Negotiations across Multiple Issues

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Motivation

- A common practice for firms wishing to collaborate is to form a joint venture.
  - A new firm is established.
  - The collaborating firms are the owners.
  - But, the new firm is granted the sole responsibility for the joint activity.
- When interested in collaborating on several independent projects, firms could form either:
  - A separate joint venture for each project.
  - A single joint venture that is responsible for all projects (linkage).
- Example for linkage: Viiv Healthcare
- This work is concerned with cooperation and issue linkage in similar settings.

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The General Problem

- A group of agents is aspiring to reach an agreement on several independent issues simultaneously.
- An agreement is a single contract that divides the aggregate payoffs of all issues.
- The agents are aware of the potential gains from each issue.
- The agents are informed only of aggregate payoffs keeping them ignorant of the payoffs breakdown by issues.
- Can such an agreement promote cooperation?
- Additional Example - Wage bargaining: An employer and a worker sign a single contract regulating the performance on several tasks.

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Methodology

- Reduced form approach to bargaining by modeling the multiple issues problem as a set of cooperative games with transferable utility.
- Protocol-independent setting, as opposed to the non-cooperative approach. Literature

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A Cooperative Game

- A cooperative game $G = (N; V)$ is:
  - A set of players $N = \{1, 2, \ldots, n\}$.
  - A characteristic function $V : P(N) \to \mathbb{R}$ where $P(N) \equiv \{S \neq \phi | S \subseteq N\}$
  - $P_i(N) \equiv \{S \cup \{i\} | S \subseteq N \backslash \{i\}\}$, $P_{-i}(N) \equiv P(N) \backslash P_i(N)$.
  - $V(S)$ is interpreted as the value attained by coalition S when operating independently.

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The Core

Definition (The Core)

$$C(V) = \left\{ x \in \mathbb{R}^n \middle| \sum_{i \in N} x_i = V(N), \forall S \in P(N) : \sum_{i \in S} x_i \geq V(S) \right\}$$

Multi Game

Definition (Multi Game)

An m-issue multi-game $\bar{G}$ is a pair $\bar{G} = (N; \bar{V})$ where $\bar{V}$ is a set of characteristic functions $\bar{V} = \{V_1, V_2, \ldots, V_m\}$ such that for every $j \in \{1, \ldots, m\}$, $V_j : P(N) \to \mathbb{R}$.

- If no confusion arises, we denote the multi-game $\bar{G} = (N; \bar{V})$ by its set of characteristic functions $\bar{V}$.

Example - Independent Issues

$$v_1(S) = \begin{cases} 0 & \text{if} \quad |S| = 1 \\ \frac{3}{4} & \text{if} \quad |S| = 2 \\ 1 & \text{if} \quad |S| = 3 \end{cases} \quad ; \quad v_2(S) = \begin{cases} 0 & \text{if} \quad |S| = 1 \\ 0 & \text{if} \quad |S| = 2 \\ 1 & \text{if} \quad |S| = 3 \end{cases}$$

- Issue 1 - “hard”, the core is empty:
  - Each pair must receive at least $\frac{3}{4}$.
  - But, the total payoff is less than $\frac{9}{8}$.
- Issue 2 - “easy”, every non-negative payoff vector whose elements add up to one is in the core.
- It is impossible to reach an agreement on all issues when they are solved independently.

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Consider the payoff vector $(\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$.

Its "justification matrices" are:

$$\mathbf{y^1} = \begin{pmatrix} \frac{2}{3} & 0 \\ \frac{1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} \end{pmatrix} \quad ; \quad \mathbf{y^2} = \begin{pmatrix} \frac{1}{6} & \frac{1}{2} \\ \frac{2}{3} & 0 \\ \frac{1}{6} & \frac{1}{2} \end{pmatrix} \quad ; \quad \mathbf{y^3} = \begin{pmatrix} \frac{1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} \\ \frac{2}{3} & 0 \end{pmatrix}$$

Every element of $\left\{x \in [\frac{1}{2}, 1]^3 \middle| x_1 + x_2 + x_3 = 2\right\}$ is a solution (and there are no other solutions).

