Pancake vortex near the sample surface

R. G. Mints and I. B. Snapiro
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel

E. H. Brandt
Institut für Physik, Max-Planck-Institut für Metallforschung, D-70506 Stuttgart, Germany

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We consider a single pancake vortex near the surface of a layered superconductor with very weak interlayer Josephson coupling. The exact solution for the magnetic field inside and outside the sample is obtained in the case of a planar surface perpendicular to the layers. From the general result we calculate the stray field of a randomly distorted Abrikosov vortex parallel to the sample surface and perpendicular to the superconducting layers. We give the mean square flux generated by this vortex through a band of given width on the sample surface. To illustrate the general formula we apply it to the case when the displacements of the pancakes arise from thermal fluctuations. [S0163-1829(96)00837-5]

I. INTRODUCTION

The most prominent of the high-temperature copper oxide superconductors consist of a periodic stack of two-dimensional CuO layers (ab planes) where the superconductivity presumably resides. These materials are extremely anisotropic. In particular, the maximum density of the superconducting current perpendicular to the layers (c direction) is much less than in the ab planes; i.e., the superconducting layers are weakly coupled. The discovery of the extremely anisotropic high-T_c superconductors stimulated many theoretical studies of vortices of layered materials with weak and very weak interlayer Josephson coupling.[1,2]

Magnetic flux in layered superconductors has several unusual properties as compared to magnetic flux in isotropic superconductors. In particular, an Abrikosov vortex line oriented nearly perpendicular to the superconducting layers may be treated as a stack of two-dimensional point vortices or ''pancakes.''[3–5] Each of these pancakes resides only in one of the superconducting layers and generates a magnetic field $\mathbf{B}$ which has components $\mathbf{B}_\parallel$ and $\mathbf{B}_\perp$ parallel and perpendicular to the ab planes. Both $\mathbf{B}_\parallel$ and $\mathbf{B}_\perp$ are proportional to the small ratio $s/\lambda \ll 1$, where s is the layer spacing and $\lambda=\lambda_{ab}$ is the penetration depth for the currents in the layers. The component $\mathbf{B}_\perp$ is spherically symmetric,

$$\mathbf{B}_\perp=\mathbf{e}_z\frac{s\Phi_0}{4\pi\lambda^2 r}\exp(-r/\lambda), \tag{1}$$

where $\mathbf{e}_z$ is the unit vector in the c direction and r is the distance from the pancake. The extension of the component $\mathbf{B}_\parallel$ is anisotropic, namely, short in the direction perpendicular to the layers and wide along the layers,

$$\mathbf{B}_\parallel=\mathbf{e}_\rho\frac{s\Phi_0}{4\pi\lambda^2\rho}\mathrm{sgn}z\left[\exp(-|z|/\lambda)-\frac{|z|}{r}\exp(-r/\lambda)\right], \tag{2}$$

$\mathbf{e}_\rho$ is the unit vector in the direction of the radius vector $\vec{\rho}=(x,y)$, $\rho=\sqrt{x^2+y^2}$, the xy plane is parallel to the ab planes, and the z axis is along the c direction.

The main contribution to the self-energy of a single pancake $\mathcal{E}$ results from the currents in the layer where the pancake is located. Indeed, in the London approximation for the case of vanishing Josephson interlayer coupling and $s\ll\lambda$ one has

$$\mathcal{E}=\mathcal{E}_B+\mathcal{E}_I, \tag{3}$$

where

$$\mathcal{E}_B=\frac{1}{2\mu_0}\int[\mathbf{B}^2+\lambda^2(\mathrm{rot}\mathbf{B})^2]dV \tag{4}$$

is the energy within the continuum approximation and

$$\mathcal{E}_I=\frac{\mu_0\lambda^2}{2s}\int I^2 dA \tag{5}$$

is the energy of the sheet current I in the pancake layer. The ratio $\mathcal{E}_B/\mathcal{E}_I$ is found from the estimates[4,5]

$$B\sim\frac{\Phi_0 s}{\lambda^2 r}\quad\text{for}\ \ r<\lambda, \tag{6}$$

$$I\sim\frac{s\Phi_0}{\mu_0\lambda^2\rho}\quad\text{for}\ \ \rho<\lambda. \tag{7}$$

It follows then from Eqs. (4) and (5) that

$$\mathcal{E}_B\propto\frac{\Phi_0^2 s^2}{\mu_0\lambda^3},\qquad \mathcal{E}_I\propto\frac{\Phi_0^2 s}{\mu_0\lambda^2}; \tag{8}$$

thus, $\mathcal{E}_I/\mathcal{E}_B\sim\lambda/s\gg1$ and $\mathcal{E}\approx\mathcal{E}_I$.

