Magnetic instabilities in hard superconductors

R. G. Mints and A. L. Rakhmanov

Institute of High Temperature of the USSR Academy of Sciences
Usp. Fiz. Nauk **121**, 499–524 (March 1977)

This review treats magnetic instabilities in hard and combined Type II superconductors. We give and discuss in detail the criteria for stability of the critical state with respect to magnetic-flux jumps. We study the effect of magnetic and thermal diffusion, as well as that of the structure of a combined superconductor, on the magnetic-field value for a flux jump. The theoretical results are compared with the existing experimental data.

PACS numbers: 74.60.—w, 74.70.Lp

CONTENTS

I. INTRODUCTION

Unusual physical properties and possible technical applications have aroused great interest in experimental and theoretical study of hard superconductors. The theme is that of superconducting materials in which the current density j can attain values of 10^5-10^6 A/cm^2 with insignificant losses, while superconductivity persists in magnetic fields up to $H=10^5-10^6$ gauss. Attaining such extremal parameters and operating of various devices under these conditions are limited in many ways by magnetic instabilities that exist in hard superconductors. This review is concerned with presenting the theory of the onset of these instabilities and with comparing it with experiment.

A. Hard superconductors

In Type II superconductors, the magnetic field penetrates in the form of quanta of magnetic flux (the value of which is $\Phi_0=\pi\hbar c/e=2\times10^{-7}$ gauss $\cdot$ cm^2) even in an external field $H_e=H_{c_1}\approx 100-1000$ gauss, which is called the first critical field. One can imagine the flux quantum itself (an Abrikosov vortex filament)[1] as consisting of two regions—the core of the vortex and its periphery. The core of the vortex consists of a practically normal metal, its size being $\xi=100-500$ Å. Persistent superconducting currents circulate in the peripheral part, which has a dimension $\delta_L=100-5000$ Å. Figure 1 shows the current and magnetic-field distributions in the vortex for materials having $\delta_L/\xi\gg1$ (for a more detailed acquaintance with the properties of Type II superconductors, see, e.g. [2]).

In the equilibrium state the vortex filaments form a net (triangular or square) having a mean density $n=B/\Phi_0$, where B is the magnetic induction inside the specimen. [1,2] Yet if the superconductor contains structural defects, the vortices can become attached to them (this phenomenon is called pinning) and form a metastable configuration of the magnetic flux (for more details on the pinning phenomenon, see, e.g., [3,4]).

Superconductors in which the vortex filaments are strongly bound to the metal lattice are called hard superconductors. Since the configuration and the energy of a vortex filament depend substantially on the temperature, the pinning force F_p also depends on the temperature. The mutual repulsion of the vortices[1,2] causes the pinning forces to depend on the density of vortex filaments, i.e., on B. If we pass a transport current through a Type II superconductor, then the interaction with it gives rise to a so-called Lorentz force that acts on each of the vortices[4,5]:

$$\mathbf{F}_L=\frac{1}{c}[\mathbf{j}\times\mathbf{\Phi}_0].$$

When acted on by this force, the magnetic flux goes into motion, energy dissipation arises, and the superconductor transforms into the resistive state. [6–8,22] Yet if the superconductor is hard, then this regime can set in only when $F_L\geq F_p(T, B)$. We can conveniently write the force F_p in the form

$$\mathbf{F}_p=\frac{1}{c}[\mathbf{j}_c\times\mathbf{\Phi}_0].$$

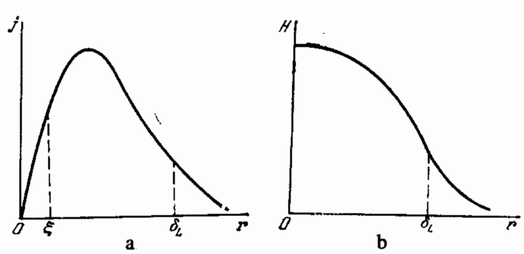

FIG. 1. Distribution of the current $j(r)$ (a) and of the magnetic field $H(r)$ (b) near the core of the vortex.

Here $j_c(T, B)$ is called the critical current density. Thus persistent currents can exist in a hard superconductor as long as $j<j_c$. Figure 2 shows relationships of j_c to B and T that are characteristic of hard superconductors.

As we have seen, the entire magnetic flux goes into motion when $j>j_c(T, B)$. A viscous-flow regime of the vortex filaments is established in the superconductor (see, e.g. [8,11–13]). Here,

$$F_L = F_p + \eta v, \tag{1.1}$$

where ηv is the viscous frictional force, η is the viscosity, and v is the velocity of motion of the vortex structure. Equation (1.1) implies that

$$j = j_c + \eta \frac{vc}{\Phi_0}.$$

We can easily derive the relation between v and the electric field E that arises upon motion of the flux by using the equation of continuity for the flux of the vortex filaments

$$\frac{\partial n}{\partial t} = -\operatorname{div}(n\mathbf{v})$$

and the Maxwell equation

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}. \tag{1.2}$$

The last two equations imply that

$$E = \frac{v}{c} B.$$

Thus:

$$j = j_c + \sigma_f E, \tag{1.3}$$

where $\sigma_f = \eta c^2/B\Phi_0 \approx \sigma_n H_{c_2}/B$. Here σ_n is the conductivity of the specimen in the normal state.

The relationship $\sigma_f \sim B^{-1}$ is well confirmed by experiment (see, e.g. [12]), and it stems from the microscopic theory.[13] For hard superconductors, $\sigma_f \sim (10^{16}\, H_{c_2}/B)$ sec^{-1}. This value is substantially smaller than the conductivity of pure metals, even in fields $B \sim H_{c_1}$. Figure

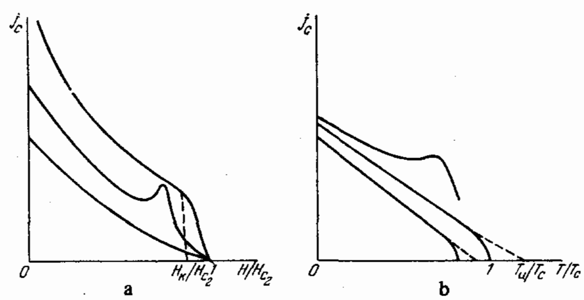

FIG. 2. Characteristic relationships of the critical current density j_c to the magnetic field (a) and the temperature (b).

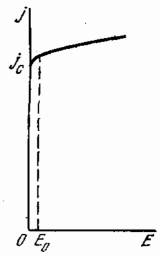

FIG. 3. Typical volt–ampere characteristic of a hard superconductor.

3[1)] shows a typical form of the volt–ampere characteristic of a hard superconductor. As we vary the electric field, the volt–ampere characteristic quickly climbs onto the linear region (for $E<E_0$; $dj/dE \gg \sigma_f$), whereas the condition $\sigma_f E \ll j_c$ holds for all actual values of the electric field E in hard superconductors. Thus we can assume that a current density close to the critical value (more exactly, exceeding it somewhat) is established in a hard superconductor in response to any applied potential difference. This concept of the critical state has been proposed in[8,14–18]. It has repeatedly been tested experimentally and it describes well the effects in hard superconductors (see, e.g., [4,17–19]).

B. Qualitative theory of flux jumps

The critical state in hard superconductors can become unstable under certain conditions. For example, assume that a fluctuation or an external agent in some volume of the superconductor has caused the temperature to rise. This diminishes the pinning forces and hence decreases the critical current. The equilibrium of the vortex net breaks down, motion of the magnetic flux sets in, and it is accompanied by generation of heat owing to the decreased energy of the superconducting currents. The temperature rise of the specimen can convert into an avalanche-like processes, i.e., it can lead to loss of stability. Such a penetration of magnetic flux into the specimen amounts to perturbations of the temperature and the electromagnetic field that arise in a correlated way, and it is called a flux jump. Hence, in a rigorous formulation of the problem, we must study the heat-conduction equation and the Maxwell equation jointly for stability.

As we know, propagation of magnetic flux and of heat flux is characterized by the corresponding diffusion coefficients: the magnetic diffusion coefficient $D_m = c^2/4\pi\sigma_f$, which involves the normal currents in the resistive state of the superconductor, and the thermal diffusion coefficient $D_t = \varkappa/\nu$ (where ν is the heat capacity and $\varkappa$ is the heat conductivity of the material). Let us introduce the parameter τ that defines the relationship between D_t and D_m: $\tau = D_t/D_m$. As we have mentioned, σ_f is relatively small in a hard superconductor. Correspondingly we have $D_t \ll D_m$, and $\tau \ll 1$ (usually even in fields $B \sim H_{c_1}$). This means that diffusion of magnetic

1)The nonlinear region ($E<E_0$) stems from an entire set of causes: inhomogeneity of pinning, structural defects of the vortex net, thermal activation, ..., [9,10] yet its size is small (in terms of electric-field value).

flux is considerably faster than the redistribution of heat. Thus, in the fundamental approximation for $\tau \ll 1$, the heating of hard superconductors during a flux jump is adiabatic. This assertion has been very convincingly confirmed experimentally (see, e.g., [20,21,28]).

