Core-Periphery R&D Collaboration Networks*

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Abstract

This work offers an explanation to the emergence of core-periphery architectures in real life R&D collaborations networks. R&D collaboration is viewed as a costly platform on which ideas flow between collaborating firms. As a consequence, R&D collaborations form a communication network where firms trade-off between the benefits of centrality and the costs of collaboration. Heterogeneity is introduced, where a high-type firm is better than a low-type firm in both innovation and execution. For intermediate collaboration costs, core-periphery networks where the core includes only the superior firms are the efficient networks. The stable networks are characterized and failures are emphasized.

Keywords: R&D Collaborations Network, Strategic Network Formation, Core-Periphery.

JEL Classification: L14, L24, D85.

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1 Introduction

Inter-firm collaborations on research and development activities produce knowledge spillovers between the involved parties. The network of R&D collaborations, where two firms are connected if they collaborate on some R&D activity, is therefore a major facilitator of knowledge dissemination (e.g. Grossman and Shapiro (1986)). This work introduces a strategic model for the formation of R&D collaborations networks by firms that strive for the fastest possible access to knowledge.

An R&D collaborations network obtains a core-periphery structure if its set of nodes can be partitioned into two subsets, the core and the periphery, such that each node in the core is directly connected to all other core members, while each periphery member is directly connected to none of the other periphery nodes.[1] Empirical evidence suggests that in many cases R&D collaboration networks are organized as core-periphery networks. Tomasello et al. (2013), for example, state that since the beginning of the 1990s "Across sectors, firms show the tendency to organize their R&D collaborations in a core of densely connected companies and a periphery of companies that are linked to the core, but only weakly interconnected

[1]The core-periphery structure is not a well-defined concept in the literature. Generally, it describes a network in which there is one group of agents that is densely connected internally, while all the other agents are sparsely connected among themselves (Borgatti and Everett, 1999). Our definition is the one used by Bramoullé and Kranton (2005) and Bramoullé (2007) (split graphs in the graph theory literature, see Foldes and Hammer (1977) and Merris (2003)). Other definitions restrict the pattern of links between the core agents and the periphery agents (e.g. Goyal (2007), Hojman and Szeidl (2008), Galeotti and Goyal (2010), Tomasello et al. (2013)). In addition, some architectures, mentioned in the literature, are special cases of core-periphery networks (e.g. the dominant group in Goyal and Joshi (2003) and Westbrock (2010) and the nested split graphs in König et al. (2012) and König et al. (2013)).

among themselves" (page 25).[2]

This work focuses on the role of R&D collaborations as platforms that provide the firm with innovative information produced elsewhere in the network. I show that if firms weave their R&D collaborations in order to get new information as fast as possible, core-periphery networks emerge as both stable and efficient for intermediate collaboration costs.

To gain intuition, suppose that periodically, one of the firms comes up with an innovative idea. Other firms get the information regarding this new idea through the collaborations network, with a delay that increases with their distance to the source. Naturally, only upon arrival can they exploit the new idea and therefore the firm's benefit from an incoming piece of information decreases with its delay relative to the initial transmission. Hence, firms establish costly R&D collaborations to be located at the centre of communication. Since all collaborations are assumed to convey information with the same speed, the firm's centrality is determined by its closeness to all other firms, measured by geodesic distances.

Firms are heterogeneous in their ability to innovate and to exploit innovation (in any other respect, such as information transmission speed or direct collaborations costs, the firms are identical). High-type firms generate innovative ideas with higher frequency than low-type firms. Therefore, I assume that all firms, all else being equal, prefer a connection (direct or indirect) with a high-type firm over a shortest path of the same length with a low-type firm. In addition, the probability

[2]See also Baker et al. (2008) on R&D collaboration among biotechnology and pharmaceutical firms (in particular, Figure 2), Autant-Bernard et al. (2007) on micro and nanotechnologies, Bojanowski et al. (2012) on the international R&D network and the brief review in Goyal (2007). The core-periphery architecture is considered to be an efficient spreader of knowledge (Borgatti (2005)). For other properties of this architecture see Chubin (1976), Krackhardt and Hanson (1993), Dodds et al. (2003), Cummings and Cross (2003), Bramoullé and Kranton (2005) and Carlsson and Jia (2013).

to transform an idea into actual innovation is higher for high-type firms than for low-type firms. Therefore, all else being equal, high-type firms find a connection to a given firm to be more valuable than a low-type firm, independent of the targeted firm's type.[3]

To capture these ideas heterogeneity is introduced into the homogeneous connections model of Jackson and Wolinsky (1996).[4] It turns out that for intermediate collaboration costs, core-periphery networks where the high-type firms form the core and the low-type firms form the periphery are both pairwise stable and strongly efficient (maximize total benefits). Within this range, the pattern of mixed collaborations, where one firm is of high-type and the other is of low-type, is sensitive to the collaboration costs. When the costs are low, the unique stable and efficient network is where each low-type firm is connected to all high-type firms, while when the costs are high, the stable and efficient network is such that low-type firms are isolated. In between these levels of collaboration costs, there is some discrepancy between pairwise stability and strong efficiency.[5] The efficient pattern is where all the low-type firms are linked to the same high-type firm (the

[3]In the literature these advantages are sometimes referred to as absorptive capacities (e.g. Autant-Bernard et al. (2007)).

[4]The type heterogeneity approach is taken by only few studies in the literature. In the context of strategic formation of directed networks, Hojman and Szeidl (2006) characterize the conditions under which a single gifted agent becomes the center of a stable star and Galeotti (2006), Galeotti et al. (2006) and Billand et al. (2011, 2012) introduce continuous heterogeneity both in the costs and in the benefits assuming there is no decay (or small amount of decay). Zeggelink (1995) introduces type heterogeneity into a dynamic network formation model where agents target an ideal number of alike friends. Bianconi and Barabási (2001) introduce individual fitness level into the standard non-strategic preferential attachment model. The other approach for introducing heterogeneity is by conditioning the linking costs on some geographic distance between the agents (e.g. Johnson and Gilles (2000), Carayol and Roux (2003), Jackson and Rogers (2005), Galeotti et al. (2006), Hojman and Szeidl (2006) and Galeotti and Goyal (2010)). These two approaches differ since linking costs heterogeneity affects only direct connections, while type heterogeneity is carried through both direct and indirect connections.

[5]For a general discussion on stability-efficiency tension in strategic network formation models see Jackson (2008).

gate) and only to it. However, in some cases, every other pattern of collaboration where every low-type firm collaborates with only one high-type firm is also stable. In other, worse, cases, no single high-type firm internalizes the importance of holding the low-type firms connected, and the only stable core-periphery network is the one where the low-type firms are completely isolated.

Clearly, the gap between high-type firms and low-type firms widens due to the strategic formation of R&D collaborations. Not only do the high-type firms possess an exogenous advantage in the production and exploitation of innovation, but they are able to translate this advantage into a positional advantage in the R&D collaborations network. The central position of the high-type firms in the R&D collaboration network enables these firms to get informed faster than any of the low-type firms on any new piece of information.[6] Therefore, while the strategic formation of R&D collaborations may have positive effect on the speed and quality of knowledge dissemination, it might also have an adverse effect on the size and competitiveness of markets.

The main strand of the literature on the strategic formation of R&D collaborations networks, initiated by Goyal and Moraga-González (2001) and Goyal and Joshi (2003), focuses on collaboration as a platform for exploiting complementarities. In these models, the firms form costly collaborations and then decide on their individual efforts (that spillover to their collaborators). Theses actions determine the firms' marginal production costs and a market competition takes place.[7] The main non-trivial architectures that emerge as stable in these models are the

[6] For related observations see Kadushin (2012) and Footnote 3 in Hojman and Szeidl (2008).

[7] Transfers, heterogeneity, direct links, government intervention and various competition structures are studied within this literature by Goyal and Moraga-González (2003), Billand and Bravard (2004), Goyal (2005, 2007), Deroian and Gannon (2006), Song and Vannetelbosch (2007), Goyal et al. (2008), Deroian (2008) and Westbrock (2010).

dominant group (a clique and isolates) and interlinked stars (connected, highly concentrated hierarchy). Goyal (2007) claims that introducing transfers and heterogeneity into such models, core-periphery networks may also emerge.