The slow decay of the sheet current $I\propto1/\rho$ makes the integral in Eq. (5) size dependent, namely,

$$\mathcal{E}\approx\frac{s\Phi_0^2}{4\pi\mu_0\lambda^2}\ln\left(\frac{L}{\xi}\right), \tag{9}$$

where L is the characteristic size of the sample in the ab plane and $\xi=\xi_{ab}$ is the coherence length in the layers. Thus, a single pancake has a diverging self-energy when $L/\xi\to\infty$

and cannot exist in the bulk of a macroscopic sample. However, a pancake vortex near a surface which is perpendicular to the layers has a finite energy, with L in Eq. (9) replaced by its distance from the surface.

Let us now consider a stack of pancake vortices. If it is perfectly aligned perpendicular to the layers, the in-plane components of their magnetic fields compensate, and the resulting magnetic field will be that of a usual Abrikosov vortex line. Similarly, a distorted stack of pancakes yields the field of a distorted vortex line calculated from the anisotropic London theory.[1,2] However, in the limit of very small or zero Josephson coupling, novel features arise from the pancake picture.[6,7] For example, it was shown that a distorted stack of pancakes is attracted to a planar specimen surface (at $x=0$) by an interaction energy decreasing only algebraically as $1/x_0^2$, where x_0 is the vortex distance from the surface.[8] This long-range attraction is caused by the dipole-dipole interaction of each pancake displacement with its image positioned at $x=-x_0$, since each displacement is equivalent to adding a pancake-antipancake pair which annihilates the original pancake and generates a new pancake at the displaced position. In order to complete these calculations one has to add to the pancake and image fields the stray-field contribution to the interaction energy. While this contribution is negligible at large distances $x_0 \gg \lambda$, where the dipole-dipole interaction $1/x_0^2$ dominates, the stray-field energy might give a non-negligible contribution if the distorted vortex is close to the surface.

In this paper we study a single pancake vortex located near the sample surface. We find an exact solution for the total magnetic field $\mathbf{B}(\mathbf{r})$ generated by a pancake inside and outside a superconducting half-space, as a first step towards calculations of the stray-field contribution to the pancake interaction. We present the modification of $\mathbf{B}(\mathbf{r})$ caused by the existence of a planar surface. Note that the stray field *outside* the superconductor is the magnetic field which is measured by common methods like Hall probes or magneto-optics near the specimen surface. Experiments which measure the magnetic field *inside* the superconductor like muon-spin resonance or nuclear magnetic resonance are much more complex and less accurate; cf. Refs. 9 and 10, and references therein. The complete solution given below modifies the magnetic field both outside and inside the superconductor as compared to the fields of the pancake and of its image alone. This problem, with similar results, has been considered by Buzdin and Feinberg.[11]

A related problem of the magnetic field of a distorted vortex lattice inside and outside a superconductor with a planar surface was considered for small vortex displacements.[12] The extension of these results to anisotropic layered superconductors and to arbitrarily large vortex displacements in principle can be obtained by linear superposition of the single-pancake field obtained below. It appears that such applications can be done only numerically except in particular cases like, e.g., the linear elastic energy of the pancake lattice near a planar surface, or some properties of randomly positioned or correlated pancakes.[9] The single-pancake results presented below are explicit expressions which enter such further calculations. As one example we calculate the mean square flux through a band on the sample surface, which is generated by a randomly distorted Abrikosov vortex line parallel to the sample surface and perpendicular to the layers.

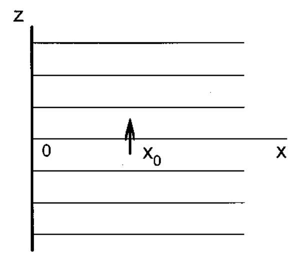

FIG. 1. A single pancake located at the point $(x_0,0,0)$ near the sample surface $x=0$ (thick line) with the superconducting layers (thin lines) perpendicular to the surface.

This paper is organized as follows. In Sec. II, we find the magnetic field generated by a pancake positioned near the sample surface. In Sec. III, we apply this result to calculate the mean square flux through a band on the sample surface, which originates from a randomly distorted Abrikosov vortex line. In Sec. IV, we summarize the overall conclusions.

II. SINGLE PANCAKE NEAR THE SURFACE

Let us consider a superconducting half-space $x \geqslant 0$ with a single pancake located at the point $(x_0,0,0)$ as shown in Fig. 1. In order to find the magnetic field generated by this pancake we write the current density and the magnetic field inside the superconductor as $\mathbf{j}=\mathbf{j}_v+\mathbf{j}_a$ and $\mathbf{B}=\mathbf{B}_v+\mathbf{B}_a$, where $\mathbf{j}_v$ and $\mathbf{B}_v$ are calculated by the method of images. The additional current density $\mathbf{j}_a$ and magnetic field $\mathbf{B}_a$ are required to satisfy the boundary conditions in the presence of a field component perpendicular to the sample surface.

