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High-field vortices in Josephson junctions with alternating critical current density

Maayan Moshe and R. G. Mints*
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel
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We study long Josephson junctions with the critical current density alternating along the junction. Equilibrium field-synchronized (FS) states are shown to exist if the applied field is from narrow intervals centered around equidistant series of resonant fields H_m. In the mth FS state, the flux per period of the alternating critical current density ϕ_i remains constant and is equal to an integer amount of flux quanta, $\phi_i = m\phi_0$. The values of H_m are much higher than the flux penetration field H_s. Two types of single Josephson vortices carrying fluxes ϕ_0 or/and $\phi_0/2$ can exist in the FS states. Specific stepwise resonances in the current-voltage characteristics might be caused by periodic motion of these vortices between the edges of the junction.

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π-shifted Josephson tunnel junctions and sequences of interchanging 0- and π-shifted Josephson junctions (see Fig. 1) are a subject of growing interest.[1–19] The properties of these complex systems have been treated for superconductor-ferromagnet-superconductor (SFS) and superconductor-insulator-ferromagnet-superconductor heterostructures,[1–8] asymmetric grain boundaries in thin films of the high-T_c superconductor $YBa_2Cu_3O_{7-x}$ (YBCO),[9–18] and YBCO/Nb zigzag junctions.[19]

It was predicted that in SFS Josephson junctions the π shift in the phase difference φ between the superconducting banks is caused by the ferromagnet interlayer.[1,2] This prediction was confirmed in recent experiments with SFS π-shifted junctions and SFS heterostructures of interchanging 0- and π-shifted fragments.[3–5] The asymmetric grain boundaries in YBCO thin films are arranged in series of facets with a variety of orientations and lengths $l \sim 10-100$ nm.[17] This spatial structure in conjunction with the d-wave symmetry of the order parameter results in grain boundary junctions with interchanging 0- and π-shifted fragments.[10,15]

The critical current density, $j_c(x)$, changes sign at each contact between 0- and π-shifted fragments (the x axis is along the junction), which results in dramatic changes in Josephson properties. In particular, the dependence of the maximum supercurrent across the junction, I_m, on the applied field H_a is strongly affected by the alternations of $j_c(x)$.[9,11,13,18] First, $I_m(H_a)$ is significantly suppressed at low fields $|H_a| \ll H_1 = \phi_0/2\lambda l$, where λ is the London penetration depth and l is the period of $j_c(x)$. Second, unlike the standard Fraunhofer pattern with a major peak at $H_a=0$, two major side peaks are observed at high fields $H_a = \pm H_1$, where $H_1 \gg H_s$ and H_s is the field of first flux penetration.[11]

In this paper, we find a series of equilibrium *field-synchronized* (FS) states existing if the applied field is from narrow intervals $\Delta H_a \sim H_s$ centered at the resonant fields $H_m = \pm m H_1$, where $m \neq 0$ is an integer. It is shown that in the mth FS state the inner flux per period of $j_c(x)$, ϕ_i, remains constant and is equal to an integer number of flux quanta, $\phi_i = m\phi_0$. We find that two high-field ($H_a \gg H_s$) Josephson-type vortices with fluxes ϕ_0 or/and $\phi_0/2$ can exist in the FS states.

We begin with a qualitative treatment of the FS states using one harmonic model for the tunneling current density $j = j_c(x)\sin\varphi$, where $j_c(x) = j_1 \sin(2\pi x/l)$, $L = Nl$, L is the length of the junction, and $N \gg 1$ is an integer. Assume that the junction is in one of the FS states and the flux ϕ_i is fixed. Since we have many vortices in the junction ($Nm \gg 1$), the field is almost uniform and the phase $\varphi(x)$ takes the form

$$\varphi(x) = 2\pi \frac{\phi_i}{\phi_0}\frac{x}{l} + \psi(x), \tag{1}$$

where the phase $\psi(x)$ is a smooth function with the typical length scale $\gg l$ and $|\psi(x)| \sim 1$. Then we have

$$j(x) = j_1 \sin\left(2\pi\frac{x}{l}\right)\sin\left(2\pi\frac{\phi_i}{\phi_0}\frac{x}{l} + \psi\right). \tag{2}$$

In general, this current density alternates rapidly with a typical length scale $\leq l$. In this case the coarse-grained approach is the right tool to describe the smooth phase $\psi(x)$.[14] If we average $j(x)$ over a distance $\mathcal{L} \gg l$, then the coarse-grained tunneling current density j_ψ is zero. This is indeed true in all cases but one. If the junction is in the FS state with $\phi_i = \pm\phi_0$, then coarse-graining of Eq. (2) leads to a nonzero result,

$$j_\psi = 0.5 j_1 \sin(\psi \pm \pi/2). \tag{3}$$

It is worth mentioning that $j_\psi \sim j_1$ and the final form of the dependence of j_ψ on the smooth phase ψ coincides with the $\pi/2$-shifted current-phase relation of a Josephson junction of conventional superconductors.