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Beliefs

- The agents do not know the breakdown of payments by issues.
- Therefore they form a belief.
- If, by this belief, there is a coalition that is under-compensated:
  - By deviating on the agent's total payoff increases.
  - True for all other members of the coalition.
  - Hence, every member has a belief that supports such a deviation.
  - The agent can rationalize the cooperation of the other members on deviating (a-la Rationalizability).
  - Therefore, the agent will not comply with the grand coalition on all issues.
- Otherwise, the agent has no reason to block the formation of the grand coalition on any one of the issues.

Efficient Decomposition Matrices

Definition (Efficient Aggregate Payoff)

The allocation $x \in \mathbb{R}^n$ is an efficient aggregate payoff vector of $\bar{V}$ if $\sum_{i=1}^{n} x_i = \sum_{V_j \in \bar{V}} V_j(N)$.

Definition (Efficient Decomposition Matrix)

The set of efficient decomposition matrices of an aggregate payoff vector x is

$$\hat{Y}(\bar{V}, x) = \left\{ y \in \mathbb{R}^{n \times m} \middle| \forall i \in N : \sum_{V_j \in \bar{V}} y_{i,j} = x_i, \forall V_j \in \bar{V} : \sum_{k=1}^{n} y_{k,j} = V_j(N) \right\}$$

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The Multi Core

Definition (The Multi Core)

An efficient aggregate payoff vector x is in the multi-core, $x \in M(\bar{V})$, if for every Agent i there exists an efficient decomposition matrix $y^i \in \hat{Y}(\bar{V}, x)$ such that $\forall V_j \in \bar{V}, \forall S \in P_i(N) : \sum_{k \in S} y^i_{k,j} \geq V_j(S)$.
We refer to y^i as a justification matrix of Agent i with regard to the payoff vector x.

Story

- Each agent i forms a belief regarding the decomposition (denoted by $y \in \hat{Y}(\bar{V}, x)$).
- If the total payment entailed in belief y to coalition S in issue V_j is lower than $V_j(S)$ $(\sum_{k \in S} y^i_{k,j} < V_j(S))$:
  - By deviating on V_j the agent's total payoff is her share of $V_j(S)$ and her payments (by y) on the remaining issues.
  - The total is greater than x_i.
  - True for all other members of S as well. Hence, every member of S has a belief that supports such a deviation.
  - Agent i can rationalize the cooperation of the other members of S in deviating on V_j.
  - Hence, given such a belief y, Agent i will not comply with the grand coalition on all issues.
- Otherwise, Agent i has no reason to block the formation of the grand coalition on any one of the issues.
- When $x \in M(\bar{V})$, Agent i has a justification for supporting x and she reasons that x will be accepted unanimously.

Alternative Solution Concepts

- In the Multi-Core agents know the individual games but are ignorant of the breakdown of payoffs.
- Agents know the individual games and the breakdown of payoffs:
  - A natural candidate - the sum over the solutions in the cores of the single issues.
  - $\sum_{V_j \in \bar{V}} C(V_j) = \left\{ \sum_{V_j \in \bar{V}} x^j \middle| x^j \in C(V_j) \right\}$.
- Agents ignorant of the individual games (and the breakdown of payoffs):
  - A natural candidate - the core of the sum of the characteristic functions.
  - $C(\sum_{V_j \in \bar{V}} V_j)$.
- In many cases the solution concept reflects the information structure rather than being an implementation choice.

Relevant Literature

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- In many cases the solution concept reflects the information structure rather than being an implementation choice.

Relevant Literature

Multi-Core vs. the Sum of Cores

Proposition

$$\sum_{V_j \in \bar{V}} C(V_j) \subseteq M(\bar{V})$$

- A matrix whose columns are allocations in the cores of the corresponding games serves as a common justification.
- The Multi-Core is strictly weaker. Example
- The gap is due to linkage.

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$$M(\bar{V}) \subseteq C(\sum_{V_j \in \bar{V}} V_j)$$

Proof

- Intuition: Deviations.
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  - Initial example: Details
  - It might be that $M(\bar{V}) = \emptyset$ and $C(\sum_{V_j \in \bar{V}} V_j) \neq \emptyset$.
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Is Issue Linkage Worthwhile?