Conceptually, König et al. (2012) is the closest work to the one introduced here. While both models view R&D collaboration as a platform for knowledge dissemination, they differ on the use the firms make with this knowledge. König et al. (2012) view firms as knowledge accumulators, while I model firms as perusing the first mover advantage. As a result in their model, profit maximization leads firms to establish R&D collaborations so that the number of their walks is maximized. They show that for intermediate collaboration costs trend, connected nested split graphs (highly hierarchical form) are strongly efficient but not pairwise stable for large networks.

The following section introduces the model, which is analysed thoroughly in the third section. In Section 4 I discuss some possible extensions.

2 The Model

Preliminaries

Let $N = \{1, 2, \ldots, n\}$ be a finite set of firms. Denote the set of all possible collaborations by $g^N = \{\{i, j\} | i, j \in N, i \neq j\}$. Denote by ij the element of g^N that contains i and j. $ij \in g$ is interpreted as firms i and j collaborate on some R&D activity, and therefore are *directly connected* in the network g. The set of all possible networks on N is $G = \{g | g \subseteq g^N\}$. I refer to g^N as the *complete R&D collaborations network* and to the empty set as the *empty R&D collaborations network*. Denote by $N(i, g) = \{j | ij \in g\}$ the set of Firm i's *neighbors* in the R&D

collaboration network g and denote its cardinality by n_i. Let $g + ij$ denote the network obtained by adding the collaboration ij to the network g and let $g - ij$ denote the network obtained by removing the collaboration ij from the network g.

A *path* p of *length* $L(p)$ between Firm i and Firm j exists in the R&D collaborations network g, if there is an ordered set of distinct firms $\{i_1, i_2, \ldots, i_{L(p)}, i_{L(p)+1}\}$ such that $\{i_1 i_2, i_2 i_3, \ldots, i_{L(p)} i_{L(p)+1}\} \subseteq g$ and $i_1 = i$, $i_{L(p)+1} = j$. p is a *shortest path* between Firm i and Firm j in the R&D collaborations network g, if there is no other path p' between them such that $L(p') < L(p)$. Denote the set of all shortest paths between Firm i and Firm j in the R&D collaborations network g by $S(i, j, g)$ and its cardinality by s_{ij}. Denote the shortest path's length by d_{ij}. If a path between Firm i and Firm j exists in the R&D collaborations network g, I say that Firm i and Firm j are *connected* in g, otherwise, Firm i and Firm j are *disconnected* in g (and d_{ij} is set to infinity). Two firms are said to be *indirectly connected* in the R&D collaborations network g if they are connected but not directly connected in g. If for every pair of firms $i, j \in N$, Firm i and Firm j are connected in the R&D collaborations network g, we say that g is *connected.* For a subset of firms, $N' \subseteq N$, the *subnetwork* $g' = \{ij | i, j \in N', ij \in g\}$ is a *component* of g if it is connected and there is no pair of firms $i \in N'$ and $k \in N \backslash N'$ such that $ik \in g$. Let $\tilde{N}(i, g)$ denote the set of firms that reside in the same component as Firm i in the R&D collaborations network g.

An R&D collaborations network g is a *star* if g is connected and $\exists i \in N$ such that $\forall kj \in g : i \in \{k, j\}$. An R&D collaborations network g is a *core-periphery* network if there is a partition of N into two subsets K, the *core*, and L, the *periphery* ($K \cup L = N, K \cap L = \emptyset$), such that $\forall i, j \in K : ij \in g$ while $\forall l, m \in L : lm \notin g$. Various classes of core-periphery networks are characterized by the pattern of the direct connections between the core firms and the periphery firms. Denote the *local core* of a periphery firm, $l \in L$, by $LC_l = \{k|kl \in g, k \in K\}$ and its size by lc_l. Similarly, denote the *local periphery* of a core firm, $k \in K$, by $LP_k = \{l|kl \in g, l \in L\}$ and its size by lp_k. An R&D collaborations network g is a *disconnected core-periphery* network if g is a core-periphery network and $\forall l \in L : LC_l = \emptyset$.[8] An R&D collaborations network g is a *maximally connected core-periphery* network if g is a core-periphery network and $\forall l \in L : LC_l = K$.[9] An R&D collaborations network g is a *minimally connected core-periphery* network if g is a core-periphery network and $\forall l \in L : lc_l = 1$.[10] An R&D collaborations network g is a *one-gate minimally connected core-periphery* network if g is a minimally connected core-periphery network and $\exists k \in K : LP_k = L$. Agent k is referred to as the *gate*.[11] Note that, given N, K and L, the disconnected core-periphery network, the maximally connected core-periphery network and the one-gate minimally connected core-periphery network are unique up to labeling. Also, note that this classification is not exhaustive - there are many core-periphery networks that belong to none of these classes. Figure 1 summarizes the core-periphery networks' classification.

[8] Many authors refer to this architecture as the dominant group architecture (e.g. Goyal and Joshi (2003) and Westbrock (2010)).

[9] Every maximally connected core-periphery network with $|K|$ core firms and $|L|$ periphery firms, can be identified also as a core-periphery network with $|K| + 1$ core firms and $|L| - 1$ periphery firms. Fortunately, this ambiguity bears no consequence on the following analysis. The maximally connected core-periphery architecture is equivalent to the two-layer inter-linked stars in Goyal and Joshi (2003), Hojman and Szeidl (2006) and Westbrock (2010). Also, it is equivalent to the core-periphery architecture as mentioned in Galeotti and Goyal (2010).

[10] This architecture is frequently referred to as hub-spokes (e.g. Hendricks et al. (1995)).

[11] This architecture is equivalent to the nested star architecture in König et al. (2012).

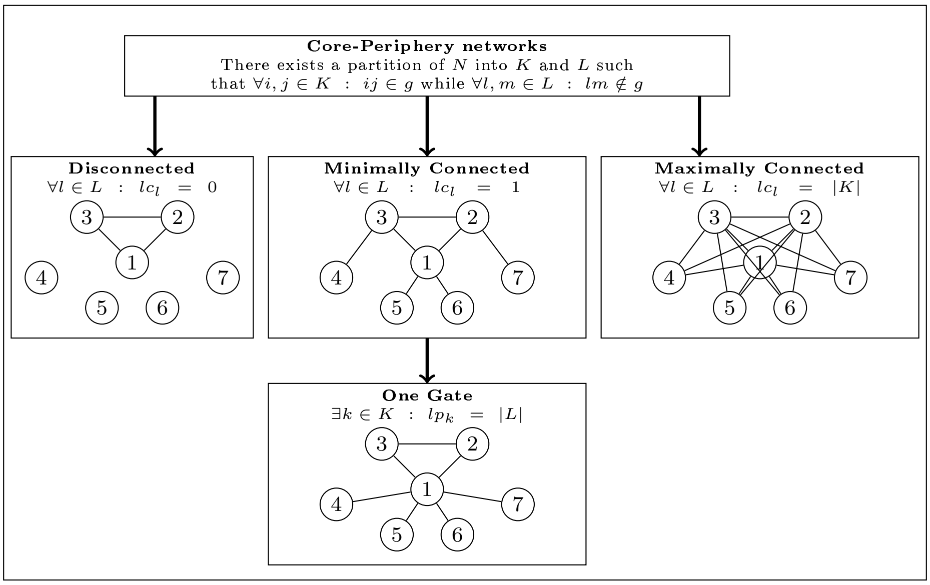

Figure 1: The classification of core-periphery networks. Firms 1, 2 and 3 belong to the core while the others belong to the periphery. The network shown in the minimally connected box is only a representative of this class. Firm 1 is the gate in the minimally connected one gate network.

The Firm's Problem

There are $n^h \geq 1$ high-type firms and $n^l \geq 1$ low-type firms ($n^h + n^l = n$). The value firm i, of type $t_i \in \{h, l\}$, gets from the R&D collaborations network g is

$$V_i(g) = \sum_{j \neq i} [\delta^{d_{ij}} \times w_{t_j}^{t_i}] - n_i \times c \tag{1}$$

$0 < \delta < 1$ is interpreted as the exogenous level of delay in knowledge spillovers in the industry. The assumption that the firms are equally good at information mediation is captured by $\delta^{d_{ij}}$ being independent of the types of the firms along the shortest path.[12] The discrete values, $w_{t_j}^{t_i}$, serve as a reduced form for the value

[12]Any distance based depreciation measure that qualifies for the analysis of distance based utility functions in Propositions 6.1 and 6.2 in Jackson (2008) provides the same results as the exponential form used here.