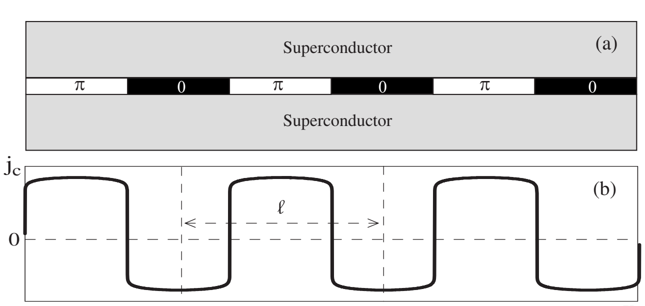

FIG. 1. Schematic diagrams of (a) tunnel junction arranged in series of interchanging 0- and π-shifted fragments; (b) spatial distribution of alternating critical current density.

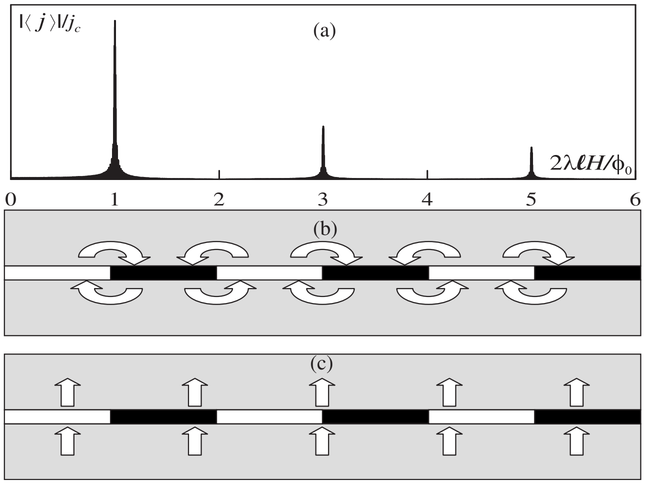

FIG. 2. (a) Coarse-grained tunneling current density dependence on the applied field for the case of a stepwise critical current density, $N=40$, and $\varphi \propto x$. Schematic diagrams of spatial distributions of the tunneling current density for junctions that are (b) not in the FS states (small local currents tend to cancel each other); (c) in the FS states (field synchronized local currents flow in the same direction).

A similar calculation of the coarse-grained current density j_ψ in the low-field region ($H_a \ll H_s$) leads to[14]

$$j_\psi = -j_1\left(\frac{l}{4\pi\Lambda_1}\right)^2 \sin 2\psi \ll j_1, \tag{4}$$

where the Josephson length is given by

$$\Lambda_1 = \sqrt{c\phi_0/16\pi^2\lambda j_1} \gg l. \tag{5}$$

Comparing Eqs. (2)–(4), we find that if the phase factor, $\sin\varphi$, modulations caused by the field and the critical current density modulations caused by the structure of the junction are *synchronized* ($\phi_i = m\phi_0$), then the critical current density $j_c(x)$ is significantly enhanced as all local tunneling currents flow in the same direction (see Fig. 2). The widths of the intervals $\Delta H_a \sim H_s \ll H_1$ of existence of the FS states are defined by minimization of the free energy. It is worth mentioning that similar *commensurate* states exist in conventional junctions with spatially modulated properties.[20,21]

Consider now a Josephson junction with $\lambda \ll l \ll \lambda_J$, where λ_J is the *local* Josephson penetration depth

$$\lambda_J = \sqrt{c\phi_0/16\pi^2\lambda\langle|j_c|\rangle}, \tag{6}$$

and the averaging over the junction length is defined as

$$\langle f\rangle = \int_0^L dx\, f(x)/L. \tag{7}$$

In this case the static spatial distribution of the phase $\varphi(x)$ is given by

$$\lambda_J^2\varphi'' - i_c(x)\sin\varphi = 0, \tag{8}$$

where $i_c(x) = j_c(x)/\langle|j_c|\rangle$ is the dimensionless critical current density. Next, we expand $i_c(x)$ in Fourier series and obtain instead of Eq. (8)

$$\lambda_J^2\varphi'' - \sum_{n=-\infty}^{\infty} i_n e^{i2\pi nx/l}\sin\varphi = 0, \tag{9}$$

where i_n are the Fourier coefficients of the Fourier-transformed $i_c(x)$.

