Parametric Recoverability of Preferences

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November 2016

New Data

- Emerging experimental literature on choices from budget sets.
- Two advantages on most previous consumer choice data
  - Large individual level data sets.
  - Controlled environment (e.g. price variation).
- For example:
  - **Risk** - Choi et al. (2007a), Choi et al. (2014), Cappelen et al. (2015).
  - **Ambiguity** - Ahn et al. (2014).
  - **Altruism** - Andreoni and Miller (2002), Fisman et al. (2007), Korenok et al. (2013), Fisman et al. (2015a), Fisman et al. (2015b), Porter and Adams (2015).
  - **Time Preference** - Andreoni and Sprenger (2012).
  - **Goods** - Harbaugh et al. (2001), Camille et al. (2011), Burghart et al. (2013).

Motivation

- These rich individual level data sets enable the elicitation of the distribution of behavioral parameters.
- We wish to provide a tool for eliciting approximate stable preferences parametrically based on the theory of Revealed Preference.
- Outline:
    - **Theory:** Introduce a loss function based on Revealed Preference theory.
    - **Data:** Choi et al. (2007a) reveals considerable differences between the proposed method and a standard distance-based method.
    - **Experiment:** Novel design to compare the two methods.
    - **Back to the data:** “Hypothesis testing”.

Data Set

$D = \left\{ \left(p^i, x^i\right)_{i=1}^{n} \right\}$ is a finite data set, where $x^i \in \Re_{+}^{K}$ is the DM's chosen bundle at prices $p^i \in \Re_{++}^{K}$ (income is normalized to 1).

Revealed Preference Relations

Definition

An observed bundle x^i is

1. **Directly Revealed Preferred** to a bundle x, denoted $x^i R_D^0 x$ if $p^i x^i \geq p^i x$.
2. **Strictly Directly Revealed Preferred** to a bundle x, denoted $x^i P_D^0 x$ if $p^i x^i > p^i x$.
3. **Revealed Preferred** to a bundle x, denoted $x^i R_D x$ if there exists a sequence of observed bundles $(x^j, x^k, \ldots, x^m)$ such that $x^i R_D^0 x^j, x^j R_D^0 x^k, \ldots, x^m R_D^0 x$ (transitive closure).

Rationalizability and GARP

Definition

A utility function $u(x)$ rationalizes D if for every observed bundle x^i, $u(x^i) \geq u(x)$ for all x such that $x^i R_D^0 x$.

Definition (Generalized Axiom of Revealed Preference)

D satisfies GARP if $x^i R_D x^j$ then $\neg \left(x^j P_D^0 x^i \right)$.

Afriat's Theorem (1967)

Theorem (Afriat (1967), Diewert (1973), Varian (1982a), Teo and Vohra (2003), Fostel et al. (2004) and Geanakoplos (2013).)

The following conditions are equivalent:

1. *There exists a non-satiated utility function that rationalizes the data.*
2. *The data satisfies GARP.*
3. *There exists a non-satiated, continuous, concave, monotonic utility function that rationalizes the data.*

- Additional condition: The existence of a piecewise linear utility function that rationalizes the data set (constructive).
- Checking data for GARP is easy (e.g. Varian (1982a)).

Inconsistent Subjects

- By Afriat's Theorem if D is inconsistent with GARP then it cannot be rationalized by a non-satiated utility function.
- The proportion of consistent subjects is substantial (above 25%).
- However, there are many subjects that do not satisfy GARP.

Generalized Revealed Preference Relations

Definition

Let $\mathbf{v} \in [0,1]^n$. An observed bundle $x^i \in \Re_+^K$ is

1. $\mathbf{v}-$*directly revealed preferred* to a bundle $x \in \Re_+^K$, denoted $x^i R^0_{D,\mathbf{v}} x$, if $v^i p^i x^i \geq p^i x$ or $x = x^i$.
2. $\mathbf{v}-$*strictly directly revealed preferred* to a bundle $x \in \Re_+^K$, denoted $x^i P^0_{D,\mathbf{v}} x$, if $v^i p^i x^i > p^i x$.
3. $\mathbf{v}-$*revealed preferred* to a bundle $x \in \Re_+^K$, denoted $x^i R_{D,\mathbf{v}} x$, if there exists a sequence of observed bundles $(x^j, x^k, \ldots, x^m)$ such that $x^i R^0_{D,\mathbf{v}} x^j, x^j R^0_{D,\mathbf{v}} x^k, \ldots, x^m R^0_{D,\mathbf{v}} x$.

Fact

Let $\mathbf{v}' \leq \mathbf{v}$*. Then:* $R^0_{D,\mathbf{v}'} \subseteq R^0_{D,\mathbf{v}}$, $P^0_{D,\mathbf{v}'} \subseteq P^0_{D,\mathbf{v}}$ and $R_{D,\mathbf{v}'} \subseteq R_{D,\mathbf{v}}$.

$GARP_v$ and v-Rationalizability

Definition

Let $\mathbf{v} \in [0,1]^n$. D satisfies the *General Axiom of Revealed Preference Given* $\mathbf{v}$ *($GARP_{\mathbf{v}}$)* if for every pair of observed bundles, $x^i R_{D,\mathbf{v}} x^j$ implies not $x^j P^0_{D,\mathbf{v}} x^i$.

Fact

Let $\mathbf{v}, \mathbf{v}' \in [0,1]^n$ *and* $\mathbf{v} \geq \mathbf{v}'$. *If* D *satisfies* $GARP_{\mathbf{v}}$ *then* D *satisfies* $GARP_{\mathbf{v}'}$.

Definition

Let $\mathbf{v} \in [0,1]^n$. A utility function $u(x)$ $\mathbf{v}-$*rationalizes* D, if for every observed bundle $x^i \in \Re^K_+$, $x^i R^0_{D,\mathbf{v}} x$ implies that $u(x^i) \geq u(x)$. We say that D is $\mathbf{v}-$*rationalizable* if such $u(\cdot)$ exists.

