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# High-field vortices in dense chains of 0- and $\pi$ -shifted Josephson junctions

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Abstract. Sequences of spatially interchanging 0- and  $\pi$ -shifted Josephson junctions are encountered in superconductor-ferromagnet-superconductor heterostructures, asymmetric grain boundaries in thin films of YBCO, and YBCO/Nb zigzag junctions. In this paper we demonstrate existence of Josephson vortices in applied fields much higher than the Josephson penetration field. The high-field vortices exist in narrow field intervals near equidistant fields  $H_n$ . When the applied field is in any of these intervals, the flux per junction is  $n\phi_0$ , where nin an integer. We show that high-field vortices are much longer than the vortices carrying flux in each of the 0- or  $\pi$ -shifted junctions. As a result the field within the vortices is much lower than the background field. High-field vortices carry one flux quanta or half-flux quanta and are free to move, unlike the semi-fluxons in low fields that are pinned by the contacts between 0- and  $\pi$ -junctions. In the presence of a transport current the high-field vortices are subjected to the Lorentz force and move to one side of the chain, reflect, move to the other side, etc. This periodic motion generates constant voltage across the chain with resonances similar to the zero-field-steps. If the vortices carry half-flux quanta, the resonances will appear at half the voltage one would expect for zero-field-steps.

## 1. Introduction

Long chains of 0- and  $\pi$ -biased Josephson junctions with the critical current density changing sign, were first considered for the ferromagnet-superconductor-ferromagnet heterostructures and later on for the grain boundaries in thin films of high-temperature superconductors (the geometry of the problem is shown in Fig.1) [1-10]. These composite tunnel junctions with spatially alternating critical current density are interesting and important for fundamental and applied physics of superconductivity [5, 6]. Indeed, spontaneous flux observed in these junctions in equilibrium was one of the first convincing evidences of the d-wave nature of the order parameter in high-temperature superconductors [7-11]. In many cases the typical length of the 0- and  $\pi$ biased fragments (~ l) is much less than the Josephson penetration depth  $\Lambda$  in each of the fragments [10]. The dense chains  $(l \ll \Lambda)$  of 0- $\pi$  junctions can be considered using the meanfield approximation [12]. In particular, this approach predicts existence of splinter vortices carrying unquantized flux [12]. Based on this prediction splinter vortices were observed at asymmetric grain boundaries in thin YBCO films [13]. It is worth mentioning that splinter vortices can propagate along the composite Josephson junctions with critical current density rapidly alternating  $(l \gg \Lambda)$  along the junction. This important feature is in contrast to the properties of the semi-fluxons, which exist if  $l \gg \Lambda$ , carry flux  $\phi_0/2$ , but are strongly pinned at the contacts between the 0- and  $\pi$ -fragments. It was also shown that in Josephson junctions



Figure 1. Schematic diagrams showing the geometry of the problem: (a) composite Josephson junction with periodically interchanging 0- and  $\pi$ -biased fragments; (b) alternating critical current density.



Figure 2. Spatial distributions of free energy. The critical current density is assumed to be sinusoidal with L = 100 l, and  $L = 8\pi\Lambda_l$ : (a) the dependence of free energy on the internal flux at  $H = H_l$ , the phase is considered to be linear; (b) the minimal value of free energy as a function of the applied field.

with rapidly alternating critical current density the maximum values of the supercurrent  $I_m(H)$ across the junction are achieved not at the applied field H = 0 but at the side-peaks located at  $H = \pm nH_l$ , where  $n \ge 0$  is an integer and  $H_l = \phi_0/2\lambda l$ . It is worth mentioning here that the field  $H_l$  corresponds to one flux quanta per each of the 0- and  $\pi$ -biased fragments [13-17]. In conventional junctions with constant critical current density,  $I_m(H)$  is given by the Fraunhofer pattern, which has a maximum at H = 0. The side peaks in the dependence  $I_m(H)$  were first observed in asymmetric Josephson grain boundaries in YBCO thin films [10].

In this paper we study composite Josephson junctions with critical current density rapidly  $(l \ll \Lambda)$  alternating along the junction. We show that if the applied field is in narrow intervals near the side peaks  $H \sim nH_l \gg H_s \sim \phi_0/4\lambda\Lambda$  ( $H_s$  is the first flux penetration field), then the equilibrium state of composite junctions is similar to the equilibrium state of conventional Josephson junctions in low fields. The main features of these equilibrium states are: (a) existence of moving fluxons, (b) magnetization curves that are similar to the magnetization curves of conventional junctions in the Meissner state, (c) steps in the I-V characteristics that are similar to the zero-field steps, and (d) specific Josephson length and flux penetration field.