Firm j provides for Firm i given both firms' types. These values are positive and independent of the path that connects the two firms (which is captured by $\delta^{d_{ij}}$). High-type firms are characterized by having better innovation and execution capabilities, than low-type firms. The advantage in innovation capabilities translates into $w_h^h > w_l^h$ and $w_h^l > w_l^l$, since both types get higher value from a high-type firm than from a low-type firm, everything else equal (including the length of the path between the firms). The advantage in execution capabilities translates into $w_h^h > w_h^l$ and $w_l^h > w_l^l$, since, given a collaborator, and holding everything else equal, a high-type firm gets higher value than a low-type firm from every collaboration. For simplicity we assume that $w_l^h = w_h^l = w^{hl}$ (see Section 4.1 for a discussion regarding the implications of this assumption). These assertions are summarized in the following,

Assumption 1. $w_h^h > w_l^h = w^{hl} = w_h^l > w_l^l > 0$.

Last, the assumption that the firms are equally good at forming and maintaining R&D collaborations is captured by $c > 0$, the homogeneous costs of partnerships. These costs may also be interpreted as a reduced form of the effects of direct spillovers on the aggressiveness of the market competition (e.g. D'Aspremont and Jacquemin (1988)). For example, low collaboration costs may represent partnerships in a sector with differentiated goods, while high costs may be used when goods are homogeneous. Next, I provide two additional restrictions that will be necessary for some of the following results.

Two Helpful Assumptions

Firm i's benefits from Firm j, as defined in Equation 1 and Assumption 1, depend both on Firm j's type and on the distance between firm i and Firm j. These benefits imply that if a firm agrees to collaborate with another firm it surely agrees to collaborate with a more distant firm, all else is equal (including the other firm's type and the indirect connections they provide). In addition, it implies that if a firm agrees to collaborate with a low-type firm it surely agrees to collaborate with a high-type firm, all else is equal (including the firms' distance and other connections). However, these benefits do not specify which of the following options provide the firm with higher value - an R&D collaboration with a high-type firm located d_h steps away or a low-type firm positioned d_l steps away where $d_l > d_h$. The following two assumptions (one for each firm type) partially expand on the original benefits structure.

Assumption 2. $(\delta - \delta^2)w_h^h > \delta w^{hl}$.

Assumption 3. $(\delta - \delta^2)w^{hl} > (\delta - \delta^3)w_l^l$.

Assumption 2 states that if a high-type firm agrees to collaborate with a low-type firm with which it has no alternative path, it surely agrees to collaborate with a high-type firm with which an alternative path of length two exists, other things being equal. Assumption 3 is weaker and it states that if a low-type firm agrees to collaborate with another low-type firm with which an alternative path of length three exists, it surely agrees to collaborate with a high-type firm with which an alternative path of length two exists, other things being equal. Alternatively, these assumptions can be viewed as an upper bound on the spillovers quality parameter.[13]

Stability and Efficiency

An R&D collaborations network g is *pairwise stable* if both conditions hold:

$$\forall ij \in g : V_i(g) \geq V_i(g - ij), V_j(g) \geq V_j(g - ij) \tag{2}$$

$$\forall ij \notin g : V_i(g + ij) > V_i(g) \Rightarrow V_j(g + ij) < V_j(g) \tag{3}$$

Thus, in a pairwise stable network, no firm would gain from terminating a partnership (Condition 2) and for every pair of non-collaborating firms, either at least one of the firms strictly loses from forming an R&D collaboration or both do not gain from establishing one (Condition 3).[14]

The *value* of the R&D collaboration network g is the sum of values of the firms, $V(g) = \sum_{i \in N} V_i(g)$. Network g is *strongly efficient* if $\forall g' \subseteq g^N : V(g) \geq V(g')$.

The following analysis mentions repeatedly core-periphery networks where the core includes all the high-type firms and the periphery includes all the low-type firms. I refer to such architectures as *separating core-periphery networks.*

3 Results

This section provides an exhaustive characterization of the model. I proceed by fixing the spillovers rate (δ) and the collaboration quality values (w_h^h, w^{hl} and

[13]Assumption 2 argues that $\frac{w_h^h}{w^{hl}} > \frac{1}{1-\delta}$ while Assumption 3 states that $\frac{w^{hl}}{w_l^l} > 1 + \delta$. For both to hold simultaneously, the spillovers parameter should satisfy $0 < \delta < \min\{1 - \frac{w^{hl}}{w_h^h}, \frac{w^{hl}}{w_l^l} - 1\}$.

[14]A simple generalization of Claim 1 in Calvó-Armengol and İlkılıç (2009) shows that Condition (2) can be relaxed to allow the firm to terminate any subset of its collaborations, without affecting the subsequent results.

w_l^l) and gradually increasing the collaboration costs (c). All proofs are relegated to the appendix.

Extremely Low Collaboration Costs

Proposition 1 shows that for extremely low collaboration costs, the complete R&D collaborations network is both the predicted and the favorable outcome.

Proposition 1. *Let $(\delta - \delta^2)w_l^l > c$. The complete network is the unique pairwise stable and the unique efficient R&D collaborations network.*

The complete R&D collaborations network is the unique pairwise stable network since the collaboration costs are low enough for every pair of firms to prefer a costly collaboration over a free indirect connection. Since the model is of positive externalities,[15] the complete R&D collaborations network is also uniquely efficient. Proposition 1 serves as a baseline for the following results by showing that when the cost of collaboration is very low, for example, in cases where the firms' products are highly differentiated, the architecture of the R&D collaborations network does not reflect any heterogeneity.

Low Collaboration Costs

Next, we analyze the case where the collaboration costs are low, but high enough for two low-type firms to refrain from collaboration provided that the pair has an alternative path of length two between them and that forming a collaboration does not shorten any of their shortest paths to other firms. Collaborations

[15] Firms never suffer from the formation of collaborations with which they are not involved. Formally, $\forall g, \forall ij \notin g, \forall k \in N \backslash \{i,j\} : V_k(g+ij) \geq V_k(g)$. For a proof see Lemma 1 in the Appendix. Similar definition can be found in Buechel and Hellmann (2012). Note that in the literature on the formation of cost-reducing R&D collaboration networks, profits decrease in other firms' collaborations since any such collaboration intensifies competition.

that involve at least one high-type firm are worthwhile, in this costs range, even if the pair shares an alternative path of length two.

Proposition 2. *Let $(\delta - \delta^2)w^{hl} > c > (\delta - \delta^2)w^l_l$. The separating maximally connected core-periphery network is the unique pairwise stable and the unique efficient R&D collaborations network.*

The separating maximally connected core-periphery network is stable and efficient since the costs are low enough to preserve every collaboration that involves a high-type firm, and too high for a pair of low-type firms to collaborate when an alternative path of length two is available. Already here, high-type firms obtain a positional advantage since every innovative idea that originates in a low-type firm, reaches all high-type firms before the other low-type firms learn about it.

Medium Collaboration Costs

Medium collaboration costs are such that a high-type firm and a low-type firm do not collaborate if they share a path of length two, provided that collaborating does not get them any closer to other firms. At the same time, if a similar pair of firms do not share any alternative path, a collaboration is worthwhile.

Proposition 3. *Suppose that Assumption 2 is satisfied and $\delta w^{hl} > c > (\delta - \delta^2)w^{hl}$.*

1. *The separating one-gate minimally connected core-periphery R&D collaborations network is:*

   (a) *The unique efficient R&D collaborations network.*

   (b) *The only network among the set of separating core-periphery R&D collaborations networks that is pairwise stable for the whole range.*

2. *If Assumption 3 holds, every separating minimally connected core-periphery R&D collaborations network is pairwise stable.*

3. *Every other pairwise stable R&D collaborations network is connected and includes all possible collaborations between high-type firms.*

Proposition 3 relies on Assumption 2 for the cliquishness of the high-type firms, on Assumption 3 for low-type firms to form an independent set and on the collaboration costs range for connectivity.[16] The efficiency result stems from the collaborations between the high-type firms, the minimality of the costs of connectivity and the short paths, of length two at most, to and within low-type firms.