1. We say that the multi-core is effective when it is strictly larger than $\sum_{V_j \in \bar{V}} C(V_j)$, and ineffective when the sets are the same.
2. We are interested in two cases:
   1. Can the Multi-Core provide a solution if *all* the problems are "hard"?
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All the problems are "hard"

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$$V_1(S) = \begin{cases} 9 & \text{if} \quad S \in \{S \subset N | \{1,2\} \subseteq S\} \\ 10 & \text{if} \quad |S| = N \\ 1 & \text{if} \quad otherwise \end{cases}$$

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A subgame of $G = (N, v)$ is a game (T, V^T) where $T \in P(N)$ and $V^T(S) = v(S)$ for all $S \subseteq T$.

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A game $G = (N, V)$ is

- *superadditive* if for every pair of disjoint coalitions $S, T \subseteq N$, $V(S) + V(T) \leq V(S \cup T)$.
- *balanced* if it has a non-empty core.
- *totally balanced* if every subgame has a non-empty core.
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All the problems are "easy" - Linkage is Ineffective

Proposition

Let $\bar{V}$ be a multi-game where every $V_j \in \bar{V}$ is convex. The multi-core of $\bar{V}$ is ineffective.

- Dragan et al. (1989) and Bloch and de Clippel (2010) show that if V is a set of convex issues, $\sum_{V_j \in \bar{V}} C(V_j) = C(\sum_{V_j \in \bar{V}} V_j)$.

Proposition

Let $\bar{V}$ be a multi-game of 3 players where every $V_j \in \bar{V}$ is balanced and superadditive. The multi-core of $\bar{V}$ is ineffective.

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The Multi Core is Effective - Example

$$V_1(S) = \begin{cases} 0 & \text{if} \quad |S| \leq 2, S \notin \{\{2,4\},\{3,4\}\} \\ \frac{1}{2} & \text{if} \quad S \in \{\{2,4\},\{3,4\}\} \\ \frac{1}{2} & \text{if} \quad |S| = 3, S \neq \{1,2,3\} \\ 1 & \text{if} \quad S \in \{\{1,2,3\},\{1,2,3,4\}\} \end{cases}$$

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- A multi-game with two totally balanced issues and four players.
- Every $x \in \sum_{V_j \in \bar{V}} C(V_j)$ must satisfy $x_1 \leq \frac{1}{4}$.
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System of Balancing Weights

Definition

For all $S \in P(N)$, let $\chi^S \in \{0,1\}^n$ denote the characteristic vector of S, so that $\chi_i^S = 1$ if $i \in S$ and $\chi_i^S = 0$ otherwise.

Definition

A function $\delta : P(N) \to \mathbb{R}_+$ is a system of balancing weights if $\sum_{S \in 2^N} \delta(S)\chi^S = \chi^N$.

- Interpretation:
  - Each agent is endowed with one unit of time.
  - A system of balancing weights is an allocation of the agents' time among the different coalitions, where $\delta(S)$ is the fraction of time devoted to coalition S.
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The core of V is non-empty if and only if every system of balancing weights, $\delta(S)$, satisfies $V(N) \geq \sum_{S \in P(N)} \delta(S) V(S)$.

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Systems of Balancing Multi-weights

Definition

A function $\tilde{\delta} : P(N) \times N \times \bar{V} \to \mathbb{R}_+$ is a system of balancing multi-weights if it satisfies the following requirements,

1. Zero to Non-members:
$\forall V_j \in \bar{V}, \forall i \in N, \forall S \in P_{-i}(N) : \tilde{\delta}(S, i, V_j) = 0.$
2. Resource Exhaustion:
$\forall V_j \in \bar{V} : \sum_{i \in N} \sum_{S \in 2^N} \tilde{\delta}(S, i, V_j)\chi^S = \chi^N.$
3. Constant Shares:
$\forall i \in N, \forall V_j, V_{j'} \in \bar{V} : \sum_{S \in 2^N} \tilde{\delta}(S, i, V_j)\chi^S = \sum_{S \in 2^N} \tilde{\delta}(S, i, V_{j'})\chi^S.$

Denote the set of all systems of balancing multi-weights by Δ.