Since $l \ll \lambda_J$, the phase φ can be written as[14,22]

$$\varphi = 2\pi\phi_i x/\phi_0 l + \psi(x) + \xi(x), \tag{10}$$

where $\psi(x)$ is a smooth function with the typical length scale $\sim\lambda_J \gg l$ and $|\psi(x)| \sim 1$, and $\xi(x)$ is a rapidly alternating function with the length scale $\sim l$. In addition, we assume that $\langle\xi(x)\rangle = 0$, and $\langle|\xi(x)|\rangle \ll 1$. Following Refs. 14 and 22 we average Eq. (9) over the junction length and obtain the equation describing the smooth (coarse-grained) phase $\psi(x)$ in the mth FS state

$$\lambda_J^2\psi'' - |i_m|\sin(\psi - \theta_m) + \gamma_m \sin 2(\psi - \alpha_m) = 0, \tag{11}$$

where $\theta_m = \arg(i_m)$, α_m, and γ_m are defined by

$$\gamma_m e^{-i2\alpha_m} = \left(\frac{l}{2\pi\lambda_J}\right)^2 \sum_{n=1}^{\infty} \frac{i_{m+n} i_{m-n}}{n^2}. \tag{12}$$

The complexity of Eq. (11) can be significantly reduced. Indeed, one can estimate $\gamma_m \sim (l/2\pi\lambda_J)^2 \ll 1$, which means that $\gamma_m \ll i_m$. As a result the third term in Eq. (11) can be neglected and Eq. (11) yields

$$\Lambda_m^2\psi'' - \sin(\psi - \theta_m) = 0, \tag{13}$$

where $\Lambda_m = \lambda_J/\sqrt{|i_m|}$. The boundary conditions to Eq. (13) are given by the field at $x=0$ and $x=L$:

$$\psi'|_{0,L} = \frac{4\pi\lambda}{\phi_0}(H_a|_{0,L} - mH_1). \tag{14}$$

It is worth noting that, in particular, Eq. (13) describes Josephson-type vortices with size $\sim\Lambda_m \gg l$ and flux ϕ_0.

It follows from Eqs. (11) and (13) that equations describing the coarse-grained phase $\psi(x)$ are the same as for Josephson junctions of conventional superconductors in the Meissner state.

Specific symmetry of $j_c(x)$ might lead to a Fourier series with some of the Fourier coefficients being zero. In this case only the third term in Eq. (11) is nonzero. In particular, if $j_c(x)$ is a stepwise function (see Fig. 1), then we have $i_{2k}=0$ and $i_{2k+1} = -\frac{2i}{\pi(2k+1)}$, where k is an integer. In the FS states with the field H_a located in the intervals ΔH_a centered at "odd" resonant fields $H_{o,k} = (2k+1)H_1$, Eq. (11) takes the form

$$\Lambda_{o,k}^2\psi'' - \sin\psi = 0, \quad \Lambda_{o,k} = \sqrt{\pi(k+1/2)}\lambda_J; \tag{15}$$

for "even" resonant fields $H_{e,k} = 2kH_1$, Eq. (11) yields

$$\Lambda_{e,k}^2\psi'' - \sin 2\psi = 0, \quad \Lambda_{e,k} = 2\pi k\beta_k\frac{\lambda_J^2}{l}, \tag{16}$$

where β_k is a constant ($\beta_1=0.9$, $\beta_2=1.1$, $\beta_3=0.9$, and $\beta_k \approx 1$ for $k>3$). It is seen from Eqs. (15) and (16) that even-field vortices carrying flux $\phi_0/2$ are by $\lambda_J/l \gg 1$ wider than odd-field vortices carrying flux ϕ_0.