Tomasello et al. (2013) describe the typical R&D collaborations network as a connected network (in many sectors more than 50% of the collaborating firms belong to the main component) where a small number of firms collaborate with a huge number of partners.[17] In addition, they find that R&D collaborations networks exhibit, in most sectors, negative assortativity - low degree firms collaborate with high degree firms. The set of separating minimally connected core-periphery R&D collaborations networks, exhibit an ideal architecture that matches these properties.

The consistency of the separating minimally connected core-periphery R&D collaborations networks with the stylized facts suggests that high-type firms may indeed prefer collaborations with isolated low-type firms over non-isolated low-type firms, trusting their tight network of collaborations with other high-type firms to

[16]Assumption 2 guarantees the attractiveness of high-type firms with a small local periphery. Assumption 3 guarantees that low-type firms are satisfied with an indirect path of length three between them.

[17]See also Baker et al. (2008) and Bojanowski et al. (2012).

provide the missing information fast enough. This trade-off is satisfactory with regard to collaborations with low-type firms that generate new ideas with low frequency. However, to enjoy the high frequency stream of innovations originating from a typical high-type firms, direct collaboration is required. Moreover, these direct collaborations provide short indirect paths to all low-type firms.

Proposition 3 exhibits the first case of tension between probable and favorable networks. Although this tension can be mitigated by a central planner, since the favorable network is also probable, it demonstrates clearly two distinct types of inefficiency. One type of inefficiency, that characterizes the minimally connected R&D networks which are not one gate, is mis-coordination, where a gatekeeper fails to emerge. Over-connectedness, the other type of inefficiency, characterizes some of the pairwise stable non core-periphery networks.[18] The following remark demonstrates one type of over-connectedness:

Remark 1. *Let $\delta w^{hl} > c > (\delta - \delta^2)w^{hl}$. Suppose that Assumption 2 holds and that Assumption 3 is strengthened to require $(\delta - \delta^2)w^{hl} > \delta w_l^l$. Let g be a pairwise stable R&D collaborations network. Any low-type firm that has no collaborations with a high-type firm, maintains at least two collaborations (no loose ends).*

Figure 2 demonstrates four examples of non core-periphery networks which are pairwise stable for some parameters consistent with the medium costs range and with Assumption 2. Network A is inefficient due to coordination failure while the other networks are over-connected. Network B demonstrates over-connectedness due to mis-coordination that leads two low-type firms to collaborate with a third

[18]Note that Buechel and Hellmann (2012) do not regard these networks as over-connected since no improvement to social welfare could result from terminating any of their collaborations. Their notion may be viewed as local over-connectivity, while we use the global over-connectivity notion of Jackson and Wolinsky (1996) which relates to the number of collaborations in the network.

low-type firm instead of collaborating with a high-type firm. Due to this mis-coordination, an attractive low-type firm emerges that forms a collaboration with two high-type firms rather than one. Network C is a pairwise stable network given parameters that satisfy the requirements of Remark 1 and therefore it is over-connected due to no-loose-ends. Network D demonstrates extreme over-connectedness where the low-type firms form a cycle of collaborations to compensate on their spread among the various high-type firms' local periphery.

High Collaboration Costs

The collaboration costs are considered high if a partnership between a high-type firm and a low-type firm is not worthwhile, even if it is the only path between them and if collaboration between two high-type firms is worthwhile even if otherwise they have a path of length two between them. Such a range of collaboration costs exists if and only if Assumption 2 is satisfied. I denote the value of the separating one-gate minimally connected core-periphery network by V(OG) and the value of the separating disconnected core-periphery network by V(DIS). Intuitively, the difference between the total value from separating one-gate minimally connected core-periphery R&D collaborations network and the total value from the separating disconnected core-periphery R&D collaborations network, is the net total industry returns from connecting all the low-type firms into the central component.

Proposition 4. *Suppose Assumption 2 is satisfied and* $(\delta - \delta^2)w_h^h > c > \delta w^{hl}$.

1. *If V(OG)>V(DIS)*

    (a) *The separating disconnected core-periphery R&D collaborations network is the only pairwise stable member of the set of separating core-periphery*

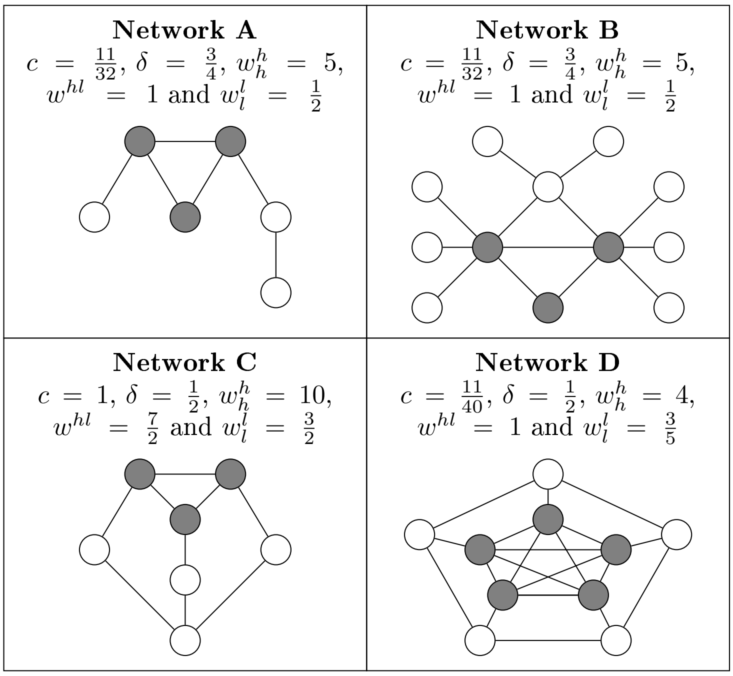

Figure 2: Examples of pairwise stable non core-periphery networks (black circles represent high-type firms and white circles stand for low-type firms). Below each title are the parameters for which the network is pairwise stable.

R&D collaborations networks.

(b) *Every other pairwise stable R&D collaborations network satisfies:*

i. *All high-type firms collaborate with each other.*

ii. *No low-type firm maintains exactly one collaboration.*

iii. *No high-type firm maintains collaborations with all other firms.*

(c) *The separating one-gate minimally connected core-periphery R&D collaborations network is uniquely efficient.*

2. *If V(OG)<V(DIS), the separating disconnected core-periphery R&D collaborations network is the unique pairwise stable and the unique efficient network.*

At relatively low costs in this range, this difference is always positive.[19] Proposition 4 points at a sever inefficiency in this case - while the socially favourable architecture is to incorporate the low-type firms into the main component, this structure is unstable. This result reflects the reluctance of the high-type firms to bear the costs of collaboration with low-type firms that do not supply additional connections, since they fail to internalize the positive externalities embedded in such collaborations.

When the increase in collaboration costs turns V(DIS) to be greater than V(OG) (the existence of such a turnaround within the interval of Proposition 4 depends on the parameters), it is socially favorable to keep the low-type firms away from the main component, and this is indeed the unique stable architecture. This strong result follows from the following. It is unstable for a pair of high-type firms not to collaborate. In addition, in a pairwise stable network all agents must have non-negative utility levels.[20] Thus, every pairwise stable network must have at least as high total value as the separating disconnected core-periphery network. As a consequence of its unique efficiency, it is also uniquely pairwise stable.

An important feature of Proposition 4 is the dependence of the turnaround collaboration costs on the sector size (n) and its composition to high-type and low-type firms. The greater the population, the higher is the social loss from isolated low-type firms and therefore the higher is also the turnaround collaboration costs. Moreover, if, for example, the number of high-type firms is larger than the number of low-type firms, the addition of a high-type firm increases the total social

[19]The difference is $2n^l \times [\delta w^{hl} + (n^h - 1)\delta^2 w^{hl} + \frac{n^l - 1}{2}\delta^2 w_l^l - c]$. Since the lower bound of the high collaboration costs interval is $c = \delta w^{hl}$, if $n > 2$ such lower range always exists.

[20]This was shown formally in a previous version. Here I use a shorter proof for Proposition 4, using a result from Buechel and Hellmann (2012). The longer, but more illuminating proof, is available from the author.