The converse limiting case of $\tau \gg 1$ can be realized in the so-called combined superconductors (see Sec. 3); and also in hard superconductors at very low temperatures ($T \lesssim 0.1$ °K). When $\tau \to \infty$ (i.e., when $D_m \ll D_t$) the superconductor is heated while the magnetic flux is frozen. For large τ, the magnetic flux slowly (within limits as $\tau \gg 1$) penetrates the specimen. Physically this involves the fact that the induced normal current compensates the decline in the superconducting current caused by the temperature rise, and evidently hinders entrance of the magnetic flux into the specimen.

Let us examine qualitatively the development of a perturbation in a hard superconductor having $\tau \ll 1$. Assume that a fluctuation (in the temperature, field, current, etc.) or an external agent has raised the temperature in some region of the superconductor by the amount θ_0. Thus a priming amount of heat $Q_0 = \nu\theta_0$ has been supplied to this site. An additional amount Q_1 of heat is released during the redistribution of the currents and the field, which is equal to

$$Q_1 = \int j_c E \, dt$$

(here we have accounted for the fact that $j_c \gg \sigma_f E$). If a flux jump does not occur, while a new equilibrium situation is established in the superconductor with a temperature elevated by the amount θ over the initial state, then we can use the law of conservation of energy for determining θ:

$$Q_1 = \nu\theta = Q_0 + Q_1 = \nu\theta_0 + Q_1. \tag{1.4}$$

In (1.4) we have accounted for adiabatic heating ($\tau \ll 1$).

In order to estimate Q_1, we shall use the Maxwell equation.[2)]:

$$\Delta \mathbf{E} = \frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}. \tag{1.5}$$

For the sake of simplicity, we shall treat the case in which $\partial j_c/\partial B = 0$ (Bean's model of the critical state[15]). Then $\partial j_c/\partial t = (\partial j_c/\partial T)\dot{\theta}$. The quantity $|\Delta \mathbf{E}|$ is $\sim E/b^2$, where b is the characteristic dimension of the specimen. Thus,

$$E \sim \frac{4\pi b^2}{c^2}\dot{\theta}\left|\frac{\partial j_c}{\partial T}\right|,$$

while

$$Q_1 = \int j_c E \, dt = \frac{1}{\gamma^2}\frac{4\pi b^2 j_c}{c^2}\left|\frac{\partial j_c}{\partial T}\right|\theta.$$

Here γ^2 is a number of the order of unity that depends on the concrete distribution of currents and of the electric field E in the specimen. Upon substituting the expression for Q_1 into (1.4), we find that

$$\theta = \frac{\theta_0}{1-(\beta/\gamma^2)}, \tag{1.6}$$

where

$$\beta = \frac{4\pi b^2 j_c}{c^2 \nu}\left|\frac{\partial j_c}{\partial T}\right|.$$

We see from the relationship (1.6) that the temperature increases without limit as $\beta/\gamma^2 \to 1$ for any value of the initial fluctuation. Hence the critical state is stable if

$$\beta < \gamma^2. \tag{1.7}$$

A criterion of stability of the form of (1.7) for a plane, semi-infinite specimen of a hard superconductor was first derived on the basis of similar qualitative arguments in[23]. Here the role of the characteristic dimension was played by the screening depth l_0 of the external magnetic field,[3)] which is determined from the condition $H(l_0) = H_e - 4\pi j_c l_0/c = 0$. Upon substituting $l_0 = cH_e/4\pi j_c$ into (1.7), we get

$$\frac{H_e^2}{4\pi\nu j_c}\left|\frac{\partial j_c}{\partial T}\right| < \gamma^2. \tag{1.7$'$}$$

Upon substituting the parameters characteristic of hard superconductors into the criteria (1.7) and (1.7′), we can easily get an estimate for the flux-jump field H_j and the maximum admissible thickness b_c of the specimen. H_j turns out to be of the order of several kilogauss (1–3 kilogauss), while b_c is of the order of several hundred microns.

Let us derive an analogous criterion for the case $\tau \gg 1$. As we have seen, heating occurs here while the magnetic flux is frozen. This is equivalent to the condition $\partial j/\partial t = 0$, whence (cf. Eq. (1.3))

$$\sigma\dot{E} + \frac{\partial j_c}{\partial T}\dot{\theta} = 0,$$

and hence,

$$E = \frac{\theta}{\sigma}\left|\frac{\partial j_c}{\partial T}\right|.$$

Thus the following power per unit volume is released:

$$\dot{Q} = \frac{j_c\theta}{\sigma}\left|\frac{\partial j_c}{\partial T}\right|.$$

Evidently the critical state will be stable if the quantity $\dot{Q}$ does not exceed the power q that is transported away by thermal conduction:

$$q = \varkappa\nabla^2 T > \frac{j_c}{\sigma}\left|\frac{\partial j_c}{\partial T}\right|\theta.$$

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2)Henceforth we shall be interested only in the case $B \gg H_{c_1}$, which permits us to assume that $B = H$.[1,2]

3)We shall assume for the sake of simplicity in this chapter that there is no magnetic field frozen in the bulk of the superconductor (see below).

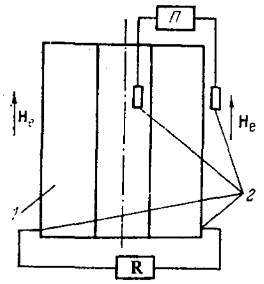

FIG. 4. Fundamental experimental scheme for studying flux jumps. H_e—external field, 1—specimen, 2—transducers, R—recording devices.

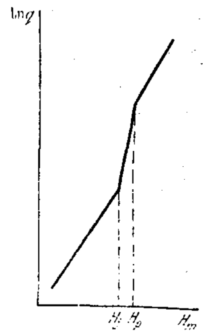

FIG. 6. Qualitative relationship of the loss power in a hard superconductor to the amplitude H_m of the external field.[24] H_j is the field for a flux jump, and H_p is the field for complete penetration of the external field into the specimen.

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$$\frac{1}{\gamma_1^2}\frac{b^2 j_c}{\varkappa\sigma}\left|\frac{\partial j_c}{\partial T}\right| < 1. \tag{1.8}$$

Here γ_1 is a number of the order of unity that is determined by the details of the temperature distribution through the specimen. We can conveniently rewrite the criterion (1.8) in the form

$$\frac{\beta}{\tau} < \gamma_1^2. \tag{1.8'}$$

Equation (1.8) has been derived under the assumption of iosthermal boundary conditions.

Hart[46,47] first derived the criterion (1.8) from similar qualitative arguments.

In the case $\tau \gg 1$, the quantities H_j and b_c substantially depend on the concrete properties of the studies materials. In particular, for combined superconductors (see Sec. 3), H_j and b_c are severalfold times larger than for a hard superconductor ($H_j \sim 10$ kilogauss, $b_c \sim 0.1$ cm).

The form of (1.8) implies that an increase in the fluctuations is damped by the normal current. Evidently, the role of the normal current consists in hindering the magnetic flux (an analog of viscous friction), and correspondingly, in diminshing the release of heat. Stability depends strongly on the dimensions of the specimen as well—thin superconductors prove to be more stable.

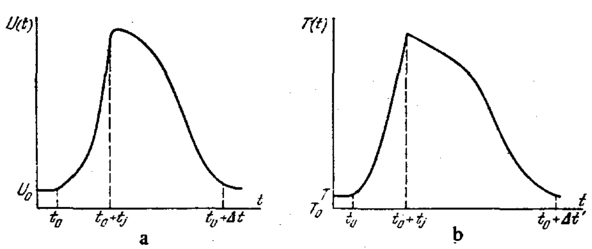

FIG. 5. (a) Time-dependence of the potential $U(t)$ recorded during a flux jump (U_0 is the potential that arises in the specimen owing to change in the external magnetic field, t_0 is the time of onset of the jump, t_j is the time for development of instability, and Δt is the relaxation time of the potential); (b) time-dependence of the temperature of the surface of the specimen (T_0 is the temperature of the refrigerant (helium), T is the initial temperature of the specimen, and $\Delta t'$ is the relaxation time of the temperature).

We note also that the geometry of the current and magnetic-field distributions can play a certain role for stable operation of various devices. This is because they evidently determine the size of the coefficient γ in each concrete case.

C. Experimental study of stability of the critical state

Schematically, experiments to study magnetic instabilities and concomitant phenomena are performed as follows. One puts the studied specimen in an external magnetic field that either increases or oscillates at a certain frequency and amplitude about a fixed value. Starting at a certain magnetic field, the stationary current and field distribution becomes unstable, and a fluctuation or external agent (which can be the change in the magnetic field itself) leads to development of a flux jump. The electric field and the temperature increase in avalanche fashion in the superconductor. In order to measure these quantities from the corresponding transducers, one takes the potential differences $U(t)$ that are induced by the motion of the vortices and the temperature $T(t)$ (see Fig. 4). Figure 5 shows a typical form of the $U(t)$ and $T(t)$ relationships. In these graphs, the flux jump process corresponds to the region of rapid rise (with a characteristic time $t_j \sim 10^{-4}-10^{-5}$ sec) in the temperature and the field strength. The further development of the signal depends on the relaxational properties of the system, and it has no direct relation to magnetic instability. Thus one measures in the experiment not only the magnetic field at which stability is lost in hard superconductors, but also the time of the flux jump, as well as the energy that is released in the form of heat (losses) (Fig. 6).[24,78,83,97]

A series of studies[25–30] has investigated flux jumps by "visual" observation of the Faraday effect with high-speed cinematography. This method has not only confirmed the known data, but has also permitted obtaining a set of new results. For example, the velocity and shape of a magnetic-flux front moving through a specimen have been determined.