Generalized Afriat's Theorem

Theorem

The following conditions are equivalent:

1. *There exists a non-satiated utility function that $\mathbf{v}-$rationalizes the data.*
2. *The data satisfies $GARP_{\mathbf{v}}$.*
3. *There exists a continuous, monotone and concave utility function that $\mathbf{v}-$rationalizes the data.*

- Afriat (1973) provides a non-constructive proof for the uniform case.
- Afriat (1987) states the theorem without a proof.
- In his unpublished PhD dissertation Houtman (1995) considers non-linear pricing (using constructive proof).
- We adapt this construction to our setting.

◂ Proof

Inconsistency Indices

- Non-parametric measure for the extent of deviation from utility maximizing behavior implied by a data set of consumer choices.
- In this work we focus on three well-known indices:
  - Varian Inconsistency Index (Varian (1990)).
  - Afriat Inconsistency Index (Afriat (1972, 1973)).
  - Houtman-Maks Inconsistency Index (Houtman and Maks (1985)).
- There are other indices in the literature.
- The indices require aggregation over observations.

Definition

$f_n : [0,1]^n \to [0, M]$, where M is finite, is an *Aggregator Function* if $f_n(\mathbf{1}) = 0$,$f_n(\mathbf{0}) = M$ and $f_n(\cdot)$ is continuous and weakly decreasing.

Varian Inconsistency Index

Example

- The minimal adjustments of the budget sets that remove cycles implied by choices.
- We follow Alcantud et al. (2010) and Varian (1990) and use the Euclidean norm of the adjustments vector (Smeulders et al. (2014) suggest the generalized mean).

Definition

Let $f : [0,1]^n \to [0, M]$ be an aggregator function. *Varian's Inconsistency Index* is,

$$I_V(D, f) = \inf_{\mathbf{v} \in [0,1]^n : D \text{ satisfies } GARP_{\mathbf{v}}} f(\mathbf{v})$$

Afriat Inconsistency Index

- Originally, “Critical Cost Efficiency Index”.
- Allows for uniform adjustments only.
- Denote the set of vectors with equal coordinates by $\mathcal{I} = \left\{ \mathbf{v} \in [0,1]^n : \mathbf{v} = v\mathbf{1}, \forall v \in [0,1] \right\}$.
- Denote a coordinate of a typical vector v $\in \mathcal{I}$ by v.

Definition

Afriat's Inconsistency Index is,

$$I_A(D) = \inf_{\mathbf{v} \in \mathcal{I} : D \text{ satisfies } GARP_{\mathbf{v}}} 1 - v$$

Houtman-Maks Inconsistency Index

- The maximal subset of observations that satisfies *GARP*.
- Identical to restrict the adjustments vector to belong to $\{0,1\}^n$ and to aggregate using the sum.

Definition

Let $f : [0,1]^n \rightarrow [0, M]$ be an aggregator function. *Houtman-Maks Inconsistency Index* is,

$$I_{HM}(D, f) = \inf_{\mathbf{v} \in \{0,1\}^n : D \text{ satisfies } GARP_{\mathbf{v}}} f(\mathbf{v})$$

Interpretation

- Behavioral interpretations to income adjustments:
  - Wasted income - Afriat (1972) and Varian (1982b, 1990, 1993).
  - Measurement error - Varian (1985),Tsur (1989),Cox (1997).
  - Consideration sets - Houtman (1995), Manzini and Mariotti (2007), Masatlioglu et al. (2012), Apesteguia and Ballester (2015) and others.
- We remain agnostic.
- Adjustments serve as a measurement tool.

Parametric Approach

- **Simplicity**:
  - The generalized Afriat theorem constructs a well behaved utility function that v-rationalizes the data.
  - But, requires $2n$ parameters.
- **Non Convex Preferences**:
  - Varian (1982b) constructs non parametric bounds for the indifference curves assuming convex preferences
  - Halevy et al. (2016) provide bounds without this assumption.
  - These bounds are “weak”.
- **Inconsistent Subjects**:
  - The generalized Afriat theorem applies for every adjustments vector v.
  - Varian’s bounds require consistency.
- **Individual Level Analysis**:
  - Non parametric revealed preferences-based random utility models are better interpreted on a population level data.

Outline

- Let u be a utility function proposed to represent the subject's preferences.
- D satisfies GARP: Mis-specification is the tension between the ranking implied by u and the (partial) ranking implied by the D.
- This requires an incompatibility measure.
- D does not satisfies GARP: the tension between the ranking implied by u and the information in D contains both mis-specification and inconsistency.
- This requires some decomposition of the incompatibility measure to mis-specification and inconsistency.

The Money Metric Vector

- Based on the Money Metric Utility Function (Samuelson, 1974).
- Suggested by Varian (1990) and Gross (1995).

Definition

The *normalized money metric vector for a utility function* $u(\cdot)$, $\mathbf{v}^{\star}(D,u)$, is such that

$$v^{\star i}(D,u) = \frac{m(x^i, p^i, u)}{p^i x^i}$$

where

$$m(x^i, p^i, u) = min_{\{y \in \Re^K_+ : u(y) \geq u(x^i)\}} p^i y$$

The Money Metric Incompatibility Index

Definition

Let $f : [0,1]^n \to [0, M]$ be an aggregator function.
The *Money Metric Index for a utility function* $u(\cdot)$ is $f(\mathbf{v}^\star(D,u))$.

Minimality of the MMI

- Let $\mathcal{U}^c$ denote the set of all locally non-satiated, acceptable and continuous utility functions on $\Re_+^K$.

Proposition

Let $D = \left\{ (p^i, x^i)_{i=1}^n \right\}$, $u \in \mathcal{U}^c$ and $\mathbf{v} \in [0,1]^n$.
$u(\cdot)$ $\mathbf{v}$-rationalizes D if and only if $\mathbf{v} \leqq \mathbf{v}^\star(D,u)$.

◂ Proof 3

- The Money Metric Index is minimal.
- The Money Metric Index is easy to compute.
- When $\mathbf{v}^\star(D,u) = \mathbf{1}$ the utility function is correctly specified.