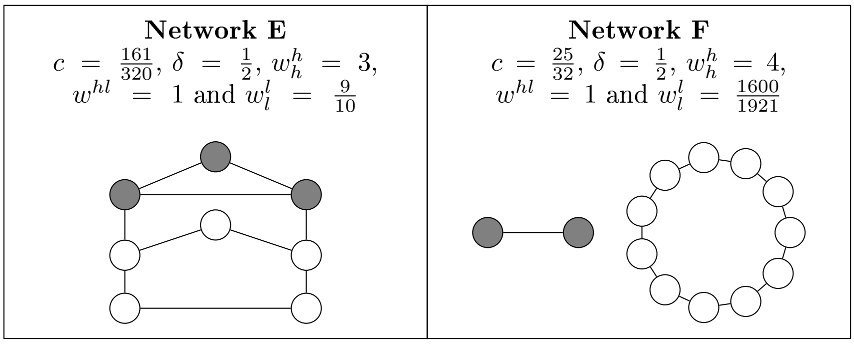

Figure 3: Examples of pairwise stable non core-periphery networks (black circles represent high-type firms and white circles stand for low-type firms). Below each title are the parameters for which the network is pairwise stable.

benefit from connectedness less than the addition of a low-type firm.[21]

Figure 3 demonstrates two examples of non core-periphery R&D collaborations networks which are pairwise stable when V(OG)>V(DIS). Network E is over-connected. Two high-type firms agree to collaborate with low-type firms since they provide additional connections while the third free-rides these collaborations. While Network E introduces a single component, Network F, is disconnected (but not over-connected). Thus, Network F, as the separating disconnected core-periphery R&D collaborations network, demonstrates an unfavourable equilibrium since the type segregation prevents information from flowing among all firms.[22] However, as opposed to the separating disconnected core-periphery R&D collab-

[21]Consider the difference V(OG)-V(DIS). An additional high-type firm provides value to all firms, but the difference depends only on the value it adds by indirect connections to the low-type firms in the case of the separating one-gate minimally connected core-periphery R&D collaborations network. An additional low-type firm adds a direct connection to the gatekeeper, and indirect connections to all other firms. By $w_l^h = w_h^l$ the indirect connections between different types are of the same value. Since an additional low-type firm creates more connections of this sort, and since it contributes other type of connections as well, an additional low-type firm is socially preferred on an additional high-type firm.

[22]Segregation usually arises in environments where the linking preferences are homophilic. Here, however, segregation emerges although low-type firms prefer a collaboration with a high-type firm over a collaboration with a low-type firm, everything else equal. Note that in this example, the low-type firms are those that refuse collaborations, since the high-type firms can provide only a single connection on top of the direct one.

orations network, the high-type firms here have no positional advantage over the low-type firms.

Extremely high linking costs

To conclude, we discuss the costs range where a collaboration between two of high-type firms is not worthwhile if they have an alternative path of length two between them and collaboration does not provide them with other shorter paths.

Proposition 5. *Suppose Assumption 2 is satisfied and $c > (\delta - \delta^2)w_h^h$. If $n^h \geq 3$ then separating core-periphery networks are neither pairwise stable nor efficient.*

Proposition 5 establishes that separating core-periphery R&D collaborations networks are not dominant when the collaboration costs are very high. The main motivation for this result is that the high costs drive high-type firms to be satisfied with indirect paths to other high-type firms in case those firms do not supply additional short paths. Assumption 2 guarantees that high-type firms also decline collaboration with an otherwise isolated low-type firm. The inefficiency results reflects the advantages of the star architecture over the complete network architecture for high-type firms when collaboration costs are very high.

Summary

Propositions 1 to 5 are summarized in Figure 4. Let us follow the vertical line where $\delta = \bar{\delta}$ from $c = 0$ to $c = 1$. Proposition 1 states that if the collaboration costs are extremely low (Area A), the unique pairwise stable network and the unique strongly efficient network is the complete network. When the collaboration costs increase into the range defined in Proposition 2 (Area B), it is efficient and (uniquely) stable to "loose" the collaborations between the low-type firms. When the collaboration costs further increase (Area C), the multiplicity of collaborations of a typical low-type firm becomes unwarranted (Proposition 3). While the separating one-gate minimally connected core-periphery R&D collaborations network is the unique strongly efficient network, if Assumption 3 holds, every core-periphery R&D collaborations network where low-type firms have a single collaboration is pairwise stable (and there are other equilibria as well). The next level of collaboration costs (Area D) is characterized by inefficiencies since while the efficient architecture does not change, the only pairwise stable core-periphery R&D collaborations network is where all the low-type firms are isolated. When the collaboration costs increase so that the efficient network becomes the separating disconnected core-periphery R&D collaborations network (Area E), it is also the unique pairwise stable R&D collaborations network (Proposition 4). Last, at extremely high collaboration costs (Area F), Proposition 5 states that separating core-periphery networks are neither pairwise stable nor strongly efficient.

4 Possible Extensions

4.1 $w_l^h \neq w_h^l$

In Section 3 it is assume that $w_l^h = w_h^l$, namely, that the benefit of a high-type firm from a collaboration with a low-type firm equals the benefit of a low-type firm from a collaboration with a high-type firm. In the efficiency analysis this assumption is of no consequence since the unit of analysis is the total benefits each connection yields.[23] In the stability analysis, this assumption may have some im-

[23] In this context w^{hl} may be thought of as the average benefit.

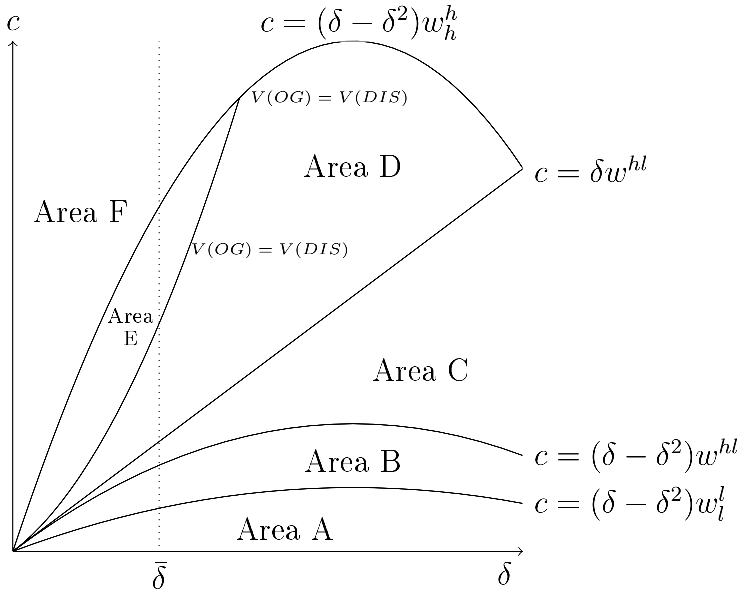

Figure 4: Summary. The collaboration quality values are $w_h^h = 4$, $w^{hl} = 1$ and $w_l^l = \frac{1}{2}$. For these values, Assumption 2 is satisfied for $\delta \in (0, \frac{3}{4})$. The spillovers rate values lie on the horizontal axis (bounded by $\delta = \frac{3}{4}$) while the collaboration costs are presented on the vertical axis (bounded by $c = 1$). Note that the only line that depends on the number of firms is $V(OG) = V(DIS)$ (in this figure $n^h = 5$, $n^l = 5$).

plications. The mutual consent requirement states that the decision to collaborate is in the hands of the least interested party. In most cases (Network F in Figure 3 is an exception) the high-type firm is the least interested since it gets no access to third-party firms.[24] Therefore, results are robust for $w_l^h \leq w_h^l$. It is only if w_h^l is significantly lower than w_l^h that the least interested party in collaboration may be the low-type firm. Such values, while counter-intuitive, may reduce the mis-coordination described in Proposition 3 since low-type firms will refuse to collaborate with high-type firm with small local periphery. In addition, such values may relieve some of the inefficiencies stated in Proposition 4 since the low-type firms better internalize the implications of the disconnected core-periphery R&D collaborations network.