The magnetic field $\mathbf{B}_v$ is a sum of the fields generated by the vortex at $(x_0,0,0)$ and antivortex at $(-x_0,0,0)$. The y and z components of the field $\mathbf{B}_v$ and the x component of the current density $\mathbf{j}_v$ vanish on the sample surface by this construction. The vector potential $\mathbf{A}_v$ of the magnetic field $\mathbf{B}_v=\mathrm{rot}\mathbf{A}_v$ satisfies the equation

$$\mathbf{A}_v-\lambda^2\Delta\mathbf{A}_v=\frac{s\Phi_0}{2\pi}\hat{\mathbf{z}}\left(\frac{\vec{\rho}-\vec{\rho}_p}{|\vec{\rho}-\vec{\rho}_p|^2}-\frac{\vec{\rho}+\vec{\rho}_p}{|\vec{\rho}+\vec{\rho}_p|^2}\right)\delta(z), \quad (10)$$

where $\vec{\rho}=(x,y,0)$, and $\vec{\rho}_p=(x_0,0,0)$. The solution of Eq. (10) satisfying the boundary condition $j_x(0,y,z)=0$ is the superposition of two pancake fields of the form (1) and (2).

The additional magnetic field $\mathbf{B}_a$ is defined inside the superconductor $(x>0)$. We introduce now the phase φ of the superconducting order parameter and the vector potential $\mathbf{A}_a$ for the field $\mathbf{B}_a=\mathrm{rot}\mathbf{A}_a$. In the London gauge $\mathrm{div}\mathbf{A}_a=0$, $A_{az}=0$, the equations for $\mathbf{A}_a$ and φ take the form

$$\mathbf{A}_a-\lambda^2\Delta\mathbf{A}_a=\frac{\Phi_0}{2\pi}\nabla_2\varphi, \quad (11)$$

$$\Delta_2\varphi=0, \quad (12)$$

where ∇_2 is the two-dimensional gradient in the xy plane and $\Delta_2={}^2/\partial x^2+\partial^2/\partial y^2$. The boundary conditions for Eqs. (11) and (12) are given by $j_x(0,y,z)=0$, which yields

$$\left\{ A_{ax} - \frac{\Phi_0 \partial\varphi}{2\pi \partial x} \right\}\bigg|_{x=0} = 0. \tag{13}$$

The stray field $\mathbf{B}_s$ is defined outside the superconductor ($x<0$) and is a potential field; i.e., $\mathbf{B}_s = \nabla\psi$, where the potential $\psi(x,y,z)$ satisfies the Laplace equation

$$\Delta\psi = 0. \tag{14}$$

The complete solution $\mathbf{B}$ is thus given by $\mathbf{B}_v + \mathbf{B}_a$ inside and by $\mathbf{B}_s$ outside the superconductor. The perpendicular component of the total magnetic field B_x satisfies the continuity condition at the surface, namely,

$$B_{sx}(-0,y,z) = B_{ax}(+0,y,z) + B_{vx}(0,y,z). \tag{15}$$

The function $B_{vx}(0,y,z)$ plays the role of the source of the stray field $\mathbf{B}_s$ outside and the additional magnetic field $\mathbf{B}_a$ inside the superconductor. It follows from the solution of Eq. (10) that[5]

$$B_{vx}(0,y,z) = -\frac{\Phi_0}{2\pi\lambda^2}\frac{s x_0}{x_0^2+y^2}\mathrm{sgn}z \times\left[\exp(-|z|/\lambda) - \frac{|z|}{r_0}\exp(-r_0/\lambda)\right], \tag{16}$$

$r_0 = \sqrt{x_0^2+y^2+z^2}$ and a factor of 2 as compared to the single-pancake solution Eq. (2) reflects the contribution of the image antivortex.

To solve Eqs. (11) and (12) we use Fourier transforms of the form

$$f(x,y,z) = \frac{1}{4\pi^2}\int \tilde{f}(x,k,q)e^{iky+iqz}dk\,dq. \tag{17}$$

The resulting equations for $\tilde{\varphi}(x)$, $\tilde{A}_x$, and $\tilde{A}_y$ are

$$\tilde{\varphi}(x) = \tilde{\varphi}(0,k,q)e^{-|k|x}, \tag{18}$$

$$\frac{d^2\tilde{A}_{ax}}{dx^2} - \gamma^2\tilde{A}_{ax} = \tilde{\varphi}(0,k,q)\frac{\Phi_0|k|}{2\pi\lambda^2}e^{-|k|x}, \tag{19}$$

$$\tilde{A}_{ay} = \frac{i}{k}\frac{d\tilde{A}_{ax}}{dx}, \tag{20}$$

where

$$\gamma = \sqrt{\lambda^{-2}+k^2+q^2}. \tag{21}$$

The solution of Eq. (19) which satisfies the boundary condition (13) and vanishes at $x\to\infty$ is

$$\tilde{A}_{ax} = -\frac{\Phi_0}{2\pi}\frac{\tilde{\varphi}(0,k,q)|k|}{(1+q^2\lambda^2)}[e^{-|k|x} + q^2\lambda^2 e^{-\gamma x}]. \tag{22}$$