The MMI for a Set of Utility Functions

Definition

Let D be a finite data set, let $f(\cdot)$ be an aggregator function and let $\mathcal{U} \subseteq \mathcal{U}^c$ be a set of continuous and locally non-satiated utility functions.
The Money Metric Index of $\mathcal{U}$ is

$$I_M(D, f, \mathcal{U}) = \inf_{u \in \mathcal{U}} f\left(\mathbf{v}^\star\left(D, u\right)\right)$$

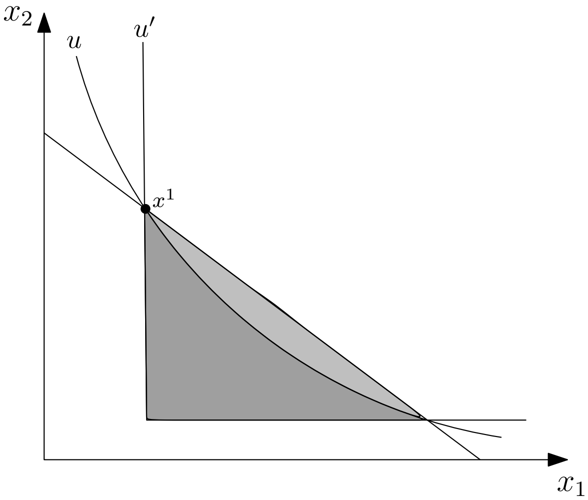
Note that for every $\mathcal{U}' \subseteq \mathcal{U}$:

$$I_M(D, f, \mathcal{U}) \leq I_M(D, f, \mathcal{U}')$$

Therefore, for every $\mathcal{U} \subseteq \mathcal{U}^c$:

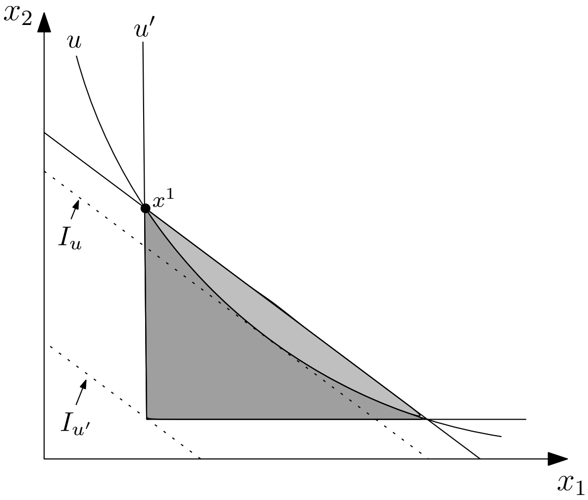
$$I_M(D, f, \mathcal{U}^c) \leq I_M(D, f, \mathcal{U})$$

Example - The Problem



Example - The MMI

Area-Based



Implications

- Consistent Subjects
  - $I_M(D, f, \mathcal{U}^c) = 0$.
  - $I_M(D, f, \mathcal{U})$ is interpreted as a measure of misspecification.
- Inconsistent Subjects
  - By Afriat's Theorem if D is inconsistent with GARP then it cannot be rationalized by any non-satiated utility function.
  - $I_M(D, f, \mathcal{U})$ no longer a measure of misspecification only, it includes inconsistency as well.

The Binary Incompatibility Vector

- All incompatibilities are treated severely.
- The Binary Incompatibility Index may be used in more general settings of choice from menus.

Definition

The *Binary Incompatibility vector for a utility function* $u(\cdot)$*,* is $\mathbf{b}^{\star}(D, u)$., is such that

$$b^{\star i}(D, u) = \begin{cases} 1, & \nexists x : p^i x^i \geq p^i x, u\left(x\right) > u\left(x^i\right); \\ 0, & \text{Otherwise.} \end{cases}$$

The Binary Incompatibility Index

Definition

Let $f : [0,1]^n \to [0, M]$ be an aggregator function.
The *Binary Incompatibility Index for a utility function* $u(\cdot)$ is $f\left(\mathbf{b}^{\star}\left(D, u\right)\right)$.

Proposition

Let $D = \left\{\left(p^i, x^i\right)_{i=1}^n\right\}$, $u \in \mathcal{U}^c$ and $\mathbf{b} \in \{0,1\}^n$. $u\left(\cdot\right)$ $\mathbf{b}$-rationalizes D if and only if $\mathbf{b} \leq \mathbf{b}^{\star}(D, u)$.

- The Binary Index is minimal.
- The Binary Index is easy to compute.
- When $\mathbf{b}^{\star}\left(D, u\right) = \mathbf{1}$ the utility function is correctly specified.

The BI for a Set of Utility Functions

Definition

Let D be a finite data set, let $f(\cdot)$ be an aggregator function and let $\mathcal{U} \subseteq \mathcal{U}^c$.
The Binary Index of $\mathcal{U}$ is

$$I_B(D, f, \mathcal{U}) = \inf_{u \in \mathcal{U}} f\left(\mathbf{b}^\star\left(D, u\right)\right)$$

Note that for every $\mathcal{U}' \subseteq \mathcal{U}$:

$$I_B(D, f, \mathcal{U}) \leq I_B(D, f, \mathcal{U}')$$

Therefore, for every $\mathcal{U} \subseteq \mathcal{U}^c$:

$$I_B(D, f, \mathcal{U}^c) \leq I_B(D, f, \mathcal{U})$$

The Decomposition of the Incompatibility Indices

Theorem

For every finite data set D and aggregator function f:

1. $I_V(D, f) = I_M(D, f, \mathcal{U}^c)$.
2. $I_{HM}(D, f) = I_B(D, f, \mathcal{U}^c)$.
3. *If* $f(\mathbf{v}) = 1 - \min_{i\in\{1,\ldots,n\}} v^i$, *then* $I_A(D) = I_M(D, f, \mathcal{U}^c)$.

Brief Overview

Implications of the Decomposition Theorem

- We get:

$$I_M(D,f,\mathcal{U}) = I_V(D,f) + (I_M(D,f,\mathcal{U}) - I_M(D,f,\mathcal{U}^c))$$
$$I_B(D,f,\mathcal{U}) = I_{HM}(D,f) + (I_B(D,f,\mathcal{U}) - I_B(D,f,\mathcal{U}^c))$$

- The former is a measure of inconsistency within choices that is independent of any parametric restriction and depends only on the DM.
- The latter is a measure of the misspecification induced by restricting the preferences to a specific parametric form by the researcher.
- Enables to compare misspecification within and between functional forms since the inconsistency index is fixed.