#### 2. Basic equations

Consider a Josephson junction with critical current density  $j_c(x)$  periodically alternating along the junction (x axis) with a period,  $l \ll \Lambda$ . At equilibrium the phase  $\varphi(x)$  is given by

$$\Lambda^2 \varphi'' = \sum_{n=-\infty}^{\infty} g_n e^{i2\pi nx/l} \sin \varphi, \qquad \Lambda^2 = \frac{c\phi_0}{16\pi^2 \lambda \sqrt{\langle j_c^2 \rangle}}, \qquad \langle f \rangle = \frac{1}{L} \int_0^L f(x) dx \quad (1)$$

with the boundary conditions  $\varphi'|_{0,L} = 4\pi\lambda H/\phi_0$  defined by the applied field.

We assume that  $l \ll \Lambda$  and claim that at equilibrium the flux inside the junction is constant if the applied field  $|H - nH_l| \leq H_{sn} = \phi_0 \sqrt{g_n} / 4\pi \lambda \Lambda$ . The outline of our reasoning is as follows. First, we write the internal flux  $\phi_i$  as  $\phi_i = \phi_0 \langle \varphi' \rangle l/2\pi$ . If  $\phi_i$  is not an integer multiple of  $\phi_0$ ,

First, we write the internal flux  $\phi_i$  as  $\phi_i = \phi_0 \langle \varphi' \rangle l/2\pi$ . If  $\phi_i$  is not an integer multiple of  $\phi_0$ , then we introduce the averaged phase as  $\tilde{\varphi} = 2\pi \phi_i x/\phi_0 l + \theta_0$  where  $\theta_0$  is a constant. If  $\phi_i = n\phi_0$ ,

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where n is an integer, then we write  $\tilde{\varphi} = 2\pi \phi_i x / \phi_0 l + \psi(x)$  and for  $\psi(x)$  we have

$$2\Lambda_n^2 \psi'' = 2\sin(\psi - \theta_n) - \gamma_n \sin 2(\psi - \alpha_n), \qquad (2)$$

 $\Lambda_n = \Lambda/\sqrt{g_n}, \ \theta_n = \arg(g_n), \ \text{and}$ 

$$\gamma_n e^{-i2\alpha_n} = \left(\frac{l}{2\pi\Lambda_n |g_n|}\right)^2 \sum_{q\neq 0} \frac{g_{n+q}g_{n-q}}{q^2} \,. \tag{3}$$

In both cases the spatial distribution of the phase  $\varphi(x)$  in equilibrium is approximately linear. This linearity "breaks" in a surface layer of the width  $\sim \Lambda_n$ .

It remains to show that near a resonant field  $H_l$ , a constant flux minimizes the free energy. If the aplied field is near  $H_l$ , then the free energy  $\mathcal{F}$  in units of  $H_{sn}^2 L/8\pi$  is given by

$$\mathcal{F} \approx \left(\frac{H_a}{H_{sn}}\right)^2 - \frac{\Lambda_n}{L} \frac{H_a}{H_{sn}} \frac{4\pi m \phi_i}{\phi_0} + \int_{-L/2}^{L/2} \left[\Lambda_n^2 \varphi'^2 - \frac{1}{2} \sin\left(\frac{2\pi n}{l}x + \theta_n\right) \cos\varphi\right] \frac{dx}{L} \tag{4}$$

where m = L/l. For a linear phase  $\varphi = 4\pi\lambda H_i x/\phi_0 + \pi/2$ , the energy is given by

$$\mathcal{F} = \frac{1}{H_{sn}^2} (H_a - H_i)^2 - \frac{1}{\pi n L/l - \pi m \phi_i/\phi_0} \sin\left(\frac{\pi n L}{l} - \frac{\pi m \phi_i}{\phi_0}\right)$$
(5)

(up to a correction of order  $l/\Lambda \ll 1$ ). If  $H_a$  is from the intervals  $-H_{sn} + nH_l < H_a < nH_l + H_{sn}$ , then the internal field that minimizes the energy is approximately  $nH_l$  which is equivalent to  $\phi_i = 2n\lambda LH_l = nL\phi_0/ml$ . When changing the applied field so it crosses  $nH_l \pm H_{sl}$ , a phase transition occurs and the internal field equals to  $\phi_i \simeq \phi_a/m = 2\lambda LH_a/m$ , see Fig. 2. Moreover, from the above reasoning it is clear that near the resonant field, the phase is given by Eq. (2).

### 3. High-field steps

An intriguing property of the phase evolution near resonant fields is the possibility of having single vortices, fluxons. Indeed, the equation for the average phase (2) is a standard sine-Gordon equation having fluxon solutions. In the vicinity of the m-th resonant field the length of a single fluxon is given by  $\Lambda_m$ . We have simulated the phase evolution near a few resonant fields (m = 1, 3, 5) for junctions with stepwise dependence of the critical current density, see Fig. 3. For even resonant fields, the junction admits fluxon solution with half-integer flux quanta [19].