Interpretation of Systems of Balancing Multi-weights

- Each agent is endowed with one unit of time per issue.
- In every issue V_j, the planner is in charge of allocating the time resources among the agents - $\{\alpha_{1j}, \ldots, \alpha_{nj}\}$ where $\alpha_{ij} \in [0, 1]^n$.
- Such allocations must satisfy $\sum_{i \in N} \alpha_{ij} = \chi^N$ (Resource Exhaustion).
- Agent i in issue V_j then chooses the amount of time, $\tilde{\delta}(S, i, j)$ to be devoted to the various coalitions S in which she participates (Zero to Non-members).
- $\alpha_{ij} = \sum_{S \in P(N)} \tilde{\delta}(S, i, j) \chi^S$ implies that the agent exhausts the resources allocated to her (Resource Exhaustion).
- The planner's allocations are identical across issues (Constant Shares).

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Non-emptiness Theorem

Theorem

The multi-core of $\bar{V}$, is non-empty if and only if every $\tilde{\delta} \in \Delta$ satisfies

$$\sum_{V_j \in \bar{V}} V_j(N) \geq \sum_{V_j \in \bar{V}} \sum_{i=1}^{n} \sum_{S \in P(N)} \tilde{\delta}(S, i, V_j) V_j(S)$$

Proof

Types of Systems of Balancing Multi-weights

Definition

A function $\tilde{\delta} : P(N) \times N \times \bar{V} \to \mathbb{R}_+$ is a system of unconstrained balancing multi-weights if it satisfies Zero to Non-members and Resource Exhaustion. (Δ_{UC}).

Definition

A system of multi-weights, $\tilde{\delta}$, satisfies Constant Allocations if $\forall V_j, V_{j'} \in \bar{V} : \tilde{\delta}(S, i, V_j) = \tilde{\delta}(S, i, V_{j'})$.

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Generalized Non-Emptiness

Definition (Extended Bondareva-Shapley condition)

A system of balancing multi weights $\tilde{\delta}(S, i, j)$ satisfies the Extended Bondareva-Shapley (EBS) condition if

$$\sum_{V_j \in V} V_j(N) \geq \sum_{V_j \in V} \sum_{i=1}^{n} \sum_{S \in P(N)} \tilde{\delta}(S, i, j) V_j(S)$$

Proposition

1. $\sum_{V_j \in \bar{V}} C(V_j) \neq \emptyset$ iff every $\tilde{\delta} \in \Delta_{UC}$ satisfies the EBS condition.
2. $M(\bar{V}) \neq \emptyset$ iff every $\tilde{\delta} \in \Delta$ satisfies the EBS condition.
3. $C(\sum_{V_j \in \bar{V}} V_j) \neq \emptyset$ iff every $\tilde{\delta} \in \Delta_{CA}$ satisfies the EBS condition.

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Interpretation of Non-Emptiness Results

- The available information in the problem is mapped to the restrictions placed upon the planner and the agents.
- Ignorance regarding the structure of the game corresponds to restricting agents to choose among identical allocations.
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Variations on the Multi-Core

- Constrain the agents to have identical beliefs over coalitional payoffs.
  - A mediator may wish to avoid incompatibilities.
  - Falls strictly between the sum of the cores and the multi-core.
- Constrain a subset of agents to hold the same beliefs.
  - A subset of agents employs a single representative.
  - Falls strictly between the sum of the cores and the multi-core.
- Consent can be achieved even if the justification matrices are such that for each issue and for each coalition only one member is satisfied.
  - If its the same member across issues, it falls between the multi-core and core of the sum of games.
  - Otherwise it may be weaker than the core of the sum of games.

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- We provide a Matlab code that implements all above mentioned solution concepts:
  - Check for non-emptiness.
  - Verify that a given payoff vector supports the formation of the grand coalition.

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Concluding Remarks

- Linking the issues together is often proposed as a mechanism for successful negotiations.
- The Multi-Core allows linkage while retaining the knowledge of the structure of the individual games.
- However, the agents are ignorant of the issue-by-issue decomposition of the aggregate payoffs.
- The Multi-Core lies between two extreme solution concepts.
- The Multi-Core may not be useful for very “easy” problems. However, it is useful for a wide set of “hard” problems.