◂ Example

Choi et al. (2007a) - Decisions under Uncertainty

- Two states of nature (equally probable, exhaustive) and two associated Arrow securities, each of which promises a payoff of one unit in one state and nothing in the other.
- Each choice problem is characterized by different security prices.
- Each subject encounters 50 choice problems (the endowment is fixed).
- Graphical interface (the chosen bundle must be on the budget line).
- 47 subjects, 12 satisfy GARP.

Screenshot Typical Subject

Choi et al. (2007a) - Functional Form

Disappointment Aversion (Gul (1991)) with CRRA VNM utility function.
In our case this reduces to

$$u(x^i) = \gamma w\left(\max\left\{x_1^i, x_2^i\right\}\right) + (1-\gamma) w\left(\min\left\{x_1^i, x_2^i\right\}\right)$$

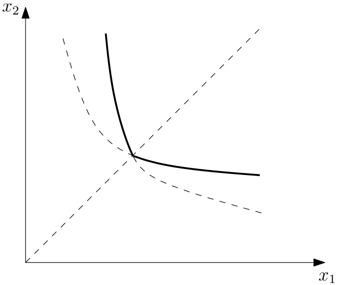
where

$$\gamma = \frac{1}{2+\beta} \qquad \beta > -1$$

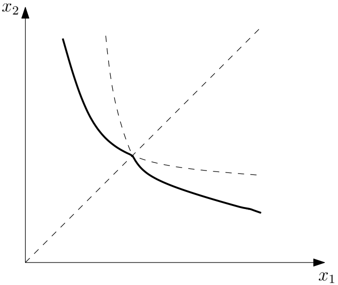
and

$$w(z) = \begin{cases} \frac{z^{1-\rho}}{1-\rho} & \rho \geq 0 \quad (\rho \neq 1) \\ ln(z) & \rho = 1 \end{cases}$$

Indifference Curves



(a) Disappointment Aversion: $\beta > 0$.



(b) Elation Seeking: $-1 < \beta < 0$.

Figure: Gul (1991) with CRRA.

- $\beta = 0$ is Expected Utility.
- $\beta = 0$ and $\rho = 0$ is Expected Value.
- We also consider $w(z) = -e^{-Az}$ where $A \geq 0$ (CARA).

Two Recovery Methods

1. NLLS (distance based loss function):

$$\min_{\beta,\rho} \sum_{i=1}^{n} \left\| x^i - \arg \max_{x : p^i x \leq p^i x^i} \left(u\left(x; \beta, \rho\right)\right) \right\|$$

   where $\|\cdot\|$ is the Euclidean norm.

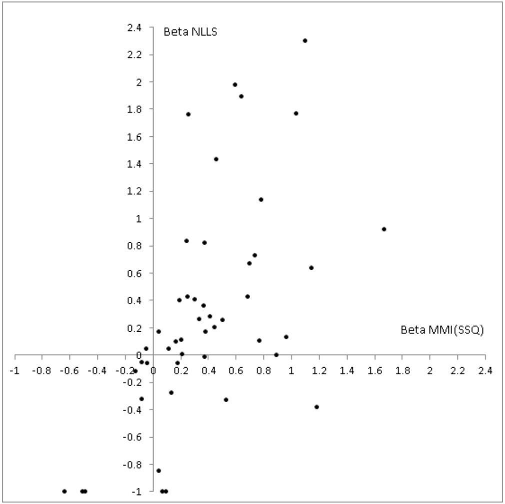
2. MMI: $I_M\left(D, f, \mathcal{U}\right)$, using the normalized average sum-of-squares aggregator, $f\left(\mathbf{v}\right) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(1 - v^i\right)^2}$.

- Reliable Varian Inconsistency Index cannot be provided for 9 of the 47.
- An unreliable index underestimates mis-specification, but is inconsequential for the recovered parameters.

Consistency vs. Mis-specification

Subject	I_V	β	ρ	I_M
320	0	-0.509	0.968	0.1322
209	0.0288	0.164	0.352	0.0563

MMI vs. NLLS


Beta NLLS
Beta MMI(SSQ)
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4

MMI vs. NLLS: Observations

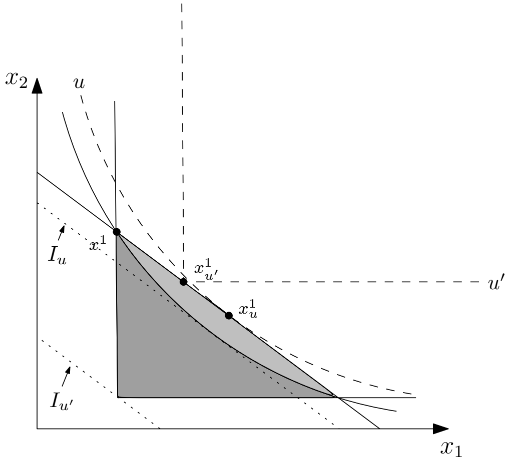
- When NLLS recovers convex preferences ($\beta > 0$) then usually MMI recovers convex preferences (But, quantitative differences).
- When NLLS recovers non-convex preferences ($-1 \leq \beta < 0$), no qualitative relation between the recovered parameters by the two methods.
- In some non-convex cases the NLLS recovers extreme elation seeking.

◂ Distributions

Motivation and Main Idea

- The parameters recovered by the MMI and NLLS are qualitatively and quantitatively different.
- We wish to compare these two methods.
- However, we must avoid using any metric in this comparison.
- Predictive success in pairwise choices is the most natural setting for such a comparison.

Evaluating based on predictions



Part 1: Linear Budget Sets

- Subjects make 22 choices from linear budget sets.
- A bundle is a portfolio of contingent assets with two equally probable states (similar to Choi et al. (2007a)).
- Budget lines are chosen so as to:
  - provide a powerful test of consistency (GARP).
  - identify local risk attitude in the neighborhood of certainty (by over sampling moderate price ratios).