It can be shown that if a low enough voltage is applied to a junction in a field-synchronized state, then the averaged phase is described by Eq. (2) with an addition of dissipation term, and of a bias current. This new equation is the conventional sine-Gordon equation which is known to have the moving fluxon solutions. A fluxon which moves with velocity v is contracted by a "Lorentz" factor  $[1 - (v/c_s)^2]^{1/2}$ . Since the current produced by a moving fluxon is proportional to the contraction and the applied voltage to the velocity, we get steps at the IV characteristics at  $V = m\phi_0 c_s/L$ , where m is the number of fluxons inside the junction, see Fig. 4. For fields with half integer fluxons, the steps will be at  $V = m\phi_0 c_s/2L$ . These "high field steps" are equivalent to the zero field steps of conventional junctions [19-22].

## 4. Conclusions

To summarize, we demonstrate existence of equilibrium high-field states for Josephson junctions with periodically alternating critical current density. We show that in these the high-field states the field in the junction is constant, there is enhancement of the maximum supercurrent, and moving single fluxons can exist. These fluxons lead to high-field steps in the IV characteristics similar to the zero-field steps.



Figure 3. The effective phase  $\tilde{\varphi}$  at three different resonant fields  $H = mH_l$ , m = 1, 3, 5. (a) The difference between the phase and the linear background mkx,  $k = 2\pi/l$ . (b) The coordinate is normalized by  $\Lambda_n$ .



Figure 4. The dependence of the voltage on the normalized bias current is shown for a junction with alternating critical current density. The applied field equals to the resonant field  $H_l$  (open circles) and to  $2H_l$ (open triangles). The first two half-steps are clearly seen.

#### References

- [1] Bulaevskii L N, Kuzii V V, and Sobyanin A A 1977 JETP Letters 25 290
- [2] Buzdin A I, Bulaevskii L N, and Panykov S V 1982 JETP Letters 35 178
- [3] Ryazanov V V, Oboznov V A, Rusanov A Yu, Veretennikov A V, Golubov A A and Arts J 2001 Phys. Rev. Lett. 86 2427
- [4] Berget F S, Volkov A F and Efetov K B 2005 Rev. Mod. Phys. 77, 1321
- [5] Hilgenkamp H, Ariando, Smilde H J H, Blank D H A, Rijnders G, Rogalla H, Kirtley J R and Tsuei C C 2003 Nature 422, 50
- [6] Goldobin E, Sterck A, Gaber T, Koelle D and Kleiner R 2004 Phys. Rev. Lett. 92 057005
- [7] Copetti C A, Rüders F, Oelze B, Buchal Ch, Kabius B and Seo J W 1995 Physica C 253 63
- [8] Van Harlingen D J 1995 *Rev. Mod. Phys.* 67 515
- [9] Tsuei C C and Kirtley J R 2000 Rev. Mod. Phys. 74 969
- [10] Hilgenkamp H and Mannhart J 2002 Rev. Mod. Phys. 74 485
- [11] Mannhart J, Hilgenkamp H, Mayer B, Gerber Ch, Kirtley J R, Moler K A and Sigrist M 1996 Phys. Rev. Lett. 77 2782
- [12] Mints R G 1998 Phys. Rev. B 57 R3221
- [13] Mints R G, Papiashvili I, Kirtley J R, Hilgenkamp H, Hammerl G and Mannhart J 2002 Phys. Rev. Lett. 89 067004
- [14] Mints R G and Snapiro I B 1994 Phys. Rev. B 49 6188; 1995 Phys. Rev. B 51 3054; 1995 Phys. Rev. B 52 9691
- [15] Smilde H J H, Ariando, Blank D H A, Gerritsma G J, Hilgenkamp H and Rogalla H 2002 Phys. Rev. Lett. 88 057004
- [16] Nappi C, Sarnelli E, Adamo M and Navacerrada M A 2006 Journal of Physics: Conference Series 43 1131
- [17] Russo M and Vaglio R 1978 Phys. Rev. B 17 2171
- [18] Gambardella U, Celentano G, Boffa V and Pace S 1998 Journal of Applied Physics 84 5363
- [19] Moshe M and Mints R G 2007 Phys. Rev B 76 140507(R)
- [20] Chen J T, Finnegan T F and Langenberg D N 1971 Physica (Amsterdam) 55 413
- [21] Fulton T A and Dynes R C 1973 Solid State Commun. 12 57
- [22] Pedersen N F Solitons (Elsevier, Amsterdam 1986)