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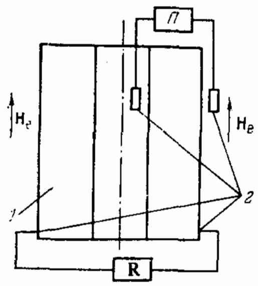

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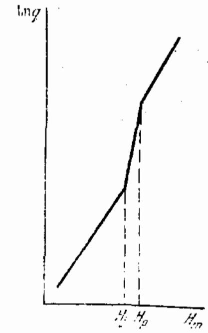

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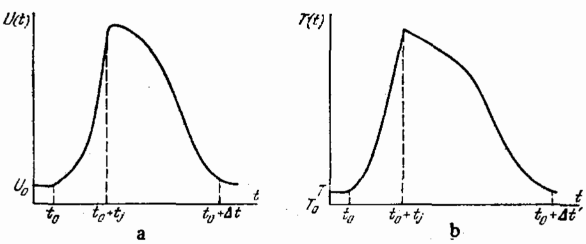

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2. THEORY OF FLUX JUMPS IN HARD SUPERCONDUCTORS

In this section we shall derive the equations that describe the development of small perturbations of the temperature and magnetic field in hard superconductors. One can solve these equations for a given magnetic-field and current distribution in a specimen and also for assigned thermal and electrodynamic boundary conditions. Evidently a system is stable if the initial perturbations decay with time. We shall determine the criteria for stability by starting with this condition.

A. Stability of the critical state (Bean's equation of state)

Let us study a specimen having a plane geometry (Fig. 7); the initial specimen temperature is T, while a small deviation in the latter is $\theta(\theta \ll T_c - T)$. In an approximation linear in θ, the heat-conduction equation has the form

$$\nu_s \frac{\partial\theta}{\partial t} = \varkappa_s \frac{\partial^2\theta}{\partial x^2} + j_c E. \tag{2.1}$$

Here $\nu_s = \nu_s(T)$, $\varkappa_s = \varkappa_s(T)$, and $j_c = j_c(T)$ are the heat capacity, the heat conductivity, and the critical current of the superconductor, respectively.

We shall use the Maxwell equation (1.5) in order to determine the electric field E. Since $j_c = j_c(T)$, we get the following expression for $\partial j/\partial t$ in the approximation linear in θ:

$$\frac{\partial j}{\partial t} = \frac{\partial j_c}{\partial T}\dot{\theta} + \sigma_f \dot{E}.$$

We shall seek $\theta(t)$ in the form

$$\theta(t) = \chi(x/b)\exp\left(\frac{\lambda t \varkappa_s}{\nu_s b^2}\right),$$

where λ is an eigenvalue to be determined. Eliminating E from (2.1) and (1.5), we can easily find the equation for χ[32,33]:

$$\chi^{IV} - \lambda(1+\tau)\chi^{II} - \lambda(\beta - \lambda\tau)\chi = 0. \tag{2.2}$$

We stress that we have accounted in (2.2) for both thermal and magnetic diffusion.

We should impose four boundary conditions on Eq.

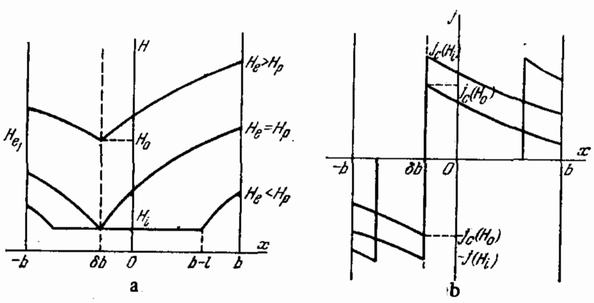

FIG. 7. Plane specimen (geometry of the problem). a) Magnetic-flux distribution; H_i is the field created by the magnetic flux frozen inside the superconductor; b) distribution of the current $j(x)$ through the cross section of the specimen.

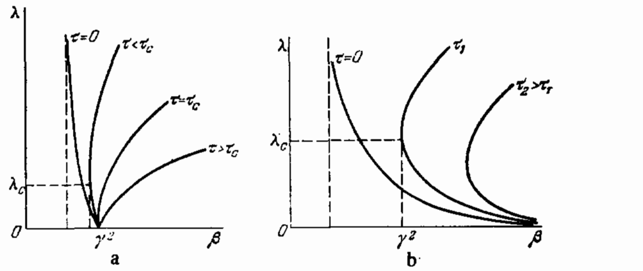

FIG. 8. Qualitative $\lambda(\beta)$ relationship for various values of τ. (a) Adiabatic boundary conditions ($W=0$), (b) finite heat removal ($0<W<\infty$).

(2.2). We shall assume the external field to be constant during the flux jump[4)]:

$$\frac{\partial H(\pm b)}{\partial t} = 0 \quad \text{or} \quad \lambda\chi'(\pm 1) - \chi'''(\pm 1) = 0. \tag{2.3}$$

Two more boundary conditions are fixed by the nature of the cooling at the surface:

$$W_0\theta(\pm b) \pm \varkappa_s \frac{\partial\theta(\pm b)}{\partial x} = 0$$

or

$$W\chi(\pm 1) \pm \chi'(\pm 1) = 0, \quad W = W_0 b/\varkappa_s, \tag{2.4}$$

where W_0 is the heat-transfer coefficient from the superconductor to the refrigerant.[5)]

When $x = \delta b$ (the definition of δ is evident from Fig. 7), or $x = b - l$ for $H_e < H_p$, $j(x)$ vanishes. ($H_p = 4\pi b(1 + |\delta|)j_c/c$ is the field for total penetration of the external field into the specimen.) Hence we can naturally require that

$$E(\delta b) = 0 \quad \text{or} \quad \lambda\chi(\delta) - \chi''(\delta) = 0. \tag{2.5}$$

Moreover, at $x = \delta b$, the temperature and the heat flux are continuous:

$$\begin{aligned} \chi(\delta+0) &= \chi(\delta-0), \\ \chi'(\delta+0) &= \chi'(\delta-0). \end{aligned} \tag{2.6}$$

A nontrivial solution of Eq. (2.2) with the boundary conditions (2.3)–(2.6) exists only for certain values $\lambda = \lambda(\beta, \tau)$. Evidently, the region of instability is defined by the condition $\lambda > 0$. Figure 8 shows the qualitative $\lambda - \beta$ relationship for the first positive value of λ for various values of τ and W. We see that instability first arises at $\beta = \gamma^2$, while the criterion for stability has the form

$$\beta < \gamma^2(\tau, W, \delta, \ldots).$$

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4)The requirement actually means that $(\partial H_e/\partial t)t_j \ll H_e$. Since $t_j \sim 10^{-4}-10^{-5}$ sec, this assumption corresponds to ordinary experimental conditions.

5)If the refrigerant is liquid helium, then in a nucleate-boiling regime at $T \approx 4.2\,°K$, we can assume that $W \approx 10^7$ erg/sec $\cdot$ cm^2 °K.

As we saw in the Introduction, the parameter β characterizes the release of heat per unit volume of the superconductor, while τ is the ratio between the thermal and magnetic diffusion coefficients. Equation (2.2) contains the combination $\beta - \lambda\tau$. This is natural, since the normal currents damp the motion of the magnetic flux and diminish the heat release. Since $\tau \ll 1$ in hard superconductors even in weak magnetic fields ($H \simeq H_{c_1}$), we shall first treat the case $\tau = 0$. In the absence of damping processes, the most "dangerous" perturbations become the "fast" ones (perturbations with $\lambda \to \infty$), since one of the stabilizing mechanisms (heat conduction) cannot operate. Actually, when $\tau = 0$ one can show that $\lambda_c \equiv \infty$ under all thermal boundary conditions ($0 \leqslant W < \infty$), while γ does not depend on W. [32,34]

For example, in the case of a plane superconductor (see Fig. 7) for $H_e \geqslant H_p$ and with arbitrary cooling, the condition for stability has the form:

$$\beta = \frac{4\pi b^2 j_c}{c^2 \nu_s} \left| \frac{\partial j_c}{\partial T} \right| < \gamma^2 = \frac{\pi^2}{4[1 + (I/I_c)]^2}; \quad 0 \leqslant W < \infty. \tag{2.7}$$

Here $I = 2|\delta| b j_c$ is the transport current flowing through the specimen, and $I_c = 2bj_c$. The parameter γ is maximal when $I = 0$, and it falls by a factor of two when $I \approx I_c$.