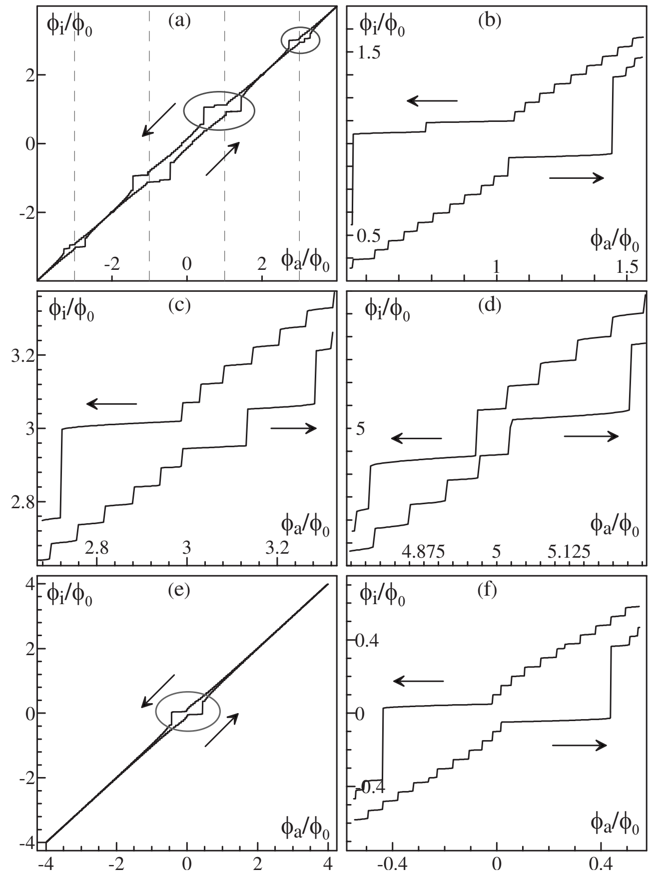

FIG. 3. (a)–(d) Magnetization $\phi_i(\phi_a)$ of a junction with stepwise critical current density and $N=40$. (a) The plateaus with ϕ_i=const reveal the first four FS states. The curves $\phi_i(\phi_a)$ in the vicinity of three FS states H_a=(b) H_1, (c) $3H_1$, and (d) $5H_1$. (e) Magnetization of a junction of conventional superconductors. (f) $\phi_i(\phi_a)$ of a conventional junction in the Meissner state.

Next, we calculate the width of the intervals ΔH_a of existence of the FS states by minimizing the free energy $\mathcal{F}$ of the junction. This can be done explicitly if we specify the spatial distribution of the critical current density $j_c(x)$. Here, for brevity, we use the one harmonic model, i.e., we assume that $j_c(x)=j_1\sin(2\pi x/l)$:

$$\mathcal{F}=\left(\frac{H_a}{H_{\mathrm{sm}}}\right)^2-\frac{\Lambda_m}{l}\frac{H_a}{H_{\mathrm{sm}}}\frac{4\pi\phi_i}{\phi_0}+\int_0^L\left[\Lambda_m^2\varphi'^2-\frac{1}{2}\sin\left(\frac{2\pi m}{l}x+\theta_m\right)\cos\varphi\right]\frac{dx}{L}, \tag{17}$$

where $\mathcal{F}$ is normalized by $H_{\mathrm{sm}}^2L/8\pi$ and the field H_{sm} is defined as $H_{\mathrm{sm}}=\phi_0/4\pi\lambda\Lambda_m$. If there are no vortices in the junction, then $\psi=0$ and $\varphi(x)=2\pi\phi_i x/\phi_0 l+\varphi_0$, where φ_0 is a constant. The minimum of the free energy, $\mathcal{F}_m=\mathcal{F}(H_m)$, is achieved for $\varphi_0=\theta_m$,

$$\mathcal{F}_m=\left(\frac{2\pi\Lambda_m}{l\phi_0}\right)^2(\phi_a-\phi_i)^2-\frac{\sin[\pi N(m-\phi_i/\phi_0)]}{\pi N(m-\phi_i/\phi_0)}, \tag{18}$$

where $\phi_a=2\lambda l H_a$. Finally, we fix the applied field H_a and minimize $\mathcal{F}_m$ with respect to the internal flux ϕ_i. It follows from this calculation that, if H_a is from the narrow interval $mH_1-H_{\mathrm{sm}}<H_a<mH_1+H_{\mathrm{sm}}$, then the flux ϕ_i is constant and equal to $m\phi_0$, i.e., $H_i\approx mH_1$.