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  - Inderst (2000), In and Serrano (2003, 2004) and In (2006) focus on settings where the agenda is endogenous.
  - Bac and Raff (1996) and Busch and Horstmann (1999b) discuss incomplete information regarding time preferences.
- Repeated games:
  - Blonski and Spagnolo (2003); Spagnolo (2001) and Perez (2005) show that linkage sustains cooperation.
  - Conconi and Perroni (2002) discuss the relation between the set of linked issues and agreements' stability.
- Mechanism design of private-values buyer-seller problem:
  - McAfee et al. (1989), Avery and Hendershott (2000), Eilat and Pauzner (2011) and Fang and Norman (2010) demonstrate that with linkage the designer has more enforcement power. ◂ Back

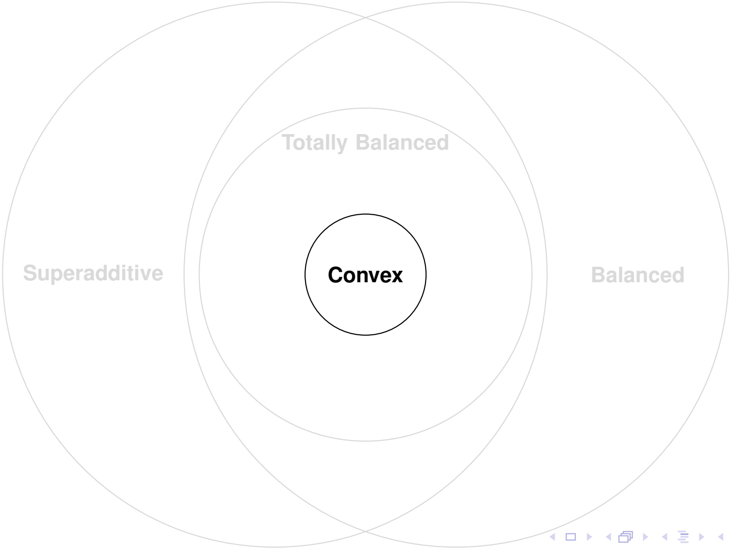
Relevant Cooperative Literature

Back

- Bloch and de Clippel (2010) - Characterizing the relation between $C(\sum_{V_j \in \bar{V}} V_j)$ and $\sum_{V_j \in \bar{V}} C(V_j)$.
- Fernández et al. (2002, 2004) - weighted sum of characteristic functions.
- Nax (2014) and Diamantoudi et al. (2013) - externalities between the issues (deviation in all issues at once).
- Assa et al. (2014) - multiple issues, one membership.

All the problems are “easy” - Facts

◂ Definitions ◂ Conclusions



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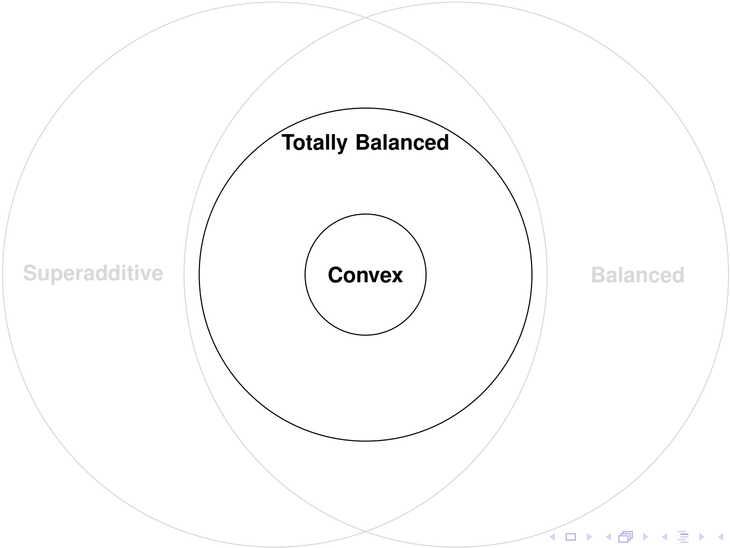