4.2 Independence of Execution and Innovation Abilities

Consider a similar model where innovative ability and execution ability are independent. Each firm may be a high-type or a low-type in its innovative ability and a high-type or a low-type in its execution abilities. Plausible collaboration preferences, in this four-types environment, should reflect the greater demand for innovation by better executors. It is expected that for a large interval of intermediate collaboration costs, the superior firms will form a densely connected core while the inferior firms will be sparsely connected. The specific stable architectures should reflect the differences between the executors (high execution and low innovation) who are relatively not attractive, but posses strong incentives to collaborate and the innovators (high execution and low innovation) that are attractive

[24] In most cost reducing models for R&D collaboration, the least interested is the firm with the least collaborations.

but have weaker incentives for collaborations. Since the mixed types have weak incentives to be densely connected internally, such a model is expected to produce a variety of architectures with multiple distinct peripheries.

4.3 Semi-Periphery

Nodes that serve as bridges between the core nodes and peripheral nodes, are sometimes referred to as semi-periphery (e.g. Mintz and Schwartz (1981), Baker et al. (2008), Goyal et al. (2006), Mahutga (2006), Autant-Bernard et al. (2007) and Cattani and Ferriani (2008)). The leading explanation for the emergence of a semi-periphery is that in some cases a small group of firms may find it beneficial to take a costly action to become the bridges between the core and the periphery that were otherwise disconnected. This costly action may be the formation of a highly expensive collaboration (see Jackson and Rogers (2005) and Galeotti et al. (2006)) or taking a costly action on top of collaboration formation (the "connectors" in Galeotti and Goyal (2010)). Semi-periphery may emerge in the current framework for different reasons. First, the elimination of Assumption 2 enables the emergence of a semi-periphery composed of the unattractive high-type firms characterized by small local periphery. Second, Network B in Figure 2 describes the emergence of a semi-periphery due to mis-coordination. Last, extended type heterogeneity, in the form of a third intermediate type, may lead to the formation of hierarchical architectures where the intermediate type firms populate the semi-periphery.

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Appendix

Lemma 1. $\forall g, \forall ij \notin g : [V_i(g+ij) - V_i(g)] + [V_j(g+ij) - V_j(g)] \leq V(g+ij) - V(g)$

Proof. By Equation 1, $\forall g, \forall ij \notin g, \forall k \in N \backslash \{i,j\} : V_k(g+ij) \geq V_k(g)$. Therefore, $\sum_{k \in N \backslash \{i,j\}} V_k(g+ij) \geq \sum_{k \in N \backslash \{i,j\}} V_k(g)$. By the definition of network's value:

$$V(g) = V_i(g) + V_j(g) + \sum_{k \in N \backslash \{i,j\}} V_k(g)$$
$$V(g+ij) = V_i(g+ij) + V_j(g+ij) + \sum_{k \in N \backslash \{i,j\}} V_k(g+ij)$$

By subtracting the first from the second and introducing the inequality, the lemma is proved. □

Lemma 2. *Let $c > (\delta - \delta^2)w^{hl}$. The separating one-gate minimally connected core-periphery R&D collaborations network has higher total value than any other connected R&D collaborations network where all the high-type firms collaborate with each other.*

Proof. In every connected network with n^h high-type firms and n^l low-type firms, there are $\frac{n^h \times (n^h - 1)}{2}$ paths between high-type firms, $n^h \times n^l$ paths between high-type firms and low-type firms and $\frac{n^l \times (n^l - 1)}{2}$ paths between low-type firms. The total value of an R&D collaboration network is the sum of the net values of its shortest paths. Since each high-type firm collaborate with all other high-type firms, the total value of the efficient network is at least $\frac{n^h \times (n^h - 1)}{2} \times (2\delta w_h^h - 2c)$. Denote the number of collaborations between a high-type firm and a low-type firm by M_1 and the number of collaborations between two low-type firms by M_2. By connectivity it must be that $M_1 + M_2 \geq n^l$. Thus, there are $n^h \times n^l - M_1$ indirect paths between high-type firms and low-type firms and $\frac{n^l \times (n^l-1)}{2} - M_2$ indirect paths between pairs of low-type firms. A property of the maximal value of a connected R&D collaborations network where all the high-type firms collaborate with each other must be that all indirect paths are of length two. Thus, the maximal total value as a function of m_1 and M_2 is:

$$\begin{aligned} V_{MAX}(M_1, M_2) = \quad & \frac{n_h \times (n^h - 1)}{2} \times (2\delta w_h^h - 2c) \quad + \\ & M_1 \times (2\delta w^{hl} - 2c) \quad + \quad 2\delta^2 w^{hl} \times (n^h \times n^l - M_1) \quad + \\ & M_2 \times (2\delta w_l^l - 2c) \quad + \quad 2\delta^2 w_l^l \times (\frac{n^l \times (n^l - 1)}{2} - M_2) \end{aligned}$$

The value of the separating one-gate minimally connected core-periphery R&D collaborations network is:

$$\begin{aligned} V_{OG} = \quad & \frac{n^h \times (n^h - 1)}{2} \times (2\delta w_h^h - 2c) \quad + \\ & n^l \times (2\delta w^{hl} - 2c) \quad + \quad 2\delta^2 w^{hl} \times (n^h - 1) \times n^l \quad + \\ & 2\delta^2 w_l^l \times (\frac{n^l \times (n^l - 1)}{2}) \end{aligned}$$

Thus,

$$\begin{aligned} & V_{MAX}(M_1, M_2) - V_{OG} = \\ & \qquad 2 \times (M_1 - n^l) \times (\delta w^{hl} - \delta^2 w^{hl} - c) + 2 \times M_2 \times (\delta w_l^l - \delta^2 w_l^l - c) \end{aligned}$$

This difference has to be non-negative. Therefore,

$$(n^l - M_1) \times (c + \delta^2 w^{hl} - \delta w^{hl}) \geq M_2 \times (c + \delta^2 w_l^l - \delta w_l^l)$$

The collaboration costs imply that $c+\delta^2 w_l^l-\delta w_l^l > c+\delta^2 w^{hl}-\delta w^{hl} > 0$. By the non-negativity of M_2 both sides of the inequality are non-negative. Therefore, it must be that either $n^l-M_1 > M_2$ or $M_1 = n^l$ and $M_2 = 0$. However, by the connectivity condition it must be that $M_1+M_2 \geq n^l$. Thus, the highest value network among the connected R&D collaborations networks where all the high-type firms collaborate with each other, must have the same total value as separating one-gate minimally connected core-periphery R&D collaborations network. In this network there must be no collaborations between low-type firms and each low-type firm must have exactly one collaboration with a high-type firm. Moreover, since the maximal value is reached when all indirect paths are of length two, all the low-type firms must be collaborating with the same high-type firm. Therefore, the separating one-gate minimally connected core-periphery R&D collaborations network has higher total value than any other connected R&D collaborations network where all the high-type firms collaborate with each other. □

Lemma 3. *Let $c > (\delta-\delta^2)w^{hl}$. The highest total value in the set of R&D collaborations networks where each high-type firm collaborates with all other high-type firms is achieved either by the separating one-gate minimally connected core-periphery R&D collaborations network or by the separating disconnected core-periphery R&D collaborations network.*

Proof. First, consider the set of disconnected networks where each high-type firm collaborates with all other high-type firms. These networks include one component that contains all the high-type firms completely connected among themselves (the h-component) and other components that include only low-type firms (l-components). By Lemma 2 the h-component is a separating one-gate minimally connected core-periphery network. In addition, since $c > (\delta - \delta^2)w_l^l$, by Proposition 6.1 in Jackson (2008), the l-components are organized either as a star (l-star) or as isolates.

Suppose that there are two components, an h-component that is organized as a separating one-gate minimally connected core-periphery network and an l-component that is organized as a star. Denote by $n^l \geq l \geq 2$ the size of the star, and denote the network that includes both components by $g(l)$.

$$\begin{aligned} V(g(l)) = \quad & \frac{n^h \times (n^h - 1)}{2} \times (2\delta w_h^h - 2c) \quad + (n^l - l) \times (2\delta w^{hl} - 2c) \quad + \\ & (n^h - 1) \times (n^l - l) \times 2\delta^2 w^{hl} \quad + \frac{(n^l - l)(n^l - l - 1)}{2} \times 2\delta^2 w_l^l \quad + \\ & (l - 1) \times (2\delta w_l^l - 2c) \quad + \quad \frac{(l-1)(l-2)}{2} \times 2\delta^2 w_l^l \end{aligned}$$

Consider now the total value of the same network where $l = 1$,

$$\begin{aligned} V(g(1)) = \quad & \frac{n^h \times (n^h - 1)}{2} \times (2\delta w_h^h - 2c) \quad + (n^l - 1) \times (2\delta w^{hl} - 2c) \quad + \\ & (n^h - 1) \times (n^l - 1) \times 2\delta^2 w^{hl} \quad + \frac{(n^l - 1)(n^l - 2)}{2} \times 2\delta^2 w_l^l \end{aligned}$$

$$V(g(1)) - V(g(l)) = 2(l-1) \times [\delta(w^{hl} - w_l^l) + (n^h - 1)\delta^2 w^{hl} + (n^l - l)\delta^2 w_l^l]$$

obviously, $\forall l > 1 : V(g(l)) < V(g(1))$.