When $H_e < H_p$, the role of the thickness of the current layer $b(1 + |\delta|)$ is played by the screening depth l_0 of the external magnetic field ($H(l_0) = 0$). Under our conditions, $l_0 = cH_e/4\pi j_c$. Upon substituting l_0 into (2.7), we find the well-known stability criterion[23,35,36]:

$$H_e < H_j = \sqrt{\frac{\pi^3 \nu_s j_c}{|\partial j_c / \partial T|}}. \tag{2.8}$$

If a magnetic flux that creates a constant magnetic field H_i in the volume is frozen in the superconductor, then we must evidently replace the criterion (2.8) with

$$H_e - H_i < H_j. \tag{2.9}$$

Upon comparing the criteria (2.7) and (2.8), we can easily understand that if a flux jump has not occurred at $H_e = H_p$, then it will not happen even when $H_e > H_p$, since the left-hand side of the inequality (2.7) does not depend on H_e. Hence the inequality (2.7) determines the critical thickness b_c of the specimen. When $b < b_c$, flux jumps do not arise in the specimen. We can conveniently rewrite the inequality (2.7) in the form

$$b^2 < b_c^2 = \frac{\pi c^2 \nu_s}{16 j_c |\partial j_c / \partial T| [1 + (I/I_c)]^2}.$$

We can study the effect of having $\tau \ll 1$ on the stability by a method that was proposed in[33,34]. The pertinent calculation leads to the following formulas (for simplicity, we shall take $I = 0$):

$$\lambda_c = \frac{\pi^2}{4} \tau^{-1/2}; \quad \gamma^2(\tau) = \frac{\pi^2}{4}(1 + 2\tau^{1/2}) \quad \text{where } W = 0, \tag{2.10}$$

$$\lambda_c = \frac{\pi^{4/3}}{2^{4/3}} \tau^{-1/3}; \quad \gamma^2(\tau) = \frac{\pi^2}{4}(1 + 2{,}2\tau^{1/3}) \quad \text{where } W = \infty. \tag{2.11}$$

Upon comparing the criteria (2.10) and (2.11) with each other, we see that the effect of thermal diffusion on stability for hard superconductors is extremely small,[6] and the role of surface cooling of the specimen is correspondingly small. [20,23] As we should expect, losses of stability for small τ involve the onset of rapidly rising (adiabatic) perturbations ($\lambda_c \gg 1$). Although the flux jump happens adiabatically, the coupled character of the propagation of temperature and electromagnetic-field perturbations leads to the following conditions[7]:

$$t_m \ll t_j \ll t_\varkappa,$$

where $t_\varkappa = b^2/D_t$ and $t_m = b^2/D_m$ are respectively the thermal and magnetic diffusion times.

Thus, while thermal diffusion fundamentally does not affect the approximation to the stability criterion, it hinders the motion of the magnetic flux, and thus governs the characteristic time t_j for development of the perturbation.

The fact that adiabatic perturbations can give rise to a flux jump permits us substantially to simplify the problem of determining the stability criterion. [34–36] In deriving the fundamental equation, we can directly omit the heat conductivity in the corresponding equation:

$$\nu_s \dot{\theta} = j_c E. \tag{2.12}$$

Upon adding the Maxwell equation (1.5) to (2.12) and eliminating the temperature θ from this system, we get the following expression for the electric field E:

$$E'' + \beta E = 0. \tag{2.13}$$

Here we differentiate with respect to the dimensionless variable x/b, while $\tau = 0$. We should impose on Eq. (2.13) only the electrodynamic boundary conditions:

$$E'(\pm b) = E(\delta b) = 0.$$

Evidently stability is lost ($\dot{\theta} > 0$) if (see (2.12)) there is a nontrivial solution of (2.13) with the boundary conditions that are imposed on it.

We note further that we can derive (2.13) also from (2.2) by taking the limit as $\lambda \to \infty$, $\tau = 0$, $\lambda\tau \to 0$. The given derivation explains the nature of the course of the processes, and it permits us to select the correct boundary conditions without taking a corresponding limit.

Since $E(\delta b) = 0$, instability develops independently in the two regions $x < \delta b$ and $x > \delta b$. The stability of the entire system is determined by the least stable region.

The heat capacity ν_s of the superconductor and the critical current density j_c are functions of the temperature. Therefore the flux-jump field H_j also depends on the temperature. [36] Figure 9 shows a characteristic

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[6]One can detect the effect of thermal diffusion experimentally from the variation in H_j for different conditions of heat transport from the surface. The expected value $\Delta H_j \sim 5$–10%.

[7]$t_j \sim \nu_s b^2/\varkappa_s \lambda_c = t_\varkappa/\lambda_c \sim t_\varkappa \tau^p \sim t_m \tau^{-q}$, $p > 0$, $q > 0$.

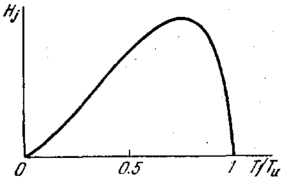

FIG. 9. H_j is a function of T.

$H_j = H_j(T)$ curve for a hard superconductor. In particular, if we assume that

$$\nu_s = \nu_0 (T/T_c)^3, \quad j_c = j_1 [1 - (T/T_u)],$$

where $T_u \sim T_c$ (see Fig. 2b), then we can easily find from (2.8) the following expression for $H_j(T)$:

$$H_j = \sqrt{\frac{\pi^3 \nu_0}{T_c^3}} \sqrt{T^3 (T_u - T)} \ . \tag{2.14}$$

Here H_j has a maximum (see Fig. 9) at $T = 0.75\ T_u$. We recall that the temperature region that is too close to T_c is not treated here. On the other hand, the approximation $\tau \ll 1$ usually does not hold ($\tau \sim T^{-3}$) at low enough temperatures ($T < 1$ °K). Yet one can show even in this case that H_j falls with decreasing T, and vanishes at $T = 0$.

B. Stability of the critical state (equation of state of general form)

As we have seen, rapidly growing (adiabatic) perturbations are weakly damped (within limits as $\tau \ll 1$) for hard superconductors, and they lead to the maximum possible heating of the specimen. Evidently this assertion depends neither on the model of the critical state nor on the geometry of the problem. Thus one can find the stability criterion in the fundamental approximation by assuming that $\tau = 0$.[8)]

Upon neglecting the density of the normal currents and the heat conductivity, we can easily derive an equation for the electric field E[37]:

$$E'' + \alpha(x) E' + \beta(x) E = 0, \tag{2.15}$$

where

$$\alpha(x) = -\frac{4\pi b}{c} \frac{\partial j_c}{\partial H}, \quad \beta(x) = -\frac{4\pi b^2}{c^2} \frac{j_c(x)}{\nu_s} \frac{\partial j_c(x)}{\partial T}.$$

We should impose the following electrodynamic boundary conditions on Eq. (2.15): $E'(\pm b) = E(\delta b) = 0$, which coincides with (2.3) and (2.5).

Before we proceed to solve (2.15), let us make a change of variable:

$$y = \frac{H_e - H(x)}{H_e - H_i},$$

whereupon (2.15) acquires the standard form

$$E'' + \tilde{\beta} E = 0, \tag{2.16}$$

where

$$\tilde{\beta} = \frac{(H_e - H_i)^2}{4\pi \nu_s T_1(H)}, \quad T_1(H) = \frac{j_c}{|\partial j_c/\partial T|}.$$

The boundary conditions for $E(y)$ are now written as:

$$E(\pm 1) = 0, \quad E'(0) = 0. \tag{2.17}$$

We note that if $j_c(H, T) = j_0(T)\phi(H)$, then T_1 does not depend on H, and hence not on y. Equation (2.16) can be solved exactly, and the stability criterion has the form

$$H_e - H_i \leqslant H_j, \quad \text{where } H_j^2 = \frac{\pi^3 \nu_s j_0(T)}{|dj_0/dT|}. \tag{2.18}$$

In the general case, one can solve (2.16) if the condition $(d/dy)(1/\sqrt{\tilde{\beta}}) < 1$ is satisfied. This is equivalent to:

$$\frac{H_e - H_i}{T_1(H)} \left| \frac{dT_1(H)}{dH} \right| < 1 \tag{2.19}$$

and it allows us to apply the WKBJ method to (2.16). Upon using the standard WKBJ solution from the boundary conditions (2.17), we can easily get the stability criterion in the form[38,39]:

$$\int_0^1 \sqrt{\tilde{\beta}}\, dy = \frac{1}{\sqrt{4\pi\nu_s}} \int_{H_i}^{H_e} \frac{dH}{\sqrt{T_1(H)}} = \int_{\delta b}^{b} \sqrt{\beta(x)}\, dx < \frac{\pi}{2}. \tag{2.20}$$

The relative accuracy with which the criterion (2.20) holds is

$$\frac{(H_e - H_i)^2}{\pi^2} \left(\frac{1}{T_1} \frac{dT_1}{dH} \right)^2 \ll 1. \tag{2.21}$$

In a weak magnetic field ($H_e \ll H_{c_2}$), $(1/T_1)(dT_1/dH) \sim 1/H_{c_2}$, and the conditions (2.19) and (2.21) are satisfied with much room to spare. Yet if $H_e \sim H_{c_2}$, then good accuracy in applying (2.20) is ensured by a small numerical coefficient in (2.21). Thus, the criterion (2.20) can be successfully applied throughout the range of external fields.

We have assumed thus far that the critical current density is a rather smooth function of the magnetic field. Yet a number of materials show a sharp break in the $j_c(H)$ relationship in the strong-field region (see Fig. 2, with a break at $H = H_k$). This situation has been treated in[37]. It turns out that a case can happen here in which flux jumps arise in two isolated external-field regions: $H_e \sim H_p$ and $H_e \sim H_k$. Under certain conditions, the critical state is least stable precisely at $H_e \sim H_k$.