We apply now the Fourier transform to Eq. (14) and take the solution

$$\tilde{\psi} = \tilde{\psi}(0,k,q)\exp(\sqrt{k^2+q^2}x), \tag{23}$$

decaying at $x\to-\infty$. To find the relation between $\tilde{\varphi}(0,k,q)$ and $\tilde{\psi}(0,k,q)$ we use the continuity conditions for the y and z components of the magnetic field. As a result we obtain

$$\tilde{\varphi}(0,k,q) = -\frac{2\pi}{\Phi_0}\frac{\mathrm{sgn}k}{q}\tilde{\psi}(0,k,q). \tag{24}$$

It follows from the continuity condition (15) that

$$\tilde{\psi}(0,k,q) = \frac{(1+q^2\lambda^2)\tilde{B}_{vx}(0,k,q)}{|k| + \gamma q^2\lambda^2 + (1+q^2\lambda^2)\sqrt{k^2+q^2}}, \tag{25}$$

where

$$\tilde{B}_{vx}(0,k,q) = i\Phi_0\frac{sq}{1+q^2\lambda^2}[\exp(-|k|x_0) - \exp(-\gamma x_0)] \tag{26}$$

the Fourier transform of the field B_{vx}, Eq. (16).

Using Eqs. (1), (2), (20), (22), (25), and (26) we obtain the complete description of the magnetic field $\mathbf{B}(x,y,z)$ of a single pancake vortex located at a distance x_0 from the sample surface. We get the following explicit formulas for the Fourier transforms of the potential $\psi(x,y,z)$ for the stray field $\mathbf{B}_s$,

$$\tilde{\psi}(x,k,q) = i\Phi_0\frac{sq[\exp(-|k|x_0) - \exp(-\gamma x_0)]}{|k| + q^2\lambda^2\gamma + (1+q^2\lambda^2)\sqrt{k^2+q^2}} \times\exp(\sqrt{k^2+q^2}x), \tag{27}$$

and of the additional magnetic field $\mathbf{B}_a$,

$$\tilde{B}_{ax}(x,k,q) = -\tilde{\psi}(0,k,q)\frac{|k|\exp(-|k|x) + \gamma q^2\lambda^2\exp(-\gamma x)}{(1+q^2\lambda^2)}, \tag{28a}$$

$$\tilde{B}_{ay}(x,k,q) = i\tilde{\psi}(0,k,q)\frac{k[\exp(-|k|x) + q^2\lambda^2\exp(-\gamma x)]}{(1+q^2\lambda^2)}, \tag{28b}$$

$$\tilde{B}_{az}(x,k,q) = i\tilde{\psi}(0,k,q)q\exp(-\gamma x), \tag{28c}$$

where

$$\tilde{\psi}(0,k,q) = i\Phi_0\frac{sq[\exp(-|k|x_0) - \exp(-\gamma x_0)]}{|k| + q^2\lambda^2\gamma + (1+q^2\lambda^2)\sqrt{k^2+q^2}}. \tag{29}$$

The components of the field $\mathbf{B}_v$ are

$$B_{vx}(x,y,z) = \frac{s\Phi_0}{4\pi\lambda^2}\mathrm{sgn}z\sum_{\sigma=-1}^{1}\frac{\sigma x - x_0}{(x-\sigma x_0)^2+y^2}\left[\exp(-|z|/\lambda) - \frac{|z|}{r_\sigma}\exp(-r_\sigma/\lambda)\right], \tag{30a}$$

$$B_{vy}(x,y,z) = \frac{s\Phi_0}{4\pi\lambda^2}\mathrm{sgn}z\sum_{\sigma=-1}^{1}\frac{\sigma y}{(x-\sigma x_0)^2+y^2}\left[\exp(-|z|/\lambda) - \frac{|z|}{r_\sigma}\exp(-r_\sigma/\lambda)\right], \tag{30b}$$

$$B_{vz}(x,y,z)=\frac{s\Phi_0}{4\pi\lambda^2}\sum_{\sigma=-1}^{1}\frac{\sigma}{r_\sigma}\exp(-r_\sigma/\lambda), \tag{30c}$$

$\sigma=\pm 1$ and $r_\sigma=\sqrt{(x-\sigma x_0)^2+y^2+z^2}$.

