with $\beta \geq 0$ (Median)** | 0.3759 | 0.295 | 0.4058 | 0.3404 | 0.4668 | 0.3022 | 0.4654 | 0.1964 |
| **Subjects with $\beta < 0$ (Median)** | -0.1047 | 0.8691 | -0.3275 | 3.8642 | -0.1575 | 0.8008 | -0.8941 | 4.0782 |

* Computed on inconsistent subjects.

** Computed on inconsistent subjects with reliable index.

Table 1: Choi et al. (2007) vs. Part 1 of the Experiment.

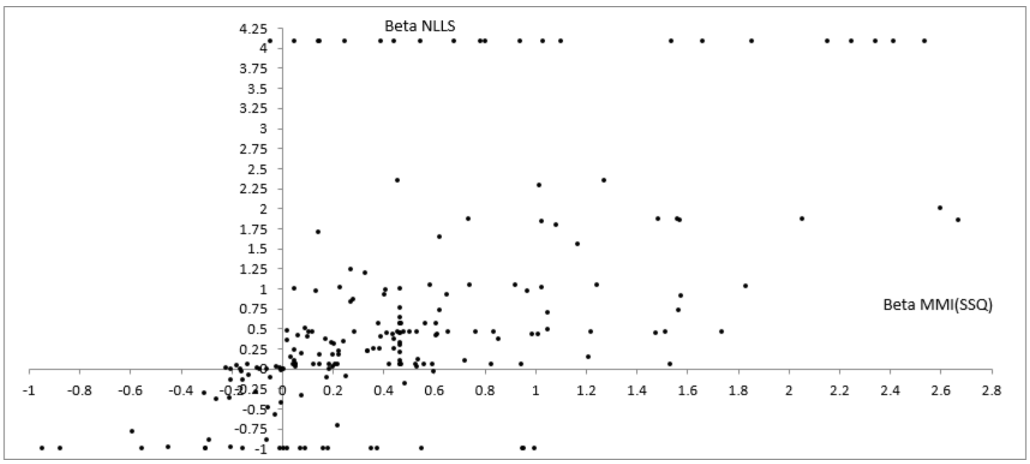



Figure 7.1: Disappointment Aversion Parameter: NLLS vs. MMI (SSQ).

Figure 7.1 replicates Figure 5.2 in the main text for the data collected in Part 1 of the experiment. Also here, when the NLLS recovery method recovers convex preferences then in most cases the MMI method recovers convex preferences as well, while when the preferences recovered by the NLLS are non-convex, there seem to be no qualitative relation between the recovered parameters by the two methods.

# 8 Pairwise Choice: Refined Results

The complete sample includes subjects and choices that arguably should not be included in a comparison between the MMI and the NLLS recovery methods.

In Section 7 of the main text we report the results using the full sample while here we refine the sample and recalculate the results reported in the main text using the refined sample.

## 8.1 The Refinement

We find two reasons to consider dropping an observation from the sample. First, the subject's choices may be too inconsistent to believe that there exists some underlying stable preference that guides her choices. Second, since the pairwise choices the subject encountered in Part 2 of the experiment were generated automatically, in some cases the two proposed portfolios were too similar for the subject to be able to thoughtfully distinguish between them.[19] Hence, our refinement scheme applies two criteria - inconsistency and similarity.

The *inconsistency refinement* removes two subjects whose Afriat Inconsistency Index is greater than 0.2.[20]

The *similarity refinement* removes observations for which there is little difference between the portfolios constructed in Part 2 of the experiment. We consider a pairwise choice to be *indefinitive* if the two sets of parameters imply similar local risk attitude (either $\min\left\{CE_{MMI}(x^R), CE_{NLLS}(x^R)\right\} > E\left[x^R\right]$ or $\max\left\{CE_{MMI}(x^R), CE_{NLLS}(x^R)\right\} < E\left[x^R\right]$) and the difference in implied certainty equivalents is very small ($|CE_{MMI}(x^R) - CE_{NLLS}(x^R)| < 0.5$).

| | # of Observations | Correct Predictions by MMI (%) | p-value |
|---|---|---|---|
| Refinement | 1489 | 804 (54.0%) | 0.0011 |

Table 2: Preliminary Results - Aggregate Level Analysis (refined sample)

| Refined Sample | | |
|---|---|---|
| X≥7 | X≤2 | p-value |
| 41 | 25 | 0.032 |

Table 3: Preliminary Results - Individual Level Analysis (refined sample)

### 8.2 Results: Refined Sample

Table 2 recalculates the aggregate level analysis reported in Table 2 in Section 7 of the main text for the refined sample. These results are almost identical to the results reported for the complete sample.

In the individual level analysis, for the similarity refinement we remove all subjects who confronted one or more indefinitive pairwise comparison in Part 2. Thus, the remaining 131 subjects are deemed sufficiently rational and exhibit a sufficient difference in predictions between recovery methods to admit a reasonable comparison.

Table 3 recalculates the individual level analysis reported in Table 3 in Section 7 of the main text for the refined sample. As the results reported for the complete sample, Table 3 also provides statistically significant evidence for the predictive superiority of the MMI recovery method over the NLLS recovery method.

[19]While in some of these cases, the similarity can be traced back to the NLLS and the MMI recovering very similar parameters, in other cases it may be a consequence of the substitutability between the two parameters, $\beta$ and $\rho$, with respect to the subject's local risk attitude.