C. Critical current of a superconducting wire

Let us treat now the stability of the critical state in cylindrical specimens.[34,40] Since we are interested in the effect of the geometry of the current and field distribution, we shall restrict ourselves to Bean's model and the case $\tau = 0$.

We shall determine in this section the maximum trans-

8)A rigorous proof of these statements is given in[38].

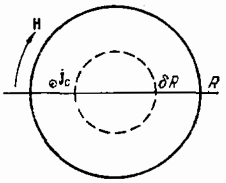

FIG. 10. Superconducting wire (geometry of the problem).

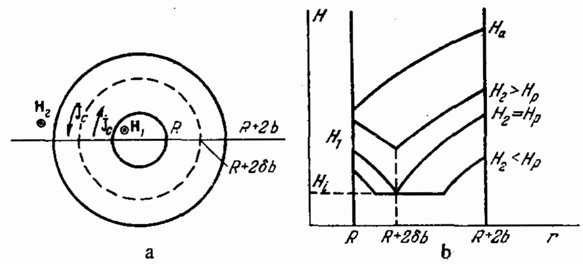

FIG. 12. Cylindrical specimen in an external magnetic field. (a) Geometry of the problem; (b) magnetic-field distribution, H_2—external field, H_1—field in the cavity of the specimen; when $H_c=H_a$, the external flux begins to enter the cavity of the tube.

port current I_m that a superconducting wire of radius R can transmit without losses (Fig. 10).

Since $\tau=0$, the onset of instability involves fast perturbations ($\lambda_c\gg 1$), and heat conduction is not essential. By analogy to the case of plane geometry, we can show that the stability criterion has the form $\beta<\gamma^2$, where:

$$\beta=\left(\frac{R}{R_0}\right)^2=\frac{4\pi R^2 j_c}{c^2\nu_s}\left|\frac{\partial j_c}{\partial T}\right|;\qquad \delta=\sqrt{1-\frac{I}{I_c}}. \tag{2.22}$$

$I_c=\pi R^2 j_c$, and I is the transport current flowing in the wire. The parameter γ is determined from the equation

$$N_1(\gamma)J_0(\delta\gamma)-N_0(\delta\gamma)J_1(\gamma)=0. \tag{2.23}$$

Here J_0, J_1, N_0, and N_1 are the Bessel and Neumann functions of zero and first order, respectively. The critical value of the transport current I_m is determined from the condition $\beta=(R/R_0)^2=\gamma^2(I_m)$. Figure 11 shows the ratio I_m/I_c found by using (2.23) (curve 1) as a function of the dimensionless radius R/R_0 of the wire. When $I\ll I_c$, the stability criterion for the wire naturally agrees with the results obtained for a plane specimen ($H_e<H_p$, where H_e is the intrinsic field of the current).

We see that I_m is always smaller than I_c. That is, a flux jump necessarily occurs with increasing current in a wire of any radius. It is not hard to understand what this involves. If a flux jump has not been elicited by fluctuations or an external agent, then the temperature of the specimen will increase by θ, and an equilibrium distribution of the currents and the field will be established in the specimen at the new temperature (see the Introduction). However, under certain conditions such an equilibrium situation may not exist. Thus, if $\delta=0$ (i.e., $I=I_c$), a state having the given transport current cannot be realized with changing (rising) temperature. Hence, as we know, the critical state is unstable for $\delta=0$—the degree of freedom needed for stability disappears near these δ values. To illustrate, let us estimate the value of R_0 for the alloy Nb–25% Zr at $T\approx 4$ °K. The parameters of interest to us are: $j_c=3\times10^5$ A/cm², $\nu_s=1.5\times10^4$ erg/cm³ °K, $\partial j_c/\partial T\sim -j_c/(T_c-T)$, where $T_c=10$ °K, whence $R_0\approx 3\times10^{-3}$ cm.

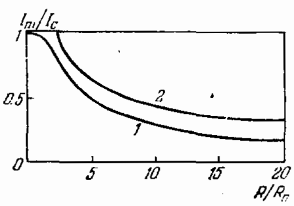

FIG. 11. Plot of I_m/I_c vs R/R_0 for a wire. Curve 1—wire without a coating, 2—wire covered with a layer of normal metal ($d>d_c'$).

D. Cylindrical specimen in an external magnetic field

In this section we shall determine the stability criterion with respect to flux jumps for a tube placed in an external magnetic field lying parallel to its axis (Fig. 12).[34] The equation for the electric field E analogous to (2.13) has the form

$$E''+\frac{1}{r}E'+\left(\beta-\frac{1}{r^2}\right)E=0, \tag{2.24}$$

where

$$\beta=\frac{4\pi b^2 j_c}{c^2\nu_s}\left|\frac{\partial j_c}{\partial T}\right|,$$

while the coordinate r is normalized to the half-thickness b of the wall of the tube. The radius of the inner cavity of the tube is R, the field in the cavity is H_1, and the external field is H_2 (see Fig. 12). For example, when $H_e>H_p$, the boundary conditions have the form

$$E'+E/r=0\quad\text{for}\quad r=\frac{R}{b},\ \ 2+\frac{R}{b},$$
$$E=0\quad\text{for}\quad j(r)=0.$$

We can easily find the equation for determining the parameter γ in the stability criterion (1.7) by substituting the solution of (2.24) into the boundary conditions.

Figure 13 shows the relationship of the parameter γ to the magnetic field. We see from this diagram that the field gradients before the jump with entering (H_2-H_i for $H_2<H_p$; H_2-H_1 for $H_2\geqslant H_p$) and exiting (H_1-H_i and H_1-H_2, respectively) magnetic flux differ. This difference vanishes with increasing inner radius R of the tube ($R\gg b$, planar limit). We note also that, in contrast to the case of the plane geometry, the size of the critical magnetic-field gradient H_j depends on the critical current density

Let us take up another special case. Let the external field H_2 have a value such that δ vanishes: $H_2=H_a$—the screening currents flow throughout the wall of the cyl-

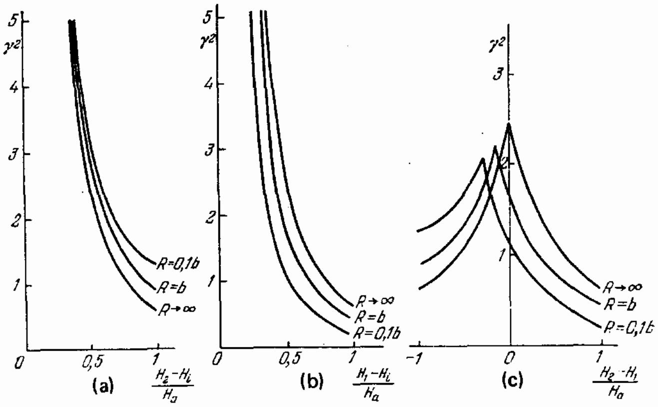

FIG. 13. Relationship of the parameter γ to the magnetic-field gradient for different values of R (tube in an external field). (a) $H_e < H_p$, flux entering; b) $H_e < H_p$, flux exiting; c) $H_e > H_p$.

inder in a single direction. We can easily see that stability declines sharply here. The reasons for this decline are the same as for a wire as $I \to I_c$. The current $I_c = 2bj_c L$ (where L is the length of the tube) flowing in the wall of the tube is determined by the field difference $H_2 - H_1$. With increasing temperature, j_c declines. If here the quantity $H_2 - H_1$ varies little, then the current density in the specimen exceeds j_c. This leads to even greater heating, and hence to loss of stability. If the inner cavity of the tube is small ($R \lesssim b$), the field H_1 also increases appreciably as the perturbation develops. Hence the stability of the system remains finite even when $H_2 = H_a$. Yet if the tube has a large inner cavity ($R \gg b$), the system completely loses one degree of freedom, and its stability falls to zero.

For quantitative analysis, let us write the equation for the electric field E in the inner cavity of the tube

$$E'' + \frac{1}{r}E' + \left(\frac{\lambda^2 \varkappa_s^2}{\nu_s^2 c^2} - \frac{1}{r^2}\right)E = 0.$$

The value of $\lambda_c \lesssim 10^3 - 10^4$, and the first term in the parentheses is small ($\sim 10^{-15}$). We shall find from the remaining equation a solution for E that is nonsingular at the origin in the form: $E = Ar$ (A = const.). If we match it at $r = R/b$ in terms of continuity of E and E' with the solution of (2.24) that satisfies the boundary condition $E'(R/b + 2) = 0$, we can easily find the relationship of the parameter γ to R/b (Fig. 14).

3. THEORY OF MAGNETIC INSTABILITIES IN COMBINED SUPERCONDUCTORS

In this chapter we shall treat the stability of the critical state in hard superconductors that exist in contact with a normal metal. The combination of normal and superconducting conductors in a specimen can be highly varied, starting with a superconductor covered with a layer of normal metal and ending with a regular structure of superconducting inclusions (fibers) in a matrix of normal metal (a combined superconductor).