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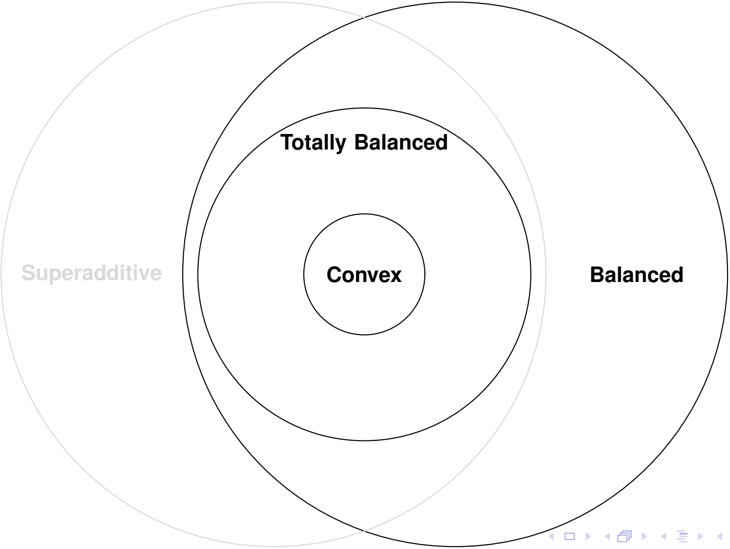

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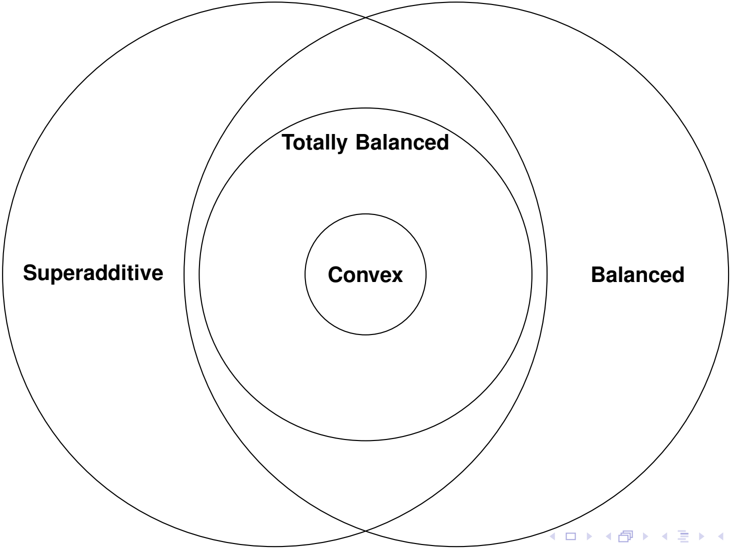

Multi-Core and the Core of Sum - Proof

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Proposition

$$M(\bar{V}) \subseteq C(\sum_{V_j \in \bar{V}} V_j).$$

- If $M(\bar{V}) = \emptyset$ the statement is vacuously true.
- Otherwise, let $x \in M(\bar{V})$,
  - $\sum_{i=1}^{n} x_i = \sum_{V_j \in \bar{V}} V_j(N)$. x is an efficient payoff vector in $\sum_{V_j \in \bar{V}} V_j$.
  - Denote the justification matrix of Player i by y^i.
  - For every coalition S, every $i \in S$ satisfies

$$\sum_{k \in S} x_k = \sum_{k \in S} \sum_{V_j \in \bar{V}} y^i_{k,j} = \sum_{V_j \in \bar{V}} \sum_{k \in S} y^i_{k,j} \geq \sum_{V_j \in \bar{V}} V_j(S)$$

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$$V_1(S) = \begin{cases} 0 & \text{if} \quad |S| = 1 \\ \frac{3}{4} & \text{if} \quad |S| = 2 \\ 1 & \text{if} \quad |S| = 3 \end{cases} \quad ; \quad V_2(S) = \begin{cases} 0 & \text{if} \quad |S| = 1 \\ 0 & \text{if} \quad |S| = 2 \\ 1 & \text{if} \quad |S| = 3 \end{cases}$$