Thus, the highest total value in the the set of disconnected networks where each high-type firm collaborates with all other high-type firms, is achieved by a network with an a-component organized as a separating one-gate minimally connected core-periphery R&D collaborations network while all the low-type firms that are not in this component are isolated. Moreover, by Lemma 2 the highest total value in the set of networks where each high-type firm collaborates with all other high-type firms is achieved by a network where all the low-type firms are either isolated or collaborate with the same high-type firm. Denote each such network by g^m where $0 \leq m \leq n^l$ denotes the number of non-isolated low-type firms.

$$V(g^m) = \frac{n^h \times (n^h - 1)}{2} \times (2\delta w_h^h - 2c) \quad + m \times (2\delta w^{hl} - 2c) \quad + \\ (n^h - 1) \times m \times 2\delta^2 w^{hl} \quad + \frac{m \times (m-1)}{2} \times 2\delta^2 w_l^l$$

The coefficient of m^2 is $\delta^2 w_l^l > 0$ and therefore $V(g^m)$ is an upward parabola in m. Thus, its maximum is achieved on one of the corners - either in $m = n^l$ (the separating one-gate minimally connected core-periphery R&D collaborations network) or in $m = 0$ (the separating disconnected core-periphery R&D collaborations network). □

Proposition 1

Proof. If $n > 2$, the net gains from terminating a collaboration are at most $c + (\delta^2 - \delta)w_l^l$ (recall that $\delta^2 - \delta$ is negative). Since $(\delta - \delta^2)w_l^l > c$, the net gains are negative and no firm wishes to terminate a collaboration. If $n = 2$ the net gains from terminating the collaboration are at most $c + (0 - \delta w_l^l)$. Again, those net gains are negative. Thus, the complete R&D collaborations network is pairwise stable. Suppose g' is some incomplete pairwise stable R&D collaborations network. Thus, g' includes at least one pair of non-collaborating firms (j and k). Denote the geodesic distance between j and k by $d \geq 2$. For both firms, the net gains from collaborating are at least $(\delta w_l^l - c) - \delta^d w_l^l$. Since $(\delta - \delta^2)w_l^l > c$, the net gains are positive. Thus, the unique pairwise stable R&D collaborations network is the complete network. Let g'' be some incomplete R&D collaborations network. As we showed above, there are two non-collaborating firms (j and k) that strictly wish to collaborate. By Lemma 1, $V(g''+jk) > V(g'')$. Hence, for any incomplete network there is a network with strictly higher total value. Therefore, the complete R&D collaborations network is the unique efficient network[25]. □

Proposition 2

Proof. Let g be a separating maximally connected core-periphery R&D collaborations network. The net gains for a high-type firm from terminating a collaboration are either at most $\delta^2 w^{hl} - (\delta w^{hl} - c)$ (if the partner is a low-type firm) or $\delta^2 w_h^h - (\delta w_h^h - c)$ (if the partner is a high-type firm). The net gains for a low-type firm from terminating a collaboration are at most $\delta^2 w^{hl} - (\delta w^{hl} - c)$ (its partner is a high-type firm). Since $(\delta - \delta^2)w^{hl} > c$, all those net gains are negative, so no firm in g wishes to terminate an existing collaboration. In addition, the net gains of a low-type firm from forming a collaboration with another low-type firm are $(\delta w_l^l - c) - \delta^2 w_l^l$. Since $c > (\delta - \delta^2)w_l^l$, those net gains are negative, meaning, no pair of low-type firms wishes to collaborate. Thus, the separating maximally connected core-periphery R&D collaborations network is pairwise stable.

To show uniqueness, consider the R&D collaborations network g' where there is a high-type firm, Firm i, that does not collaborate with all other firms, for example,

[25] Using Lemma 1 above and Theorem 1 of Buechel and Hellmann (2012) it is straightforward to show that the complete network is efficient. However, to show uniqueness the rest of the proof is needed.

suppose that Firm i does not collaborate with Firm j. Establishing a collaboration between Firm i and Firm j provides both firms with a net gain of either at least $(\delta w_h^h - c) - \delta^2 w_h^h$ or at least $(\delta w^{hl} - c) - \delta^2 w^{hl}$. Since $(\delta - \delta^2) w^{hl} > c$, these net gains are positive. Thus, g' is not pairwise stable and in every pairwise stable the R&D collaborations network the high-type firms collaborate with all other firms. Consider the R&D collaborations network g'' where all high-type firms collaborate with all other firms and there is at least one pair of low-type firms that form an R&D collaboration. The net gains of each of these two firms from terminating their collaboration are $\delta^2 w_l^l - (\delta w_l^l - c)$. Since $c > (\delta - \delta^2) w_l^l$, both firms are better off terminating their collaboration. Therefore, g'' is not pairwise stable and the separating maximally connected core-periphery R&D collaborations network is the unique pairwise stable network.

To show unique efficiency, let g''' be the R&D collaborations network where there is a high-type firm, Firm i, that does not collaborate with all other firms, for example, suppose that Firm i does not collaborate with Firm j. Consider the network $g''' + ij$. As shown above, both firms wish to collaborate since their net gains from such a collaboration are positive. By Lemma 1, $V(g''' + ij) > V(g''')$. Therefore, every efficient R&D collaborations network must belong to the class of R&D collaboration networks in which high-type firms collaborate with all other firms. Let g'''' be a member of this class such that there exists a pair of low-type firms that collaborate with each other. As shown above both firms are strictly better off if this collaboration is terminated. The high-type firms are unaffected by such termination since they collaborate with all firms. In addition, the low-type firms are also unaffected by such termination since they collaborate with the high-type firms and have a short two-length path to every other low-type firms through any of the high-type firms. Thus, $V(g'''' - ij) > V(g'''')$. Thus, the separating maximally connected core-periphery R&D collaborations network is the unique pairwise stable and the unique efficient R&D collaborations network. $\square$

Proposition 3

Proof. First I will show that the separating minimally connected core-periphery network is pairwise stable when assuming both Assumption 2 and Assumption 3. For a separating minimally connected core-periphery network to be pairwise stable four conditions should be met. First, the net gains of a high-type firm from terminating a collaboration with another high-type firm are at most $\delta^2 w_h^h - (\delta w_h^h - c)$ if $n^h > 2$ and $0 - (\delta w_h^h - c)$ if $n^h = 2$. By Assumption 2, no high-type firm wishes to terminate a collaboration with another high-type firm. Second, the net gains of a low-type firm from establishing a collaboration with another low-type firm are at most $(\delta w_l^l - c) - \delta^3 w_l^l$. By Assumption 3, no low-type firm wishes to establish a collaboration with another low-type firm. Third, consider the termination of an existing collaboration between a high-type firm and a low-type firm. It suffices to consider only the net gains of the high-type firm since none of its paths to other firms are affected by such termination. The net gains of a high-type firm from terminating a collaboration with a low-type firm are $0 - (\delta w^{hl} - c) < 0$. Thus, no member of a collaborating pair of a high-type firm and a low-type firm, wishes to terminate the partnership. Fourth, consider the formation of a new collaboration between a high-type firm and a low-type firm. It suffices to consider only the net gains of the high-type firm since none of its paths to other firms are affected by such collaboration. The net gains of a high-type firm from forming a new collaboration with a low-type firm are $(\delta w^{hl} - c) - \delta^2 w^{hl} < 0$. Thus, no high-type firm wishes to form a new collaboration with a low-type firm, and the separating minimally connected core-periphery network is pairwise stable.