The presence of a normal metal having good conductivity leads to strong damping of fast perturbations in the specimen. Since they are precisely what leads to flux jumps in hard superconductors, the stability of the critical state should increase. On the other hand, if a region of the superconducting circuit drops out of service for any reason, the normal metal shunts the damage, and it thus hinders a catastrophic transition of the whole system to the normal state.[41] Thus the study of stability of the critical state in superconductors that exist in contact with a normal metal seems highly interesting.

A. Contact of the superconductor with normal metal and stability of the critical state

The method proposed in Chap. 2 permits us to treat magnetic instabilities in hard superconductors covered with a layer of normal metal of arbitrary thickness d.

We shall assume that the normal metal of the coating has a heat conductivity[9)] $\varkappa_n$ that satisfies the relationship $\varkappa_n \gg \varkappa_s$, while the heat capacity satisfies $\nu_n \sim \nu_s$. Since $j_c \sim 10^4 - 10^6$ A/cm^2, we have $\sigma_n E \ll j_c$, in any case in the development of a small perturbation, and the heat release in the normal metal ($\sigma_n E^2$) is considerably less than in the superconductor ($j_c E$). From what we've said, we can evidently assume when $d \ll b$ that the thermal conditions at the superconductor–coating boundary and at the coating-outer medium boundary coincide. However, if $d \gg b$, the coating sharply improves the heat removal from the superconductor, and it actually leads to isothermal conditions ($\theta = 0$) at the superconductor–coating boundary. Consequently, both when $d \ll b$ and when $d \gg b$, we can restrict the treatment within the normal metal to electrodynamic processes alone.

First let us study the stability of the critical state in a specimen having a plane geometry (see Fig. 7). The Maxwell equation (1.5) for the electric field E in the coating has the form

$$E'' + \lambda\tau' E = 0. \qquad (3.1)$$

Here $\tau' = \sigma_n \tau / \sigma_f \gg \tau$. Just as in Chap. 2, we seek the relationship of the field E to the time t in the form $E(t) \sim \exp(\lambda t \varkappa_s / \nu_s b^2)$. Evidently the electric field E and its derivative E' are continuous at the normal metal–superconductor boundary. Moreover, the thermal boundary condition $\theta' \pm W\theta = 0$ is satisfied. In addition, we shall assume that the magnetic field does not vary at the normal metal–outer medium boundary during the time of the jump, i.e., $\partial H/\partial t = E' = 0$.

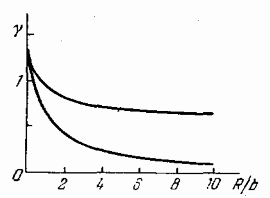

FIG. 14. Graph of the function $\gamma(R/b)$ (tube in an external field). Upper curve—$H_2 < H_a$ ($H_2 = H_a = 0$), lower curve—$H_2 > H_a$.

[9)] Quantities that refer to the normal conductor are denoted with the subscript n, and those that refer to the superconductor with s.

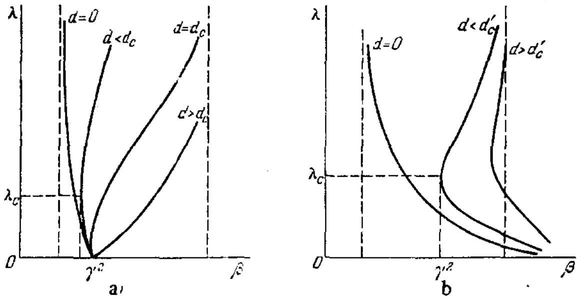

FIG. 15. Curves of $\lambda(\beta)$ for various values of d. a) Adiabatic conditions at the boundary of the superconductor; b) isothermal cooling.

As we know,[42] in a normal metal an ac electromagnetic field penetrates only to the depth of the skin layer δ_{sk}. We see from (3.1) that in our notation $\delta_{sk}(\lambda) = b(\lambda\tau')^{-1/2}$. If $\lambda=\lambda(\beta)\gg 1$, then we have $\delta_{sk}\to 0$, and the electric field E vanishes at the superconductor–coating boundary. With such a boundary condition, we can easily find from (2.2) (with $\tau=0$) that the $\lambda(\beta)$ curve asymptotically approaches the straight line $\beta=\pi^2$ (Fig. 15). Thus the presence of the normal metal strongly deforms and shifts the $\lambda=\lambda(\beta)$ curve for $\lambda\gg 1$, independently of the thickness of the coating and the thermal boundary conditions.

In the region $\lambda\ll 1$, the effect of the coating on the perturbation spectrum is less appreciable (if the coating does not change the heat-removal regime from the superconductor). In particular, under adiabatic conditions the position does not change of the point $\beta=\beta_0$ at which $\lambda(\beta_0)=0$,[32] since screening currents are not induced in the normal metal when $\lambda=0$.

Thus, with $d\ll b$ and with external thermal insulation, the coating cannot raise the value of γ^2 beyond $\gamma^2=3$. Let us see how the transition occurs from $\gamma=\pi/2\,(d=0)$ to $\gamma=\sqrt{3}$. As we increase the thickness of the coating (in the region $d\ll b$), the $\lambda=\lambda(\beta)$ curve is deformed and it shifts to the right from the line $\beta=3$, while asymptotically approaching the line $\beta=\pi^2$ (see Fig. 15a). Starting at some value $d=d_c=2b/105\tau'$, the entire branch $\lambda(\beta)>0$ lies in the region $\beta>3$.[32]

Under isothermal boundary conditions, perturbations having $\lambda\ll 1$ can appear only in the region $\beta\gg 1$ (see Fig. 15b). Thus we can increase γ^2 to the value $\gamma^2=\pi^2$ by using a coating. When $\pi^2\tau'\gg 1$ (which we know to be satisfied for characteristic values of τ'), Eq. (2.2) and the boundary conditions imply that $\lambda_c\gg 1$. The quantities λ_c and γ depend on the damping properties of the normal metal. The substantial increase in γ with increasing d occurs in a range of d below some critical value d'_c. Then λ_c and γ cease to depend on d, and they are determined solely by the values of τ'[40]:

$$\lambda_c=\pi^4\tau',\quad \gamma^2=\pi^2\left(1-\frac{1}{\pi^2\tau'}\right),\quad d>d'_c. \tag{3.2}$$

This effect has been detected experimentally in[98]. Evidently the critical thickness of the coating is of the order of the depth of the skin layer δ_{sk} for $\lambda=\lambda_c$. Upon defining d'_c as $3\delta_{sk}$, we can easily get an estimate $(I=0)$[40]:

$$d'_c=\frac{3b}{\pi^2\tau'}=\frac{3c^2\nu_s b}{4\pi^3\varkappa_s\sigma_n}. \tag{3.3}$$

Let us see how instability develops when there is good heat removal from the volume of the superconductor. Since here $\lambda_c\gg 1$, a perturbation of the magnetic field and the temperature, grows sharply while the electric field E vanishes simultaneously at the two boundaries of the specimen. Hence the total magnetic flux in the superconductor is unchanged. This means that at the onset the magnetic flux rapidly (within a time of the order of $t_\varkappa/\lambda_c$) becomes redistributed within the superconductor, and then the final distribution of the magnetic field and the currents is slowly established (within the diffusion time of the magnetic field through the normal metal).[32,90,97] Onishi[43] has observed this effect experimentally.

As we have seen, the stability of the critical state is in many ways determined specifically by the perturbations having $\lambda_c\gg 1$. Instability with respect to them is absolute in the sense that γ cannot (within limits as $\lambda_c\gg 1$) become elevated by any external agents (improved heat removal, increased thickness or conductivity of the coating, etc.). Thus the maximum attainable values of γ are determined by the onset of growing perturbations having $\lambda\gg 1$.

The effect of the geometry of the current and field distribution, as well as the role of the $j_c(H)$ relationship, can also be treated for specimens coated with a normal metal.

Figure 11 (curve 2) shows the $I_m(R/R_c)$ relationship for a wire (see Fig. 10) coated with a normal metal[40] $(d>d'_c)$. In contrast to the case of a wire lacking a normal coating, the parameter γ does not vanish as $\delta\to 0$ $(I\to I_c)$, since here one of the degrees of freedom of the system does not disappear—the currents that compensate the decline in j_c arise in the coating. If the radius R of the wire is less than $R_c\approx 2.4R_0$, flux jumps do not arise up to $I=I_c$. In the example that we treated earlier of Nb–25% Zr (Sec. C of Chap. 2), the characteristic value is $\varkappa_s\doteq 4\times 10^3$ erg/cm·sec·°K. If a wire made of this material is covered with a pure metal (Cu or Al), then $\tau'\approx 1$, while we get the following estimates for R_c and d'_c: $R_c\simeq 7\times 10^{-3}$ cm; $d_c(R_c, I_c)\approx 5\times 10^{-3}$ cm.

B. Flux jumps in combined superconductors

We shall treat in this section the problem of stability of the critical state of a combined superconductor (a matrix of a normal metal containing a regular structure of superconducting regions embedded in it, or fibers in the critical state). The number N of superconducting fibers in the cross section of the specimen varies from 2–3 to several tens and even hundreds. One can use as the matrix either metals of good conductivity (Cu, Al), or various alloys of lesser conductivity.[41,44,79,87,94] In such a combined superconductor, instability can be associated not only with losses of stability of any of the superconducting regions, but also with onset of collective effects—loss of stability of the entire distribution of current and flux as a whole. Naturally, the case of greatest interest here is that of materials having $N \gg 1$. We shall derive the conditions below under which a flux jump develops in a combined superconductor made of stabilized fibers.