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Non Emptiness - Proof (Part 1)

- Linear program:

$$\min_{x \in \mathbb{R}^n} \sum_{i=1}^{n} x_i$$

$$\text{subject to:} \quad \forall i, l \in N : \sum_{V_j \in \bar{V}} y^i_{l,j} = x_l$$

$$\forall i \in N, \forall V_j \in \bar{V}, \forall S \in P_i(N) : \sum_{l \in S} y^i_{l,j} \geq V_j(S)$$

The multi-core is non-empty iff $\sum_{i=1}^{n} \bar{x}_i \leq \sum_{V_j \in \bar{V}} V_j(N)$.

- Some Algebra to eliminate the payoff vector.
- The asymmetric dual problem:

$$\max_{z \in \mathbb{R}^{nm2^{n-1}}} b'z$$

$$\text{subject to:} \quad A'z = c \quad , \quad z \geq 0$$

By the Strong Duality Theorem, the multi-core is non-empty iff every $z \in \mathbb{R}_+^{nm2^{n-1}}$ such that $A'z = c$ satisfies $b'z \leq \sum_{V_j \in \bar{V}} V_j(N)$.

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Non Emptiness - Proof (part 2)

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- Let $Z = \{z \in \mathbb{R}_+^{nm2^{n-1}} | A'z = c\}$.
- It turns out that Z is identical to Δ.
- b is a vector of characteristic functions' values.
- Therefore, the multi-core is non-empty if and only if every system of balancing multi-weights satisfies

$$\sum_{V_j \in \bar{V}} V_j(N) \geq \sum_{V_j \in \bar{V}} \sum_{i=1}^{n} \sum_{S \in 2^N} \tilde{\delta}(S, i, V_j) V_j(S)$$

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3 Agents Balanced Superadditive - Proof (Part 1)

- Definitions:
  - Let $F : P(N) \to \mathbb{R}_+$ be a system of weights.
  - Let $W^F = \sum_{S \in 2^N} F(S)\chi^S$.
  - F_1 and F_2 are W-equivalent if $W^{F_1} = W^{F_2}$.
  - Γ is the set of all W-equivalence classes.
  - For every $\gamma \in \Gamma$, the agents' weights are denoted by W^γ.
  - For every V and γ, $T_V^\gamma \equiv \max_{F \in \gamma} \sum_{S \in P(N)} F(S)V(S)$.
- Insight 1:
  - Let $F \in \gamma$.
  - Construct F' by subtracting weight α from S and from T $(S \cap T = \emptyset)$ and add weight α to $S \cup T$.
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- Definitions:
  - Let $F : P(N) \to \mathbb{R}_+$ be a system of weights.
  - Let $W^F = \sum_{S \in 2^N} F(S)\chi^S$.
  - F_1 and F_2 are W-equivalent if $W^{F_1} = W^{F_2}$.
  - Γ is the set of all W-equivalence classes.
  - For every $\gamma \in \Gamma$, the agents' weights are denoted by W^γ.
  - For every V and γ, $T_V^\gamma \equiv \max_{F \in \gamma} \sum_{S \in P(N)} F(S)V(S)$.
- Insight 1:
  - Let $F \in \gamma$.
  - Construct F' by subtracting weight α from S and from T ($S \cap T = \emptyset$) and add weight α to $S \cup T$.
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  - Then, $F' \in \gamma$.
  - If V is superadditive and balanced:
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- Insight 3 (uses insights 1 and 2)
  - Let V be a balanced, superadditive, three agents game.
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- W.L.O.G: Let γ be such that $W^\gamma[1] \geq W^\gamma[2] \geq W^\gamma[3]$.
- Let $x \in M(\bar{V})$.
- Using Insight 3 and the existence of a justification matrix for Agent 1 we show that $\sum_{i \in N} W^\gamma[i] x_i \geq \sum_{V_j \in \bar{V}} T^\gamma_{V_j}$.
- This is true for every class of systems of weights.
- By a result from Gayer et al. (2014), x can be decomposed to elements in the cores of the individual games.
- $x \in \sum_{V_j \in \bar{V}} C(V_j)$.
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