We use now the exact solution (27) to calculate the stray field generated by a pancake located far from the surface; i.e., we suppose that $x_0\gg\lambda$. We further assume that $|x|,|z|\ll x_0$. In this case the stray-field potential ψ reduces to

$$\psi(x,y,z)\approx-\frac{\Phi_0}{\pi^2\lambda}\frac{sx_0}{x_0^2+y^2}F(x,z), \tag{31}$$

where

$$F(x,z)=\int_0^\infty du\frac{\exp(ux/\lambda)\sin(uz/\lambda)}{1+u^2+u\sqrt{1+u^2}}. \tag{32}$$

This yields the magnetic field at the sample surface in the two limiting cases $|z|\ll\lambda$,

$$B_x(0,y,z)\approx-\frac{\Phi_0}{4\pi\lambda^2}\frac{sx_0}{x_0^2+y^2}\mathrm{sgn}z, \tag{33a}$$

$$B_y(0,y,z)\approx\frac{\Phi_0}{\pi^2\lambda^2}\frac{sx_0yz}{(x_0^2+y^2)^2}\ln\frac{\lambda}{|z|}, \tag{33b}$$

$$B_z(0,y,z)\approx-\frac{\Phi_0}{\pi^2\lambda^2}\frac{sx_0}{(x_0^2+y^2)}\ln\frac{\lambda}{|z|}, \tag{33c}$$

and $|z|\gg\lambda$,

$$B_x(0,y,z)\approx-\frac{2\Phi_0}{\pi^2\lambda^2}\frac{sx_0\lambda^3}{z^3(x_0^2+y^2)}, \tag{34a}$$

$$B_y(0,y,z)\approx\frac{2\Phi_0}{\pi^2\lambda^2}\frac{sx_0\lambda^2 y}{z(x_0^2+y^2)^2}, \tag{34b}$$

$$B_z(0,y,z)\approx\frac{\Phi_0}{\pi^2\lambda^2}\frac{sx_0\lambda^2}{z^2(x_0^2+y^2)}. \tag{34c}$$

In the region $|x|\gg\lambda$ the expression (31) for the scalar potential ψ simplifies to

$$\psi(x,y,z)\approx-\frac{\Phi_0}{\pi^2\lambda}\frac{s\lambda x_0 z}{(x_0^2+y^2)(x^2+z^2)}. \tag{35}$$

The symmetry of the spatial distribution of the magnetic field $\mathbf{B}(x,y,z)$ follows from Eqs. (27)–(30),

$$B_x(x,y,z)=B_x(x,-y,z),$$

$$B_x(x,y,z)=-B_x(x,y,-z),$$

$$B_y(x,y,z)=-B_y(x,-y,z),$$

$$B_y(x,y,z)=-B_y(x,y,-z),$$

$$B_z(x,y,z)=B_z(x,-y,z),$$

$$B_z(x,y,z)=B_z(x,y,-z). \tag{36}$$

In Figs. 2(a)–2(d) we visualize the field generated inside and outside the superconductor by a single pancake vortex residing at distances $x_0=0.5,\ 1,\ 2,$ and 10 from the sample surface in units of the penetration depth λ. The arrows indicate the direction of the field $\mathbf{B}(x,0,z)$ in the plane $y=0$. The x and z axes are drawn as dashed lines. The vertical z axis denotes the position of the sample surface $x=0$, and the pancake position is marked by a small circle at $(x_0,0,0)$.

The following features of the magnetic field of a pancake are seen from these figures.

First we note that $\mathbf{B}$ is continuous across the sample surface as it should be. Inside the sample the component B_x parallel to the superconducting layers performs a jump on the plane $z=0$; namely, it has a finite magnitude at $z=\pm 0$ but changes sign as one crosses the layer $z=0$. The resulting sharp bend of $\mathbf{B}$ is a known feature of the pancake field[3–5] and reflects the finite sheet current $\mathbf{I}(x,y)$ in the pancake layer, $\mathbf{B}_\parallel(x,y,+0)=-\mathbf{B}_\parallel(x,y,-0)=\mu_0\mathbf{I}(x,y)\times\mathbf{e}_z/2$. This jump is, of course, absent outside the sample since there is no current. Notice that the z component of $\mathbf{B}$ inside the sample changes sign somewhere between the pancake position and the surface. Without the stray-field contribution, this sign change would occur only at the surface since one has $B_{vy}(0,y,z)=B_{vz}(0,y,z)=0$. As a consequence, the measurable magnetic field outside the sample in the pancake plane $z=0$ is oriented *antiparallel* to the vortex and is finite at the surface due to the stray-field contribution.

A particular interesting field pattern is seen when the pancake is far from the surface, $x_0\gg\lambda$; see Fig. 2(d). The magnetic field lines then ''flow'' along the surface and flow back into the superconductor. Note that this ''U turn'' occurs at $|z|$ values large compared to λ and thus the magnitude of $\mathbf{B}$ is small there. It should be noted here that the current density has no z component in the considered model of extremely weak Josephson interlayer coupling. The current flows only in the xy planes, and near the surface it flows only in the y direction.