For a quantitative description of collective effects, we must derive the equation for propagation of a small perturbation through a combined superconductor, as averaged over regions whose dimensions include enough structural elements of the combined superconductor, yet are smaller than the dimensions of the specimen itself. Evidently such an equation is valid up to the point where its solution varies over distances larger than the characteristic dimension of the structure, while the time for equalizing the perturbation over the scale of the structure is much less than the corresponding time of variation of the entire solution. After averaging, we get the equations for θ and E:

$$\bar{\nu}\dot{\theta} = \bar{\varkappa}\,\Delta\theta + \mathbf{j}_0\mathbf{E} \tag{3.4}$$

and the relationship of the current density j to the field E:

$$\mathbf{j} = \mathbf{j}_0 + \sigma\mathbf{E}. \tag{3.5}$$

The quantities $\bar{\nu}$, j_0, $\bar{\sigma}$, and $\bar{\varkappa}$ that enter into (3.4) and (3.5) are the averages of the heat capacity, the critical current density (here we assume that the superconducting currents flow in the same direction inside the region of averaging), and the electric and thermal conductivities, respectively. Let us denote the relative concentration of the superconducting metal as x_s, and that of the normal metal as $x_n (x_s + x_n = 1)$. Then

$$\bar{\nu} = x_n\nu_n + x_s\nu_s, \quad j_0 = x_s j_c, \quad \bar{\sigma}_n = x_s\sigma_f + x_n\sigma_n.$$

The averaged value of the thermal conductivity transverse to the structure of the combined superconductor is determined by the details of the structure. As we can easily verify, a good estimate is $\bar{\varkappa} = (1 - \sqrt{x_s})\varkappa_n$.

For the following treatment we must choose a model of the critical state. Here we shall restrict the choice to Bean's model. We can generalize to the case of an arbitrary $j_c(H)$ relationship by the methods presented in Sec. B of Chap. 2.

Upon eliminating E from Eqs. (3.4) and (3.5), we can easily derive the equation for θ[45]:

$$\theta^{\mathrm{IV}} - \lambda(1 + \bar{\tau})\,\theta'' + \lambda(\lambda\bar{\tau} - \bar{\beta})\,\theta = 0. \tag{3.6}$$

In (3.6) the spatial derivatives are taken with respect to the variable $\mathbf{r}/b$, where b is the characteristic dimension of the specimen, while the $\theta(t)$ and $E(t)$ relationships are taken in the form θ, $E \sim \exp\{\lambda t\bar{\varkappa}/\bar{\nu}b^2\}$. In order to determine $\lambda = \lambda(\bar{\beta}, \bar{\tau})$, we must impose the usual thermal and electrodynamic boundary conditions on (3.6).

As a rule, $\bar{\tau}$ for combined superconductors is greater than unity, and it can attain values up to 10^2–10^4. If $\bar{\tau} \gg 1$, then heat redistribution runs much faster than the diffusion of magnetic flux (see the Introduction). Hence $\lambda_c \ll 1$, and the thermal boundary conditions play an essential role.

For example, let us examine the problem of stability of a plate made of a combined superconductor having isothermal conditions at its boundary. The $\lambda = \lambda(\bar{\beta}, \bar{\tau})$ curves have a form analogous to that illustrated in Fig. 8b. The value of γ^2 at which a root $\lambda > 0$ first appears is $(\bar{\tau} \gg 1)$[45]:

$$\gamma^2 = \frac{\pi^2}{4}\bar{\tau} + 3\left(\frac{\pi}{2}\right)^{4/3}\bar{\tau}^{2/3}, \tag{3.7}$$

while the value of λ_c is:

$$\lambda_c = \left(\frac{\pi}{2}\right)^{4/3}\bar{\tau}^{-1/3}. \tag{3.8}$$

Analogously, when $\bar{\tau}^{-1} \ll W \ll 1$, we can easily get the corresponding expression for λ_c:

$$\lambda_c = \left(\frac{W}{2}\right)^{2/3}\bar{\tau}^{-1/3}.$$

In the fundamental approximation with $\bar{\tau} \gg 1$, the criterion (3.7) coincides with the known expression[46,47] that has been found from qualitative arguments.

Let us now derive the criterion for applicability of (3.6). The characteristic structural scale of the combined superconductor is $b/\sqrt{N}$, while the minimal scale of variations in the solutions is the depth δ_{sk} of the skin layer in the normal metal. In the frequency region $\lambda \sim \lambda_c$ that is of interest to us, we evidently have $\delta_{sk} \sim b/\sqrt{\lambda_c\bar{\tau}}$. Hence the following condition must be satisfied:

$$N \gg \lambda_c\bar{\tau}. \tag{3.9}$$

In temperature equalization, the thermal diffusion in the superconducting fiber occurs slowest of all, and the corresponding time t_s proves to be of the order of

$$t_s \sim \frac{\nu_s x_s b^2}{\varkappa_s N},$$

while the characteristic time t_j of a flux jump is:

$$t_j \sim \frac{\bar{\nu}b^2}{\lambda_c\bar{\varkappa}}.$$

We get from the condition $t_j \gg t_s$ the second condition for applicability of (3.6):

$$N \gg \frac{\lambda_c\bar{\varkappa}\nu_s}{\varkappa_s\bar{\nu}}. \tag{3.10}$$

In particular, when $W \ll 1$, Eqs. (3.9) and (3.10) imply that

$$N \gg W\bar{\tau}^{2/3}, \quad N \gg W\bar{\tau}^{-1/3}\frac{\bar{\varkappa}\nu_s}{\varkappa_s\bar{\nu}} \qquad (\bar{\tau} \gg 1).$$

The condition $\bar{\tau} \gg 1$ permits us in many cases to simplify substantially the problem of studying stability.[45] In fact, if we substitute the explicit current-field relationship (3.5) into the Maxwell equation (1.5), we get

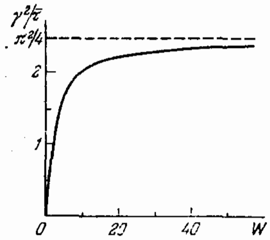

FIG. 16. Relationship of $\gamma^2\sqrt{\bar{\tau}}$ to the heat removal W for a combined superconductor ($W \gg \bar{\tau}^{-1}$, $\bar{\tau} \gg 1$).

$$\Delta E = \lambda\bar{\tau}\left(E + \frac{\theta}{\bar{\sigma}}\frac{dj_0}{dT}\right).$$

If the heat removal is not too small ($W \gg \bar{\tau}^{-1}$), then, as we can easily show by using Eq. (3.6) with the appropriate boundary conditions, the characteristic time of the flux jump is much smaller than the magnetic-diffusion time in the specimen (see the Introduction). Hence we find that $\lambda_c\bar{\tau} \gg 1$ (in particular, with ideal refrigeration, $\lambda_c\bar{\tau} \sim \bar{\tau}^{2/3}$). Since ΔE is a finite quantity, we have in the fundamental approximation:

$$\frac{\partial j}{\partial t} = \frac{\partial}{\partial t}\left(\bar{\sigma}E + \theta\frac{dj_0}{dT}\right) = 0.$$

This gives the relation between θ and E. Upon substituting the found relationship into the heat-conduction equation, we get an equation for θ in the stated approximation:

$$\Delta\theta + \left(\frac{\bar{\beta}}{\bar{\tau}} - \lambda\right)\theta = 0. \qquad (3.11)$$

Hart[46,47] has derived this equation from qualitative arguments.

We should evidently impose on Eq. (3.11) the usual thermal boundary conditions, whereupon we can easily find the corresponding solution for specimens having various geometries and current and magnetic-field distributions; existence of solutions having $\lambda > 0$ implies loss of stability. Evidently a solution exists in all cases if

$$\sqrt{\frac{\bar{\beta}}{\bar{\tau}} - \lambda} = \gamma_1. \qquad (3.12)$$

Here γ_1 is some eigenvalue of the problem to be solved. Eq. (3.12) implies that $\lambda = \bar{\beta}/\bar{\tau} - \gamma_1^2$, and hence,

$$\gamma^2 = \gamma_1^2\bar{\tau}.$$

The condition $\partial j/\partial t = 0$ that we have used implies that instability sets in at a frozen magnetic flux in the fundamental approximation with $\bar{\tau} \gg 1$. One can also derive (3.11) directly from (3.6) in the limit as $\bar{\tau}$, $\lambda\bar{\tau} \to \infty$. The given derivation merely explains the nature of the course of the process.