[20]In fact, these two subjects also have the highest number of GARP violations. Moreover, we provide an exact calculation of the Varian Inconsistency Index for all but three subjects (for whom we report overestimates, see Section 2.1.2). These three subjects include the pair with the extreme Afriat Inconsistency Index values. The approximated Varian Inconsistency Index values for these two subjects are substantially greater than 0.1 for the minimum, MEAN and the SSQ aggregators. No other subjects have Varian Inconsistency Index greater than 0.1.

| | # of Observations | # Correct Predictions by MMI | % Correct Predictions by MMI | p-value |
|---|---|---|---|---|
| DDA | 1025 | 528 | 51.5% | 0.1744 |
| IDA | 464 | 276 | 59.5% | <0.0001 |

Table 4: Refined Sample Results by Group - Aggregate Level Analysis.

| DDA | | | IDA | | |
|---|---|---|---|---|---|
| X≥7 | X≤2 | p-value | X≥7 | X≤2 | p-value |
| 19 | 16 | 0.3679 | 22 | 9 | 0.0147 |

Table 5: Refined Sample Results by Group - Individual Level Analysis.

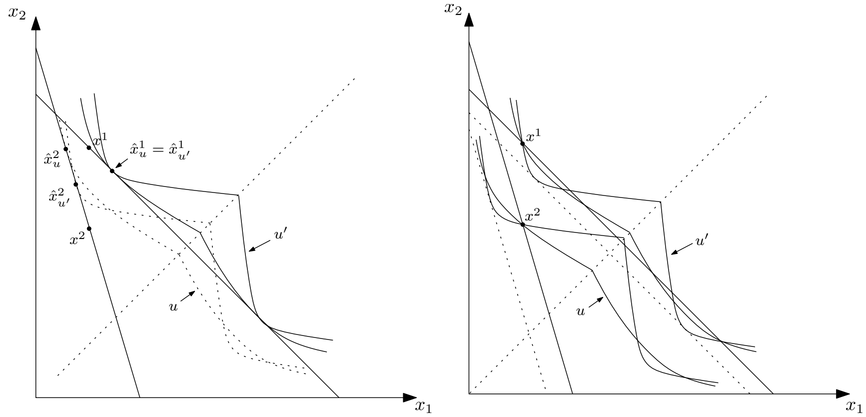
## 8.3 Disappointment Aversion: Refined Sample

The Definite Disappointment Averse (DDA) group is composed of those subjects for which both methods recover $\beta \geq 0$, whereas Indefinite Disappointment Averse (IDA) group is composed of those subjects for which $\beta$ is negative for one or both recovery methods. After the inconsistency refinement we are left with 148 subjects in the DDA group and 53 subjects in the IDA group.

In the aggregate analysis we treat the whole set of observations as a single data set with 1332 observations for the DDA group and 477 for the IDA group. Then we remove all the indefinitive pairwise comparisons. Table 4 demonstrates that, also when using the refined sample, the MMI recovery method remains a better predictor in both cases, but while its advantage is insignificant in the DDA group, it is highly significant in IDA group.

In the individual level analysis, using the refinement we are left with 84 subjects in the DDA group and 47 subjects in the IDA group. Table 5 demonstrates that also here, although the MMI recovery method predicts better than the NLLS recovery method in both DDA and IDA, the difference in predictive accuracy within the DDA group is insignificant. However, this difference within the IDA group is substantial and statistically significant.

Next, consider the Definite Elation Seeking (DES) group that includes those subjects for whom both recovery methods recover $\beta < 0$. After the refinement is applied, for the aggregate analysis the DES group includes 248 observations. The MMI recovery method predicted correctly 156 of the choice




(a) NLLS (b) MMI

Figure 9.1: MMI vs NLLS - Non-convex Preferences Recovery

problems, which amount to 62.9% of the observations. Hence, the difference between the recovery methods within the DES group is even more substantial than in the whole IDA group and it is highly statistically significant ($p$-value smaller than 0.0001).

The individual results for the DES group are similar - for 16 out of the 25 subjects that survive the refinement one method predicted correctly more than two thirds of the pairwise choices. It turns out that in 13 of the 16 cases, it was the MMI (81.3%, $p$-value 0.0106).[21]

# 9 Recovery of Non-Convex Preferences

Figure 9.1 demonstrates how the MMI and NLLS may recover different sets of parameters for the same data set. Suppose we take two observations, $x^1$ and $x^2$, and try to determine which of two utility functions – $u$ and $u'$, is a better fit for the data. Define $\hat{x}_v^i$ as the utility maximizing choice from budget line $i$ given utility function $v$.

The left panel shows that the NLLS recovery method selects $u'$ over $u$, as

[21] We exclude Subject 1702 from the DES group since $\beta_{NLLS} \approx 0$. For similar reason we excluded also the definitive observations of Subject 604 from the previously mentioned aggregate analysis.

the distance between the utility maximizing bundle and the observed choice is identical at $x^1$, and smaller for $u'$ at $x^2$. This arises from the lower price elasticity (higher non-convexity) implied by $u'$. The right panel demonstrates that the MMI selects $u$ over $u'$ using minimal budget set adjustment. The farther the observed portfolio is from the certainty line, the smaller is the adjustment required for the "flatter" (less non-convex) $u$ compared to $u'$.

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