The stray field $\mathbf{B}_s$ outside the superconductor is a smooth field which is sharply peaked near the surface point $x=y=z=0$ closest to the pancake. The decay of $\mathbf{B}_s$ at the surface $x=0$ for large distances y,z is given by the formulas (33) and (34).

Figure 3 shows cross sections of the magnetic field in the planes (a) $y=0$ and (b), (c) on the sample surface $x=0$ for $x_0=\lambda=1$ in units of $s\Phi_0/\pi^2\lambda^3$. One can see that the total field near the origin is decreased substantially as compared with the field $\mathbf{B}_v$ of the vortex and its image $B_{vx}(0,0,+0)=-s\Phi_0/2\pi\lambda^3$ or $-\pi/2$ in the units of the graph. At the origin this reduction is exactly by a factor of 2 since $B_{ax}(0,0,+0)=-B_{sx}(0,0,-0)=\pi/4$.

III. FLUX THROUGH THE SAMPLE SURFACE PRODUCED BY A RANDOMLY DISTORTED ABRIKOSOV VORTEX

A stack of pancakes arranged to a perfectly straight Abrikosov vortex line parallel to the sample surface and perpendicular to the layers has its field parallel to the surface and thus has no stray field. Any nonuniform displacement of the pancakes results in a stray field and in a mean square flux through the sample surface.