For a hard superconductor, stability breaks down independently in different regions of the specimen that differ in direction of current. In the studied case where $\bar{\tau} \gg 1$, instability sets in immediately throughout the volume of the superconductor ($H_e \geq H_p$), since $t_j \gg t_\kappa$, and the temperature in the specimen can become equalized. Hence, in the fundamental approximation with $\bar{\tau} \gg 1$, stability does not depend on the transport current, and is determined solely by the dimension $2b$ of the region through which the current is flowing (i.e., by the amount of heat release).[45]

By using (3.11) and the thermal boundary conditions, we can easily obtain γ (see (1.7)) for an arbitrary quality of heat removal. We have the following relationship for determining $\gamma(W)$ for a flat plate (see Fig. 7):

$$\mathrm{tg}\,(\gamma\bar{\tau}^{-1/2}) = \frac{W\bar{\tau}^{1/2}}{\gamma}. \qquad (3.13)$$

Figure 16 shows a graph of $\gamma^2/\bar{\tau}$ as a function of W. In particular, if $W \ll 1$ (we recall that $W \gg \bar{\tau}^{-1}$), Eq. (3.13) implies that

$$\gamma^2 = W\bar{\tau}. \qquad (3.14)$$

As $W \to \infty$, Eq. (3.13) yields the fundamental approximation of Eq. (3.7).[10)]

Let us examine now the conditions under which collective effects arise in a combined superconductor made of stabilized elements ($b' < b_c'$; see Chap. 2, A). In this case, Eq. (3.14) implies that for

$$N > N_c \approx W\frac{\bar{\tau}\nu}{\pi^2\nu_s x_s}\left(\frac{b_c}{b'}\right)^2 \qquad (W \ll 1)$$

the system as a whole becomes unstable. The characteristic value is $W = 10^{-2}$, and when $\bar{\tau} = 10^3$, N_c proves to be of the order of unity. Thus prevention of flux jumps requires special measures: twisting or transposition.[41,44,80,86,90,93]

4. EXPERIMENTS ON FLUX JUMPS, COMPARISON OF THEORY WITH EXPERIMENT

A considerable number of studies has been devoted to experimental study of flux jumps. However, the overwhelming majority of them are hard to compare with theory, and they show a substantial scatter of data from study to study. This situation is not fortuitous, and it mainly involves lag in the onset of instability. In fact, onset of a flux jump requires a fluctuation or an appropriate external agent. This priming perturbation must be great enough to transform a considerable fraction of the volume of the superconductor into a flux-flow regime.

Therefore one must in the course of the experiment initiate flux jumps (e.g., by mechanical shock[46]) in order to determine the true stability boundary H_j of the critical state. This fact has not been taken into account in the vast majority of the studies, and instability has often been initiated by random factors.

As we have noted in the Introduction, the external

10)We note that the usual conditions of cooling with liquid helium a superconductor having a copper or aluminum matrix correspond to $W < 1$. One can realize the case of $W \gg 1$, e.g., by putting the specimen into a massive enough copper envelope.

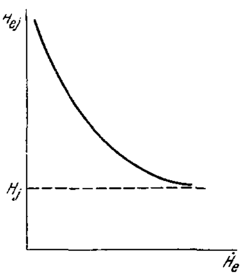

FIG. 17. Typical relationship of H_{ej} to $\dot{H}_e$.

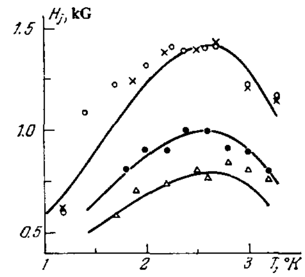

FIG. 18. Variation of H_j with temperature (comparison of theory and experiment). The experimental points for four different specimens are plotted. The theoretical curves are plotted as the solid lines, and the unknown parameters are chosen by least squares.

magnetic field is varied according to a definite law in experiments to study the stability of the critical state. An entire set of studies[24,27,30,35,49–60,85,91] has dealt with the effect of the rate of growth $\dot{H}_e$ of the magnetic field on the magnetic-field value H_{ej} at which one observes a flux jump. Figure 17 shows a characteristic relationship between H_{ej} and $\dot{H}_e$. As we can easily estimate, the relationship $\dot{H}_e t_j \ll H_e$ is known to hold under experimental conditions. Thus, as the theory implies, the true stability boundary (the field H_j) does not depend on $\dot{H}_e$. This assertion was first formulated in[35], and Harrison *et al.*[30] arrived at the same conclusion from their experiments. The role of an ac external field is reduced only to initiating flux jumps. We can easily understand that this initiation will be effective enough ($H_{ej} \simeq H_j$) in the absence of other "priming" agents if the electric field E that is caused by the variation in H_e exceeds the value E_0 in a large enough part of the volume (see Fig. 3; $E > E_0$ corresponds to transition to a flux-flow regime). This explanation easily allows us to understand also the climb of the $H_{ej}(\dot{H}_e)$ relationship up to a constant value that has been found in a number of experiments.[11)]

Now let us examine the fundamental results of the experimental studies of magnetic instabilities in hard superconductors and compare them with the theoretical results.

As we have seen, flux jumps arise only when $\partial j_c/\partial T < 0$. Hence, in the region of the peak effect (see, e.g.,[3,4]), where $\partial j_c/\partial T > 0$, the critical state is absolutely stable.[61,62,79] An entire series of studies has firmly established[62–66] that stability increases with decreasing value of $|\partial j_c/\partial T|$, and flux jumps are absent when $\partial j_c/\partial T > 0$.

The heat capacity of the specimen influences the stability very effectively. An entire set of experiments has investigated the corresponding relationship in detail. In particular, porous superconductors have been studied.[67] Helium becomes superfluid and flows into the pores below the λ-point. The heat capacity rises, and H_j is correspondingly increased (see[23,67,68,89]). It has been shown in[77] that $H_j \sim \sqrt{\nu_s}$.

The dependence of stability on the specimen temperature has been studied in[20,53–57,69,84,95]. Figure 18 gives data obtained in[54,55] for synthetic specimens (porous glass with In pressed into the pores). At rates of introduction of the external field $\dot{H}_e > 10^2$ gauss/sec, the flux-jump field ceases to depend on $\dot{H}_e$, and we can naturally assume here that $H_{ej} \approx H_j$ (see[54,55]). The $H_j(T)$ curve constructed by using (2.14) shows good agreement with experiment.

It was shown in[23] that stability does not depend on the value of j_c in the external-field region where $H_e \leq H_p$ (see the criteria (2.8) and (2.8′)). As j_c varies by a factor of about three (which corresponded to increasing the magnetic field H_e from 5 to 30 kilogauss), H_j declined by only 5% (which might, e.g., be explained by the relationship of $\partial j_c/\partial T$ to H: $\partial j_c/\partial T \sim -j_c/T(1 - H/H_{c_2})$. A number of subsequent experiments have observed periodic flux jumps with increasing external field.[20,68,95] The increase ΔH_e in the external field between successive jumps depended little on H_e. This evidently confirms the conclusion that H_j is independent of j_c.

The existing experimental data do not allow us to elucidate the effect of the geometry of the current and field distribution on stability (see Chap. 2). Existence of a dependence of H_j on the prehistory involves the finite nature of the specimen in two dimensions. Hence the existence of the effect does not depend on the concrete shape of the conductor. This phenomenon has been treated qualitatively in a number of cases (see, e.g.,[70,71]).

A dependence of the stability on the size of the transport current I ($H_e > H_p$) has been found in[57]. Flux jumps were lacking when $I = 0$, while they appeared when $I > 0$, which agrees with the predictions of the theory.

Sutton[72] has studied the stability of a specimen consisting of two superconducting plates having differing critical current densities j_c (Fig. 19), where $j_c(x<0)$

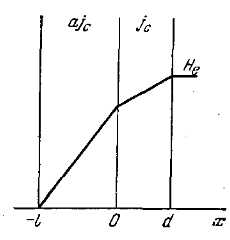

FIG. 19. Geometry of the two-layer specimen.

[11)] The increase in H_{ej} that has been noted in certain studies in the region of very high H_e is apparently to be explained by heating of the specimen during entrance into it of the external flux.[50,56]

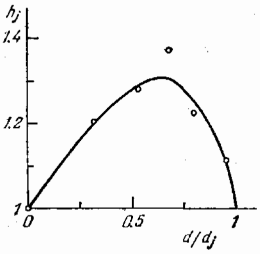

FIG. 20. Relationship of the ratio $h_j = H_j^2/H_j^1$ to d/d_j for the two-layer specimen. The theoretical curve is plotted as the solid line; the parameter $a=2.7$.

$= aj_c(x>0)$. The stability boundary in this case is evidently a function of the thickness d of the outer layer (we can consider the inner superconductor to be semi-infinite) and of the parameter a. The ratio (h_j) of the flux-jump field H_j^2 of the two-layer specimen to the flux-jump field H_j^1 of one of the specimens alone was measured experimentally as a function of the quantity d/d_j (Fig. 20, where $d_j = cH_j^1/4\pi j_c$). The corresponding problem can be easily solved by using (2.13). The solid line in Fig. 20 shows the theoretical curve that we obtained. The value $a=2.7$ was chosen by least squares. As we see, the theory satisfactorily describes the experimental relationship. We note further that the maximum value in $H_j^2 = 2H_j^1$ ($d \sim d_j$, $a \gg 1$) for a two-layer specimen.

The effect of the external thermal conditions on the position of the stability boundary has been studied in[24,28,92]. In line with the theoretical predictions, H_j depends weakly on the nature of the heat removal.

A large number of experiments have measured the time of development of instability.[20,28–30,53,73–75] To the accuracy with which the experimental and theoretical results can be compared, the agreement between them is satisfactory. We note also that they have observed in[30,76] an increase in the characteristic time of a flux jump with increasing conductivity of the normal coating.

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Translated by M. V. King