◂ Power

Part 1.5: Recovery and Pairwise Choice Construction

For each subject, in the background and without her knowledge:

- We recover parameters using the MMI and NLLS:
  - DA-CRRA functional form.
  - Similar loss functions to those used earlier.
- Then, we construct pairwise choice sets designed to separate the two sets of parameters.
  - Each pair included one *risky* portfolio, where outcomes differed across states, and one *safe* portfolio.

Part 2: Pairwise Choice

- Subjects make choices from 9 pairwise menus (represented as points in the coordinate system).
- By construction, for all choice problems, one of the portfolios is preferred by one set of parameters and the other portfolio by the other set of parameters.
- Recall that each choice is between a risky portfolio and a safe (certain) portfolio. We over-sampled low-variability portfolios to identify local risk attitudes.

Details

- Location: Experimental Lab at the Vancouver School of Economics (ELVSE) in October 2014 and February 2015.
- Who: 203 UBC undergraduate students.
- Duration: approximately 45 minutes including instructions, the experiment, and payment.
- Each subject made 31 choices. One of these choices was selected randomly to be paid (the state was determined by a coin flip).
- Cost: average payment was $29.53 CAD including a $10 show-up fee

Results

- We first report the results of the second part.
- We report results at both the individual level and the aggregate level.
- Our report includes all subjects and all their choices (a refinement that provides similar results is reported in the draft).

Aggregate Results

203 subjects:

	# of Observations	Correct Predictions by MMI (%)	*p*-value
Complete Sample	1827	986 (54.0%)	0.0004
Low-variability	1218	652 (53.5%)	0.0074
High-variability	609	334 (54.8%)	0.0093

p-value: probability that X or more out of x choices are predicted correctly by chance alone (coin flip)

Individual Results

- X: number of correct prediction my MMI.
- Decisive subject: $X \in \{0, 1, 2, 7, 8, 9\}$.
- The probability for a subject being decisive by chance is 18%.
- For 103 out of 203 subjects, one prediction method is decisively better than the other (likelihood under random prediction is close to 0).

$X \geq 7$	$X \leq 2$	p-value
61	42	0.0378

Classification by Disappointment Aversion

- Let us divide the subjects into two groups:
  1. The Definite Disappointment Averse (DDA) group - 150 subjects with $\beta_{MMI}, \beta_{NLLS} \geq 0$.
  2. The Indefinite Disappointment Averse (IDA) group - 53 subjects with $\beta_{MMI} < 0$ or $\beta_{NLLS} < 0$ or both.

DDA and IDA: Aggregate Analysis

	# Observations	# Correct Predictions by MMI	% Correct Predictions by MMI	*p*-value
DDA	1350	706	52.3%	0.0484
IDA	477	280	58.7%	< 0.0001

DDA and IDA: Individual Analysis

DDA (150)			IDA (53)		
$X \geq 7$	$X \leq 2$	p-value	$X \geq 7$	$X \leq 2$	p-value
38	30	0.1981	23	12	0.0448

Elation Seeking

- The Definite Elation Seeking (DES) group: 29 subjects with $\beta_{NLLS}, \beta_{MMI} < 0$.
- MMI predicts correctly: 163/261 (62.5%, $p < 0.0001$).
- 20 of 29 subjects are decisive.
- MMI decisively better predictor in 15/20 ($p = 0.0207$).
- Thus, the MMI recovers a significantly more accurate representation of subject preferences when the underlying preferences are non-convex.
- For 21 of 29 subjects: $\beta_{NLLS} < \beta_{MMI} < 0$ (for 19/21 the difference is more than 0.1).
- For 6 of 8 subjects for which $\beta_{MMI} < \beta_{NLLS} < 0$, the difference is less than 0.1.

Illustrative Discussion

- Consider the case where choices exhibit non-convex preferences (maybe due to some underlying procedure) and the DA family is mis-specified.
- The NLLS usually picks parameters that imply greater non-convexity than those recovered by the MMI.
- Very informally:
  - NLLS implies “closer is better” achieved by extreme non-convexities.
  - MMI implies “smoother is better” that requires weak non-convexities.
- In fact, as the subject’s choices drift farther from the certainty line, the greater is the difference between the recovered parameters.
- Bottom Line: The parameters recovered by the MMI are considerably more successful in prediction.

Examples Illustration

Decomposition Revisited

- Non Nested Model:
  - Suppose $\mathcal{U}$ and $\mathcal{U}'$ are two parametric families.
  - Then, their respective MMI loss indices are $I_M(D, f, \mathcal{U}')$ and $I_M(D, f, \mathcal{U})$.
  - Recall, they share the same level of inconsistency ($I_V(D, f)$).
  - By the Decomposition Theorem, the data set D may be better approximated by $\mathcal{U}$ or $\mathcal{U}'$ depending on the magnitude of the MMI loss index.
- Nested Models:
  - By the monotonicity of the MMI, an additional parametric restriction on preferences increases misspecification.
  - Then, the difference between the MMI indices is a measure of the marginal misspecification implied by the restriction.
- We will use both the data of Choi et al. (2007a) and the data of Part 1.

Evaluating Misspecification

	Part 1 of the Experiment		Choi et al. (2007a)	
Original Sample	203 subjects		47 subjects	
Consistent	92 (45%)		12 (26%)	
Dropped	3 (1.5%)		9 (19%)	
Inconsistency Level	at most 6%		at most 2.5%	
Utility index	**CRRA**	**CARA**	**CRRA**	**CARA**
# of Subjects with at most 5% misspecification	136 (68%)	127 (63.5%)	26 (68.4%)	23 (60.5%)
# of Subjects with at least 10% misspecification	4 (2%)	10 (5%)	3 (7.9%)	6 (15.8%)
Subjects for whom misspecification is more than 90% of the MMI	149 (74.5%)	153 (76.5%)	26 (68.4%)	27 (71.1%)
Subjects for whom misspecification is less than 50% of the MMI	0 (0 %)	0 (0 %)	1 (2.6%)	1 (2.6%)

- Mis-specification: $I_M(D, f, \mathcal{U}) - I_V(D, f)$ where f is the SSQ aggregator.
- The sample slightly over-represents the less inconsistent subjects.

