Combinatorics V63.0240.
Instructor: Dr. Ron Peled.

Homework Assignment 1

The assignment is due in class on the 29'th of January. Starred exercises are not mandatory but it is recommended that you try them.

1. (a) What is the magic sum of a magic cube of size $n \times n \times n$?
   (b) Question 15 in section 1.8 of the book.

2. Consider an $n \times n$ square board. Recall that a *proper coloring* of it with k colors is an assignment of one of the k colors to each of the squares of the board such that no two adjacent squares have the same color. Two squares which touch at a corner are not considered adjacent and may have the same color.
   (a) Show that there are exactly 2 proper colorings of the board with colors black and white.
   (b) (*) Show that there are at least $2^{n^2/2}$ proper colorings of the board with colors black, white and green.
   (c) (*) Show that there are at most $3 \times 2^{n^2-1}$ proper colorings of the board with colors black, white and green.

3. How many ways are there to properly color an $n \times 1$ square board with k colors?

4. (*) Questions 34 and 35 in section 1.8 of the book
   Hint: first consider the game with 100 replaced by a small number.

5. Solve questions 2, 4, 6 and 14 of section 2.7 of the book.

6. (a) How many different "words" can be made from the letters t,r,e,e,s? from the letters c,o,m,m,i,t,t,e,e?
   (b) (*) How many different "words" can be made from the letters s,h,a,k,e,s,p,e,a,r,e?

7. In how many ways can one make a dice? That is, in how many non-equivalent ways can one assign the numbers 1,2,3,4,5,6 to the faces of

a cube, where we consider two such dice to be equivalent if one can be rotated in space to give the other.
Hint: Determine for a given assignment of the numbers to the faces how many other assignments can be obtained from it by a rotation. Then use the division principle.