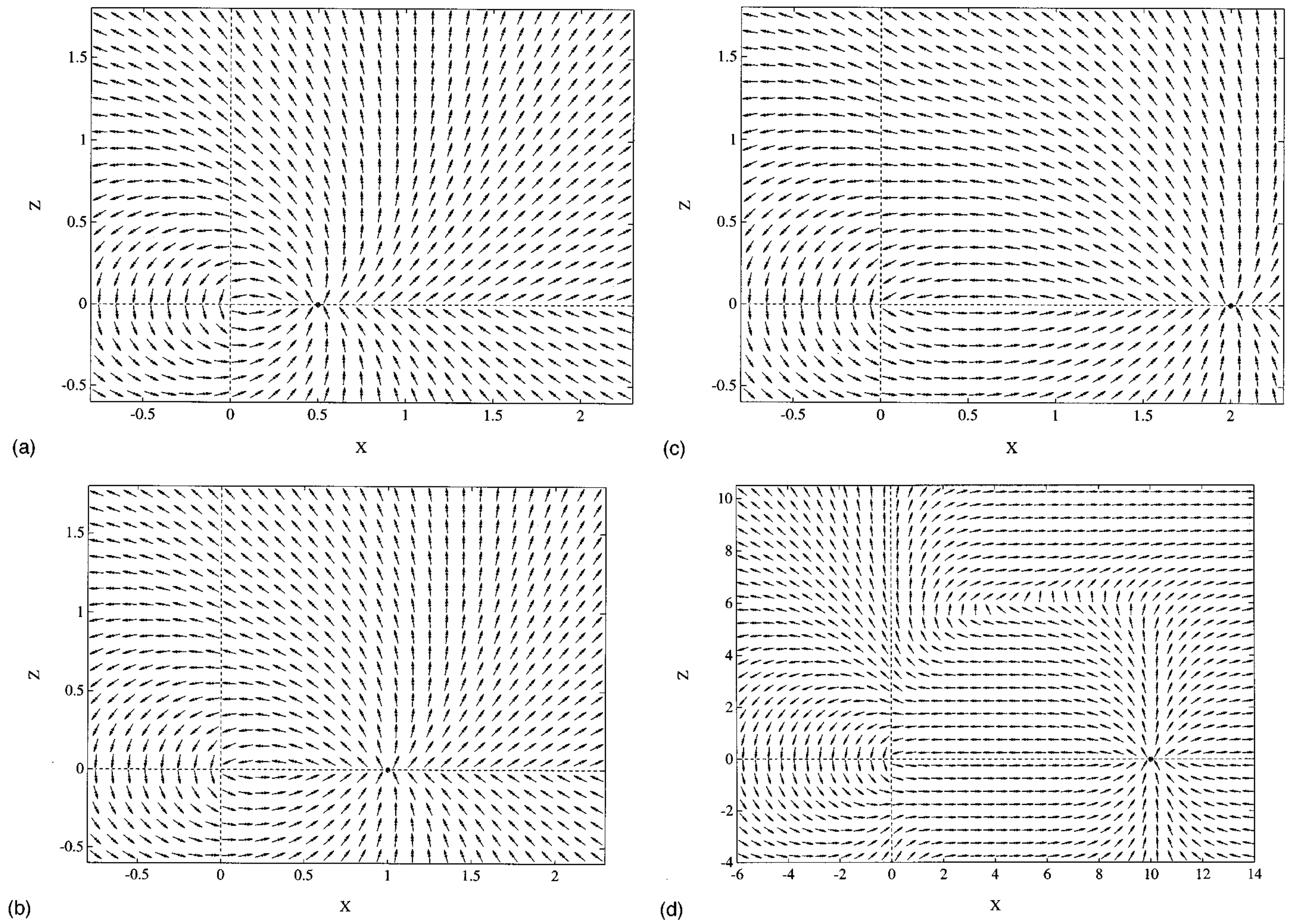

FIG. 2. Visualization of the magnetic field of a pancake positioned at a distance x_0 from a planar surface (vertical dashed line) perpendicular to the superconducting layers. In units of the penetration depth λ one has (a) $x_0=0.5$, (b) $x_0=1$, (c) $x_0=2$, and (d) $x_0=10$. The arrows (of constant length) denote the field direction. The pancake is marked by a circle.

To characterize these quantities we calculate the mean square flux through a band of an arbitrary width a on the sample surface, which is generated by a randomly distorted Abrikosov vortex line parallel to the sample surface and perpendicular to the layers; i.e., the function

$$K(a)=\langle\Phi^2(a)\rangle, \tag{37}$$

where

$$\Phi(a)=\int_0^a\int_{-\infty}^{\infty}B_x(0,y,z)\,dy\,dz. \tag{38}$$

Let us first find the flux through a band of width dz on the sample surface, which is generated by a pancake located at the point $(x_0,0,0)$; i.e., the quantity

$$\frac{d\Phi_p}{dz}=\int_{-\infty}^{\infty}B_x(0,y,z)\,dy. \tag{39}$$

It follows from Eq. (27) that

$$\begin{aligned}\frac{d\Phi_p}{dz}&=\int_{-\infty}^{\infty}\frac{\partial\psi}{\partial x}dy\\&=i\frac{\Phi_0 s}{2\pi\lambda^2}\int_{-\infty}^{\infty}du\frac{u}{\sqrt{1+u^2}(|u|+\sqrt{1+u^2})}\\&\quad\times\exp\left(iu\frac{z}{\lambda}\right)\left[1-\exp\left(-\sqrt{1+u^2}\frac{x_0}{\lambda}\right)\right].\end{aligned} \tag{40}$$

Next we consider a distorted Abrikosov vortex line with pancakes at $\mathbf{r}_n=(x_0+u_{xn},u_{yn},z+ns)$, where $n=0,\pm1,\pm2,\ldots$, and the displacements of the pancakes u_{xn},u_{yn} are assumed to be small; i.e., $|u_{xn}|,|u_{yn}|\ll x_0$. We treat here random and isotropic displacements with ensemble averages $\langle u_{xm}\rangle=\langle u_{ym}\rangle=0$, $\langle u_{xm}u_{yn}\rangle=0$, $\langle u_{xm}u_{xn}\rangle=f_x(|m-n|)$, and $\langle u_{ym}u_{yn}\rangle=f_y(|m-n|)$.

The flux through a band of width dz on the sample surface generated by this distorted vortex line is given by the sum over pancake contributions,

$$\frac{d\Phi}{dz}=\sum_n\frac{d\Phi_p}{dz}\Big|_{(x_0+u_{xn},u_{yn},ns)}. \tag{41}$$

This quantity depends only on the displacements u_{xn}. Keeping the term linear in u_{xn} and changing the sum to an integral we obtain