Now, Assumption 2. For the separating one-gate minimally connected core-periphery network to be pairwise stable the same four conditions should be met. However, the second consideration is slightly different. The net gains of a low-type firm from establishing a collaboration with another low-type firm are only $(\delta w_l^l - c) - \delta^2 w_l^l$. By the collaboration costs, no low-type firm wishes to establish a collaboration with another low-type firm (Assumption 3 is not needed). In every other minimally connected core-periphery network, there is at least one pair of low-type firms that will gain $(\delta w_l^l - c) - \delta^3 w_l^l$ by collaborating. There may be collaboration costs in this range such that these gains are positive and these networks are not pairwise stable. Other separating core-periphery networks are not pairwise stable in the whole range, either due to connectivity or to multiple collaborations of low-type firms.

There are networks that are pairwise stable and do not belong to the set of separating minimally connected core-periphery networks. Let g be a network in which there is a pair of high-type firms that do not collaborate. The net gains of each such firm from establishing a collaboration are at least $(\delta w_h^h - c) - \delta^2 w_h^h$. By Assumption 2, g is not pairwise stable. Thus, the set of pairwise stable networks is a subset of the set of all R&D collaborations networks in which each pair of high-type firms collaborate. Next, let g' be an R&D collaborations network where each pair of high-type firms collaborate and there is at least one pair of firms with no path between them. Thus, one component of this network includes at least all the high-type firms while all the other components include only low-type firms.

The net gains from establishing a collaboration for a high-type firm and a low-type firm located in a different component are at least $(\delta w^{hl} - c) - 0 > 0$, for each of them. Thus, the set of pairwise stable R&D collaborations networks is a subset of the set of all connected networks in which each pair of high-type firms collaborate.

To conclude, I show that the separating one-gate minimally connected core-periphery network is the unique efficient R&D collaborations network. First, Let g be an R&D collaborations network in which there exists a pair of non-collaborating high-type firms (i and j). As was shown above, their net gains from establishing a collaboration are positive. By Lemma 1 the efficient network is a member of the set of R&D collaborations networks where all high-type firms collaborate with each other. Second, Let g' be a member of this set where there is at least one pair firms with no path between them. As shown above, there is at least one pair of a high-type firm and a low-type firm who have no path between them in g' and their net gains from forming a collaboration are positive. Again, by Lemma 1 the efficient R&D collaborations network is a member of the set of connected networks where all high-type firms collaborate with each other. By Lemma 2 the separating one-gate minimally connected core-periphery network is the unique efficient R&D collaborations network. □

Remark 1

Proof. Let g be a pairwise stable R&D collaborations network where each pair of high-type firms collaborates and there is at least one low-type firm which does not collaborate with a high-type firm. This firm maintains at least one collaboration since every pairwise stable network is connected. Suppose it maintains exactly one collaboration (to low-type Firm j). The net gains of Firm j from terminating this collaboration are $0-(\delta w_l^l - c)$. By the additional assumption and by the collaboration costs, those net gains are positive and g is not pairwise stable. Therefore any low-type firm which does not collaborate with any high-type firm must maintain at least two collaborations. □

Proposition 4

Proof. For the separating disconnected core-periphery R&D collaborations network to be pairwise stable three conditions should be met. First, if $n^h > 2$, the net gains for a high-type firm from terminating a collaboration with another high-type firm are $\delta^2 w_h^h - (\delta w_h^h - c)$ while if $n^h = 2$ it is $0 - (\delta w_h^h - c)$. Thus, by the collaboration costs range, no high-type firm wishes to terminate a collaboration with another high-type firm. Second, the net gains for a low-type firm from initiating a collaboration with another low-type firm are $\delta w_l^l - c) - 0$. By the collaboration costs range, no low-type firm wishes to initiate a collaboration with another low-type firm. Last, the net gains for a high-type firm from collaborating with a low-type firm are $(\delta w^{hl} - c) - 0$. By the collaboration costs range, no high-type firm wishes to initiate a collaboration with a low-type firm.

Next I show the efficiency result. Let g be a network in which there exists a pair of high-type firms, Firm i and Firm j, that do not collaborate. These firms net gains from establishing the collaboration ij are at least $(\delta w_h^h - c) - \delta^2 w_h^h$. By the collaboration costs, these net gains are positive. By Lemma 1, $V(g + ij) > V(g)$. Hence, the efficient network is a member of the set of networks where each pair of high-type firms collaborate. By Lemma 3 the efficient network is either the separating one-gate minimally connected core-periphery R&D collaborations network or the separating disconnected core-periphery R&D collaborations network. By the relation between $V(OG)$ and $V(DIS)$ efficiency results are determined.

Next, I show that when $V(OG) < V(DIS)$ there are no pairwise stable networks other than the separating disconnected core-periphery R&D collaborations network. Let g be a pairwise stable network in which there is a pair of of high-type firms that do not collaborate. As shown above, their net gains from collaborating are positive. Hence, every pairwise stable network must be a member of the set of networks where each pair of high-type firms collaborates. By Theorem 1 of Buechel and Hellmann (2012) and Lemma 1 above, no pairwise stable network is a super-network of the separating disconnected core-periphery R&D collaborations network. Since every other member of the set of networks where each pair of high-type firms collaborate is a super-network of the separating disconnected core-periphery R&D collaborations network, none of them are pairwise stable. Hence, the separating disconnected core-periphery R&D collaborations network is the unique pairwise stable network when $V(OG) < V(DIS)$.

However, when $V(OG) < V(DIS)$ there are networks that are pairwise stable other then the separating disconnected core-periphery R&D collaborations network. First, let g be a pairwise stable network in which there is a pair of non-collaborating high-type firms. As shown above, their net gains from establishing a collaboration are positive. Therefore, every pairwise stable network must be a member of the set of networks where each pair of high-type firms collaborates. Next, let g' be a network in which each pair of high-type firms collaborates and there is a low-type firm with exactly one collaboration (with Firm j). The net gains of Firm j from terminating this collaboration are at least $0 - (\delta w^{hl} - c)$.

By the collaboration costs these net gains are positive. Therefore, every pairwise stable network must be a member of the set of networks where each pair of high-type firms collaborates and each low-type firm is either isolated or has at least two collaborations. Third, by Theorem 1 of Buechel and Hellmann (2012) and Lemma 1 above, no pairwise stable network is a super-network of the separating one-gate minimally connected core-periphery R&D collaborations network. Therefore, every pairwise stable network must be a member of the set of networks where each pair of high-type firms collaborates, each low-type firm is either isolated or has at least two collaborations and there is no firm that collaborates with every other firm. Last, suppose g'' is a pairwise stable separating core-periphery network which is not the separating disconnected core-periphery R&D collaborations network. Then, there is at least one low-type firm that maintains at least two collaborations with high-type firms. The net gains of each of these from terminating this collaboration are $\delta^2 w^{hl} - (\delta w^{hl} - c)$. By the collaboration costs these net gains are positive. The separating disconnected core-periphery *R&D* collaborations network is the only pairwise stable member of the set of separating core-periphery *R&D* collaborations networks. □

Proposition 5

Proof. Let g be a separating core-periphery R&D collaborations network other than the separating disconnected core-periphery R&D collaborations network. Thus, there is at least one high-type firm which collaborates with a low-type firm. The net gains of the high-type firm from terminating this collaboration are at least $0 - (\delta w^{hl} - c)$. By Assumption 2 those net gains are positive and g is not pairwise stable. In addition, the separating disconnected core-periphery R&D collaborations network is not pairwise stable since the net gains for a high-type firm from terminating its collaboration with one of its fellow high-type firms are $\delta^2 w_h^h - (\delta w_h^h - c)$ which are positive (using $n^h > 2$).

By Lemma 3 the highest total value in the set of networks where each high-type firm collaborates with all other high-type firms is either the separating one-gate minimally connected core-periphery network or the separating disconnected core-periphery network. In particular, one of these two networks maximize the total value among the set of core-periphery networks. Let g' be a star network encompassing all the firms centred around a high-type firm. $V(g') - V(OG) = (n^h - 1)(n^h - 2)[\delta^2 w_h^h - (\delta w_h^h - c)]$ therefore the separating one-gate minimally connected core-periphery network is not efficient. By Proposition 1 of Jackson and Wolinsky (1996), the separating disconnected core-periphery network has lower total value than either the empty network or the network where all the high-type firms form a star while the low-type firms are isolates. □