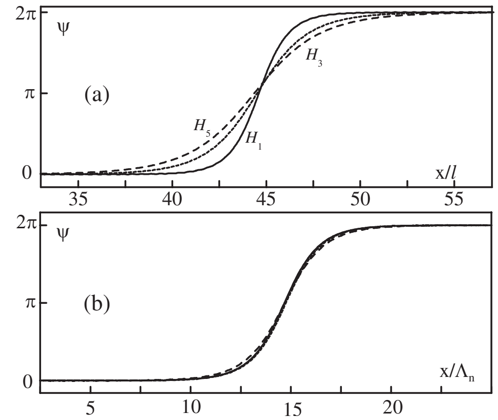

FIG. 4. (a) Phase $\psi(x)$ distribution for vortices carrying flux ϕ_0 at three "odd" resonant fields H_1, $3H_1$, and $5H_1$. (b) The same three curves are shown to collapse into one curve if the coordinate is normalized by Λ_1, Λ_3, and Λ_5.

The above theoretical analysis can be supported by numerical simulations. To treat both the statics and dynamics of the phase difference $\varphi(x,t)$ we introduce time dependence into Eq. (8) and arrive at

$$\ddot{\varphi}+\alpha\dot{\varphi}-\lambda_J^2\varphi''+i_c(x)\sin\varphi=0, \tag{19}$$

where $\alpha\sim 1$ is a decay constant and the term $\alpha\dot{\varphi}$ describes dissipation. As a result of this dissipation the system ends up in one of the stable stationary states which is a solution of Eq. (8). We used the finite-difference explicit method (see Ref. 23 for details) and boundary conditions $\varphi'=4\pi\lambda H_a/\phi_0$ at the edges of the junction to solve Eq. (19), assuming that the alternating critical current density $j_c(x)$ is stepwise.

The magnetization curves (dependencies of ϕ_i on ϕ_a) obtained by numerical simulations are shown in Fig. 3. The plateaus with ϕ_i=const are clearly seen for the series of the four FS states. The field inside the junction is constant for each of the FS states in contrast to junctions of conventional superconductors for which the internal field is constant only in the Meissner state. It is seen in Fig. 3 that the magnetization curve of a junction in the FS state is the same as the magnetization curve of a junction of conventional superconductors but with the field H_a biased by mH_1. Our numerical simulations confirm that the width of the plateau in which the internal field is constant is proportional to $\sqrt{|i_m|}=\lambda_J/\Lambda_m$, as it follows from the above theoretical analysis.

Next, we simulated numerically the dynamics and statics of single Josephson-type vortices in the FS states using Eq. (19) (see Figs. 4 and 5). We observed stable Josephson-type vortices carrying fluxes ϕ_0 (for odd resonant fields) and $\phi_0/2$ (for even resonant fields). The width of vortices with flux ϕ_0 scales proportionally to $\sqrt{|i_m|}$ and the width of vortices with flux $\phi_0/2$ is by a factor $\lambda_J/l\gg 1$ larger than that of ϕ_0 as follows from Eqs. (15) and (16).

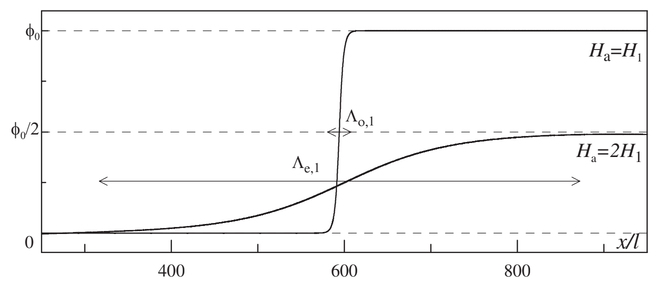

FIG. 5. Phase $\psi(x)$ distribution for the first odd, $H_a=H_1$, and even, $H_a=2H_1$, resonant fields describing vortices with fluxes ϕ_0 ($H_a=H_1$) and $\phi_0/2$ ($H_a=2H_1$).

In addition, in the numerical study we found that periodic motion of single vortices between the edges of the junction produces steps in the current-voltage characteristics at an equidistant series of voltages $V_m=m\phi_0 c_s/L$, where m is the number of fluxons inside the junction and c_s is the Swihart velocity.[24] These high-field steps are similar to the zero-field steps in junctions of conventional superconductors.[25–27] In detail, the study of the effect of high-field vortices on the current-voltage characteristics of the FS states will be presented elsewhere.

To summarize, we find a series of equilibrium FS states in Josephson junctions with periodically alternating critical current density. The FS states exist if the applied field is from narrow intervals centered at equidistant series of fields. In FS states the flux in the junction is fixed and the maximum supercurrent across the junction is significantly enhanced. Two types of single high-field vortices with flux ϕ_0 or/and $\phi_0/2$ exist in FS states.

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*mints@post.tau.ac.il

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