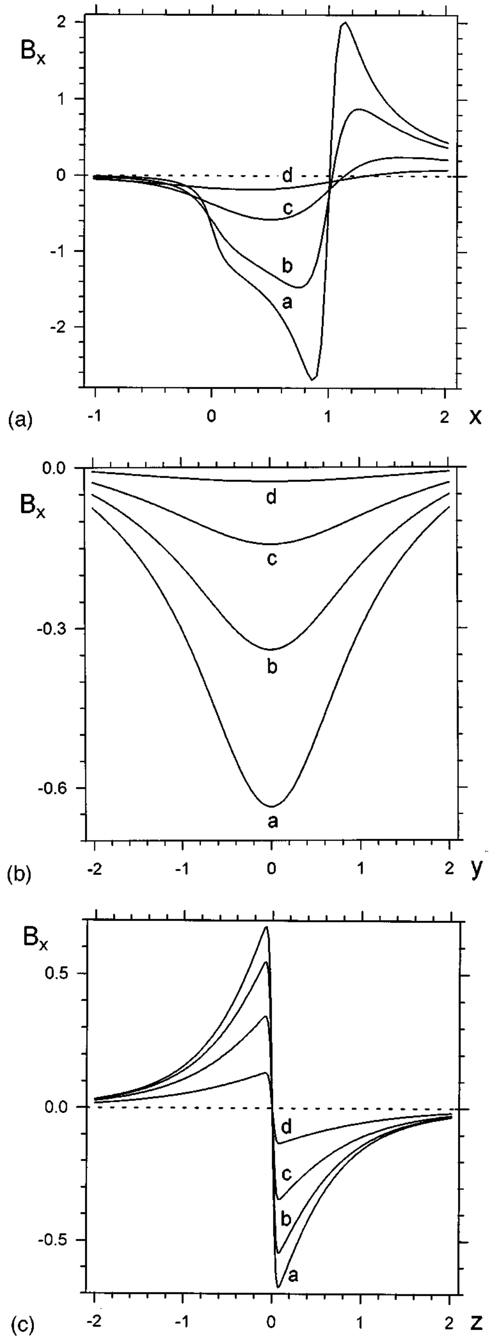

FIG. 3. The x component of the magnetic field generated by a pancake for $x_0=\lambda$ in units $s\Phi_0/\pi^2\lambda^3$. (a) $B_x(x,0,z)$ for $z=0.1$ (curve a), 0.2 (curve b), 0.5 (curve c), 1 (curve d); (b) $B_x(0,y,z)$ for $z=0.1$ (curve a), 0.5 (curve b), 1 (curve c), 2 (curve d); (c) $B_x(0,y,z)$ for $y=0.1$ (curve a), 0.5 (curve b), 1 (curve c), 2 (curve d). The dashed lines in graphs (a) and (c) indicate the z axis.

$$\frac{d\Phi}{dz}=i\frac{\Phi_0}{2\pi\lambda^3}\int_{-\infty}^{\infty}du\frac{u}{|u|+\sqrt{1+u^2}}\exp\left(i\frac{uz}{\lambda}-\sqrt{1+u^2}\frac{x_0}{\lambda}\right)\int_{-\infty}^{\infty}d\xi u_x(\xi)\exp\left(-i\frac{u\xi}{\lambda}\right). \quad (42)$$

Applying the Fourier transform to Eq. (42) we find

$$\tilde{\Phi}(p)=\frac{\Phi_0}{\lambda}\frac{\tilde{u}_x(p)}{|p|\lambda+\sqrt{1+p^2\lambda^2}}\exp\left(-\sqrt{1+p^2\lambda^2}\frac{x_0}{\lambda}\right). \quad (43)$$

Using the Fourier transform $\tilde{\Phi}(p)$ we present Eq. (37) for $K(a)$ in the form

$$K(a)=\frac{2}{L}\int_{-\infty}^{\infty}\frac{dp}{2\pi}|\tilde{\Phi}(p)|^2[1-\cos(pa)], \quad (44)$$

where L is the sample thickness.

As an example we apply Eq. (44) to the case when the displacements of pancakes arise from the thermal fluctuations of an Abrikosov vortex. For $x_0 \gg \lambda$ we get[8]

$$|\tilde{u}_x(p)|^2=\frac{8\pi L k_B T\lambda^4}{\Phi_0^2\ln(1+p^2\lambda^2)}. \quad (45)$$

Using Eqs. (44) and (45) we find that for any value of a and $|p|\lambda \ll 1$

$$K(a)=16k_BT\int_0^{\infty}\frac{dp}{p^2}\exp\left(-p^2x_0\lambda-\frac{2x_0}{\lambda}\right)[1-\cos(pa)]. \quad (46)$$

It follows from Eq. (46) that

$$K(a)\approx 4\sqrt{\pi}k_BT\frac{a^2}{\sqrt{x_0\lambda}}\exp\left(-\frac{2x_0}{\lambda}\right), \quad \frac{a^2}{x_0\lambda}\ll 1, \quad (47)$$

$$K(a)\approx 8\pi k_BT|a|\exp\left(-\frac{2x_0}{\lambda}\right), \quad \frac{a^2}{x_0\lambda}\gg 1. \quad (48)$$

Note that the characteristic length separating the coherent limit $K(a)\propto a^2$ from the diffusive limit $K(a)\propto a$ is $\sqrt{x_0\lambda}$.

IV. CONCLUSIONS

In conclusion, we have calculated the magnetic field of a pancake vortex positioned at arbitrary distance x_0 from the planar surface of a layered superconductor with layers perpendicular to the surface and with vanishing Josephson interlayer coupling. The magnetic stray field outside the specimen was fully accounted for. From this general solution the magnetic field of a stack of such pancakes, forming a vortex line, is obtained by linear superposition. As an application of the general results we give the variance of the magnetic flux through a band of arbitrary width a on the specimen surface. This width may be interpreted as the width of a strip-shaped detector scanning the surface of the superconductor in the ac plane, moving along the c axis, and measuring the stray field of a distorted vortex line parallel to the surface and (in average) to the c axis. This stray-field variance decays exponentially with the vortex distance x_0 from the surface, and it increases with the strip width a first quadratically and then linearly when a exceeds $\sqrt{x_0\lambda}$.

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