Expected Utility

- Expected utility is nested within the disappointment aversion model, satisfying the restriction that $\beta = 0$.
- Relative measure of additional misspecification:

$$\gamma = \frac{I_M(D, f, EU) - I_M(D, f, DA)}{I_M(D, f, DA) - I_V(D, f)}$$

- Expected utility is rejected if $\gamma > 10\%$.
- Re-samplings were calculated, but cannot be interpreted as confidence sets.
- Subjects with incomputable Varian Index were dropped, as well as subjects for whom DA is not a reasonable model.

	Part 1 of the Experiment	**Choi et al. (2007a)**
CRRA	40.8% (80 of 196)	32.4% (11 of 34)
CARA	44.7% (85 of 190)	45.2% (14 of 31)

Model Selection DA-CRRA vs. DA-CARA

- We calculate the extent of mis-specification implied by each functional form and select the functional form that better represents the decision maker's preferences.
- Absolute measure of additional mis-specification:

$$I_M(D, f, DA - CARA) - I_M(D, f, DA - CRRA)$$

	Part 1 of the Experiment	Choi et al. (2007a)
Full Sample	71.4% (145 of 203)	80.9% (38 of 47)
Restricted Sample	88% (103 of 117)	80% (24 of 30)

- The second row includes subjects whose Varian Index is computable and the difference in mis-specification is greater than 10%.

Concluding Remarks

- A novel interpretation of some inconsistency indices.
- A general recovery method based on minimizing the incompatibility between the ranking information encoded in choices and the ranking induced by a candidate model.
- Application of this methodology to individual level risk data.
- A comparison to a distance-based method shows considerable differences in elicited preferences.
- Novel experimental design to compare the two methods by their predictive success.
- The proposed method predicts better than the NLLS, especially when preferences are non-convex.
- Mis-specification is more “important” than inconsistency.
- Roughly 40% are well approximated by Expected Utility.
- Next step: The integration of stochastic component.

Thanks


THANK
YOU
GRACIAS
ARIGATO
SHUKURIA
DANKSCHEEN
TASHAKKUR ATU
YAQHANYELAY
SUKSAMA
EKHMET
GRAZIE
MEHRBANI
BIYAN
SHUKRIA
TINGKI
BOLZIN
MERCI
GOZAIMASHITA
EFCHARISTO
JUSPAXAR
KOMAPSUMNIDA
MAAKE
PALDIES

Proof: First Step

Back

- First, suppose there exists a locally non-satiated utility function $u(\cdot)$ that $\mathbf{v}$-rationalizes D.
- If D does not satisfy $GARP_{\mathbf{v}}$ then there are two observed bundles x^i, x^j such that $x^i R_{D,\mathbf{v}} x^j$ and $x^j P^0_{D,\mathbf{v}} x^i$.
- Therefore, $u(x^i) \geq u(x^j)$ and by local non-satiation $u\left(x^j\right) > u\left(x^i\right)$. Contradiction.
- It is left to be shown that if D satisfies $GARP_{\mathbf{v}}$ then there exists a well behaved utility function that $\mathbf{v}$-rationalizes D.

Proof: Relation-Rationalize

Back

- We say that $\succeq$ **v**-relation-rationalizes D if $R^0_{D,\mathbf{v}} \subseteq \succeq$ and $P^0_{D,\mathbf{v}} \subseteq \succ$.
- We use Szpilrajn (1930) extension theorem to show that D satisfies $GARP_{\mathbf{v}}$ if and only if there exists a transitive and reflexive $\succeq$ such that $\succeq$ **v**-relation-rationalizes D.
- Thus, we have to show that for every data set D and adjustments vector **v**, if $\succeq$ is transitive and reflexive and **v**-relation-rationalizes D then there exists a well behaved utility function that **v**-rationalizes D.
- Our proof is constructive.

Proof: Construction Lemma

Back

- Consider a family of real functions $\{(z_i)_{i=1}^n\}$ (one for each observation).
- Define: $x^i R x^j \Leftrightarrow z_i(x^j) \leq 0$ and $x^i P x^j \Leftrightarrow z_i(x^j) < 0$.
- Suppose $\succeq$ is transitive and reflexive and satisfies $R \subseteq \succeq$ and $P \subset \succ$.
- We provide an algorithm to construct

$$f(x) = \min_{i \in \{1,\ldots,n\}} f_i + \lambda_i z_i(x)$$

  such that $\lambda_i > 0$ and $f(x^i) \geq f_i$.
- To complete the proof we have to:
  1. Choose $\{(z_i)_{i=1}^n\}$ such that $\succeq$ **v**-relation-rationalizes D.
  2. Show that $f(x)$ **v**-rationalizes D and is well behaved.

Proof: Initial Functions Choice

Back

- We choose $z_i(x) = \frac{1}{v_i} p^i x - p^i x^i$ if $x \neq x^i$ and zero otherwise.
- Since R is $R^0_{D,\mathbf{v}}$ and P is $P^0_{D,\mathbf{v}}$ we get that $\succeq$ $\mathbf{v}$-relation-rationalizes D.
- Also, it is easy to show that $f(\cdot)$ $\mathbf{v}$-rationalizes D.
- However, z_i are discontinuous at x^i when $v_i < 1$ and therefore f is not continuous.

Proof: Adaptation

Back

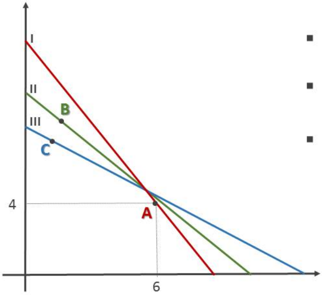
- We redefine $\hat{z}_i(x) = \lim_{y \to x} z_i(y)$. then $\hat{z}_i(x) \geq z_i(x)$ for $x = x^i$ and $\hat{z}_i(x) = z_i(x)$ otherwise.
- We consider $\hat{f}(x) = \min_{i \in \{1,\ldots,n\}} f_i + \lambda_i \hat{z}_i(x)$ where f_i and λ_i are the same as in f.
- We show that $\hat{f}(x)$ **v**-rationalizes D, it is continuous, acceptable, monotonic and concave.

Example - Data

Back

3 given budget sets, marked I, II & III

The chosen allocations marked A, B & C

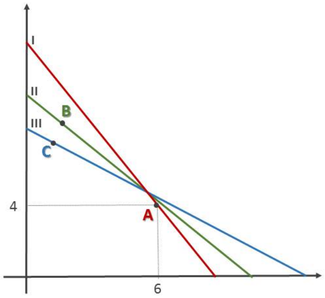


- **Obs I:** Income = 10, $P_1 = 1$, $P_2 = 1$ — **A** (6 , 4)
- **Obs II:** Income = $9\frac{7}{27}$, $P_1 = \frac{1}{2}$, $P_2 = \frac{4}{3}$ — **B** ($2\frac{14}{27}$, 6)
- **Obs III:** Income = $11\frac{7}{8}$, $P_1 = \frac{1}{4}$, $P_2 = 2$ — **C** ($1\frac{11}{14}$, $5\frac{10}{14}$)

Example - Violations

Back

In the chosen allocations one can identify the following violations:



R^0 / P^0

- B R^0 A
- B R^0 C
- A R^0 B
- A R^0 C
- C R^0 A

R

- B R A
- B R C
- A R B
- A R C
- C R A
- C R B

GARP Violations

- (B, A)
- (A, B)
- (A, C)
- (C, A)
- (C, B)

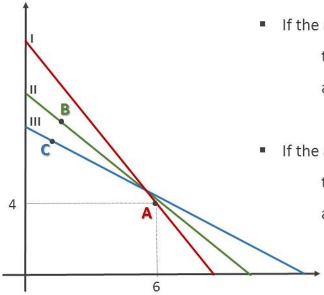
Two alternatives to resolve the violations (there are others):

1. Move line I so it will go through C. Then vI_c=0.75.
2. Move both lines II and III so they will go through A. Then vII_a=0.9 and vIII_a=0.8.

Example - Two Options

Back

Two possible aggregators to calculate the severity of the violations



- If the aggregator is $\mathbf{F} = \sum(1 - v)$

  then option (1) gives (1-0.75) = 0.25

  and option (2) gives [(1-0.9) + (1-0.8)] = 0.3

  Hence, option 1 is chosen, since it requires less adjustments

- If the aggregator is $\mathbf{G} = \sum (1 - v)^2$

  then option (1) gives $(1\text{-}0.75)^2 = 0.0625$

  and option (2) gives $[(1\text{-}0.9)^2 + (1\text{-}0.8)^2] = 0.05$

  Hence, option 2 is chosen, since it requires less adjustments

Proof

back

- Suppose $u(\cdot)$ $\mathbf{v}^{\star}(D,u)$ - rationalizes D.
- Hence, If $\mathbf{v} \leqq \mathbf{v}^{\star}(D,u)$ then $u(\cdot)$ $\mathbf{v}$ - rationalizes D.
- The other direction - Suppose that $\mathbf{v}$ is such that $u(\cdot)$ $\mathbf{v}$ - rationalizes D and for observation i, $v^i > v^{\star i}(D,u)$.
- Let $x^{i\star}$ be the minimizer of the money metric and note that it is strictly feasible under v^i and $u(x^{i\star}) \geq u(x^i)$.
- By the non satiation of $u(\cdot)$ there exists a bundle that is strictly feasible under v^i and is strictly better than x^i.
- Contradiction to $u(\cdot)$ $\mathbf{v}$ - rationalizes D.

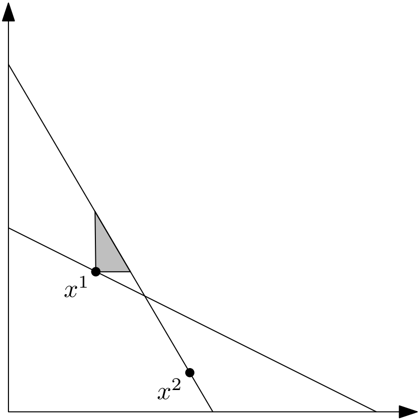
Area Based Parametric Recoverability

- Intersection Incompatibility Index.
- Apesteguia and Ballester (2015) suggest the Consumer Setting Swaps Index as an extension of the Minimal Swaps Index.
- A corresponding inconsistency measure, a decomposition theorem and a broader family of utility functions are required.
- Area Inconsistency Index - eliminate the area of overlap between the budget set and those bundles which are revealed preferred or monotonically dominate the bundle (Heufer (2008, 2009)).
- Two remarks:
    - Computation of integrals is much harder than linear adjustments.
    - Biased towards non-convex preferences.

Inconsistency Index

Area Inconsistency Index

Back



Proof: $I_V(D, f) \leq I_M(D, f, \mathcal{U}^c)$

The Converse

- If $I_V(D, f) = 0$ then $I_V(D, f) \leq I_M(D, f, \mathcal{U}^c)$.
- Otherwise, if $I_V(D, f) > 0$, suppose that $I_V(D, f) > I_M(D, f, \mathcal{U}^c)$.
  - There exists $u \in \mathcal{U}^c$ such that $f(\mathbf{v}^\star(D, u)) < I_V(D, f)$.
  - $u(\cdot)$ $\mathbf{v}^\star(D, u)$-rationalizes D.
  - By the extended Afriat theorem, D satisfies $GARP_{\mathbf{v}^\star(D,u)}$.
  - $I_V(D, f)$ cannot be the infimum of $f(\cdot)$ on the set of $\mathbf{v}$ such that D satisfies $GARP_{\mathbf{v}}$. Contradiction.

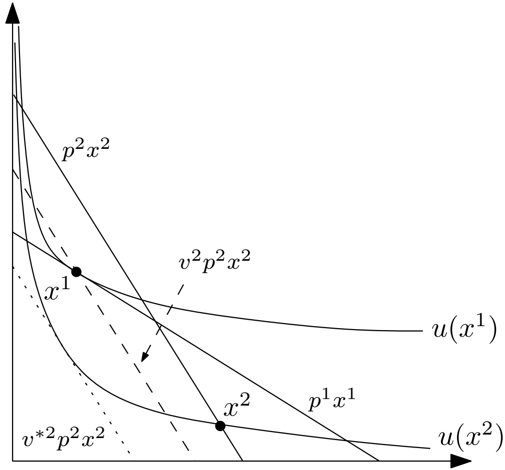
Proof: $I_V(D,f) \geq I_M(D,f,\mathcal{U}^c)$

Back

- By the extended Afriat theorem, D satisfies $GARP_{\mathbf{v}}$ if and only if there exists $u \in \mathcal{U}^c$ that $\mathbf{v}$-rationalizes D.
- Hence, D satisfies $GARP_{\mathbf{v}}$ if and only if $\mathbf{v} \leq \mathbf{v}^{\star}(D,u)$.
- Since $f(\cdot)$ is weakly decreasing, D satisfies $GARP_{\mathbf{v}}$ if and only if $f(\mathbf{v}^{\star}(D,u)) \leq f(\mathbf{v})$.
- Therefore, D satisfies $GARP_{\mathbf{v}}$ if and only if $I_M(D,f,\mathcal{U}^c) \leq f(\mathbf{v})$.
- $I_M(D,f,\mathcal{U}^c) \leq \inf_{\mathbf{v}\in[0,1]^n : D \text{ satisfies } GARP_{\mathbf{v}}} f(\mathbf{v})$
- $I_V(D,f) \geq I_M(D,f,\mathcal{U}^c)$.

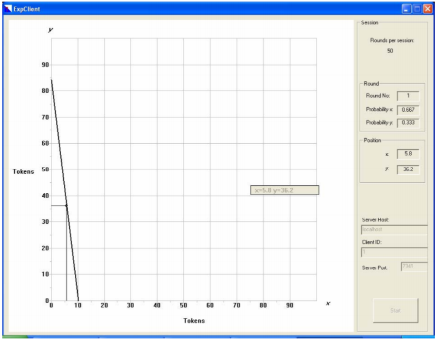
Example - Decomposition of MMI

Back



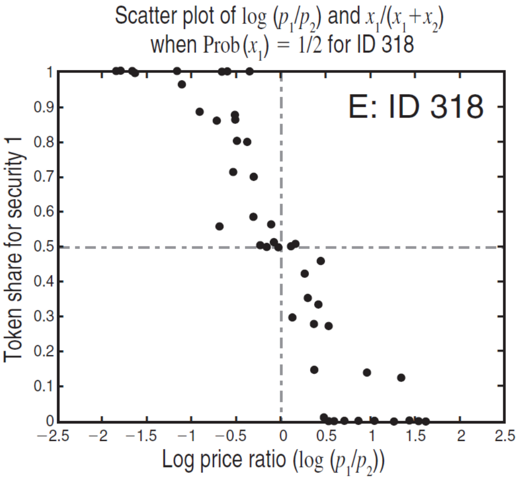
Screenshot

Back



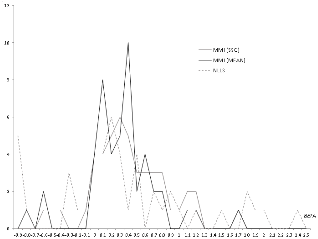
Typical Subject

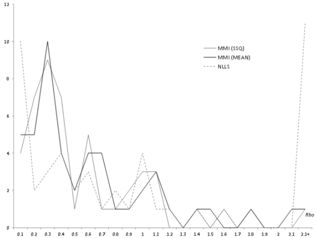
Back



CRRA Parameters: Distributions (Choi et al. (2007a))

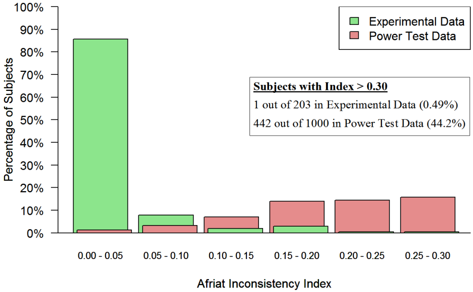
Back





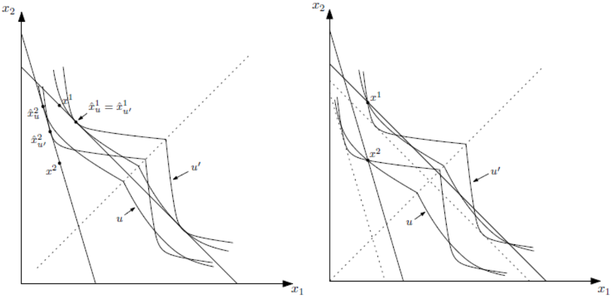
power

Back


100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Percentage of Subjects
Experimental Data
Power Test Data
Subjects with Index > 0.30
1 out of 203 in Experimental Data (0.49%)
442 out of 1000 in Power Test Data (44.2%)
0.00 - 0.05
0.05 - 0.10
0.10 - 0.15
0.15 - 0.20
0.20 - 0.25
0.25 - 0.30
Afriat Inconsistency Index

Non-convex Preferences: MMI vs. NLLS

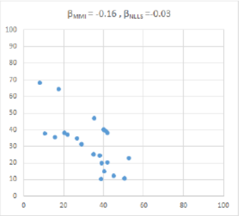
Back



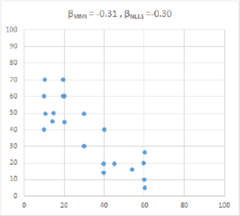
(a) NLLS (b) MMI

Non-convex Preferences: 4 Examples

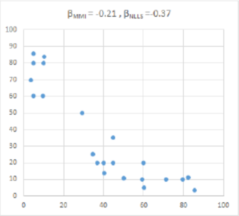
Back



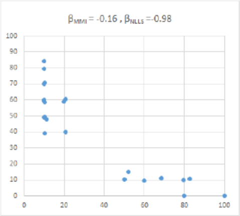
(a) Subject 1203



(b) Subject 1512



(c) Subject 2203



(d) Subject 301