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Self-organized criticality effect on stability: magneto-thermal oscillations in a granular YBCO superconductor

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PACS. 74.60Ge – Flux pinning, flux creep, and flux-line lattice dynamics.
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Abstract. – We show that the self-organized criticality of Bean's state in each of the grains of a granular superconductor results in magneto-thermal oscillations. We find that the frequency of these oscillations is proportional to the external magnetic field sweep rate $\dot{B}_e$ and is inversely proportional to the square root of the heat capacity. We demonstrate experimentally and theoretically that this dependence is influenced mainly by the granularity of the superconductor.

The magnetic-flux dynamics in a superconductor with a strong pinning potential for vortices is an important field of application for the self-organized criticality theory [1]-[5]. In the scope of its ideas the pinned vortices space distribution arises as a result of a subsequent local vortex avalanches establishing the critical state. Bean's critical-state model [6] successfully describes the irreversible magnetization in type-II superconductors by introducing the critical current density j_c. In the framework of Bean's model the value of the slope of the stationary magnetic-field profile is less than or equal to $\mu_0 j_c$. It makes the spatial distribution of vortices similar to the sand particles spatial distribution in a sandpile [7].

The stationary critical state becomes unstable under certain conditions when the local vortex avalanches result in a global flux jump driving the system to the normal state [8]. This instability can be preceded by a series of magneto-thermal oscillations [9]. These oscillations have been reported earlier [10]-[12] but no systematic experiments accompanied by an adequate theory have been made.

Each of the local vortices avalanches establishing the critical state produces a heat pulse and a temperature rise decreasing the critical current density for a certain time interval and, thus, changing the initial conditions for the subsequent avalanches. In other words, the heat pulses produced by the vortex avalanches result in a correlation mechanism specific for the self-organized criticality of magnetic-flux motion in superconductors. In this letter we demonstrate that this mechanism results in the magneto-thermal oscillations arising close to the threshold of the superconducting-state stability. Our study is focused on the dependence of the frequency of these oscillations on the temperature and magnetic-field sweep rate in case of a granular superconductor.

We begin with a theoretical consideration and propose a one-dimensional model of a granular superconductor treating it as a stack of superconducting slabs having the width $2b_i$ ($i = 1, 2, 3, \ldots, N$) randomly distributed with a certain mean value b. We assume that: *a*) there is no electrical contact between the slabs and there is an ideal thermal contact between them; *b*) the external magnetic field $B_\mathrm{e}(t)$ is parallel to the sample surface ($\mathbf{B}_\mathrm{e} \parallel z$-axis) and the sweep rate $\dot{B}_\mathrm{e}$ is constant; *c*) the critical state arises simultaneously in the entire superconductor, *i.e.* in each of the slabs the magnetic, $B_\mathrm{i}(x,t)$, and electric, $E_\mathrm{i}(x,t)$, fields arise simultaneously. We also suppose that close to the instability threshold most of the slabs are saturated, *i.e.* B_e is higher than Bean's saturation field $B_\mathrm{p} = \mu_0 j_\mathrm{c} b$.

Magneto-thermal oscillations in the critical state are coupled oscillations of temperature and electric field. These oscillations are observed at low values of magnetic-field sweep rates $\dot{B}_\mathrm{e}$ with the background electric field in the slabs corresponding to the flux creep regime. In this regime the dependence of the current density j on the electric field E takes the form

$$j = j_\mathrm{c} + j_1 \ln\left(\frac{E}{E_0}\right), \tag{1}$$

where E_0 is the voltage criterion at which j_c is defined, and the characteristic current density j_1 ($j_1 \ll j_\mathrm{c}$) determines the slope of the j-E curve. In the framework of self-organized criticality of Bean's state the function $j_1(T)$ seems to tend to a non-zero value for $T \ll T_\mathrm{c}$ [4], [5]. We will consider the latter case as it is in good agreement with numerous experimental data [13] and assume that the ratio $n = j_\mathrm{c}/j_1 \gg 1$ is a certain constant for $T \ll T_\mathrm{c}$.

We use heat diffusion and Maxwell equations to determine the frequency of small-amplitude magneto-thermal oscillations. In the case of small temperature $\delta T = \exp[\gamma t]\,\theta(x)$ and electric field $\delta E = \exp[\gamma t]\,\epsilon(x)$ perturbations these equations read [8], [9]

$$\frac{nE_\mathrm{i}}{\mu_0 \gamma j_\mathrm{c}}\,\epsilon'' - \epsilon = -\frac{nE_\mathrm{i}}{j_\mathrm{c}}\left|\frac{\partial j_\mathrm{c}}{\partial T}\right|\theta, \tag{2}$$

$$\kappa\theta'' - \gamma C\theta = -j_\mathrm{c}\epsilon\,, \tag{3}$$

where C is the heat capacity per unit volume, κ is the heat conductivity, and γ is complex.

Let us clarify the following calculation qualitatively. Suppose that the initial temperature of the superconductor T_0 increases by a small perturbation θ_0 arising due to a heat pulse with the energy δQ_0. This temperature increase leads to a decrease of the superconducting currents. The reduction of these screening currents results in an additional flux penetration inside the superconductor. This flux motion induces an electric-field perturbation ϵ_0 producing an additional heat release δQ_1, an additional temperature rise θ_1, and, consequently, an additional reduction of the superconducting currents. At certain conditions this process results in a critical-state instability.

The critical state is stable if the heat release, δQ, arising in the process of electric-field and temperature perturbations development is less than the heat flux to the coolant. The value of δQ depends on both $\dot{B}_\mathrm{e}$ and B_e for the unsaturated grains and only on $\dot{B}_\mathrm{e}$ for the saturated grains. In our experiments most of the grains are saturated in the magnetic-field region corresponding to the magneto-thermal oscillations. Therefore, the heat release in the unsaturated grains, δQ_u, is small compared with the heat release in the saturated grains,

δQ_s. However, the term δQ_u is the only magnetic-field–dependent term in the heat balance equation and, thus, it determines the value of the global flux jump field B_j. In other words, the relatively small heat release in the unsaturated grains is tuning the superconducting state in a granular superconductor to the instability.

Under conditions of our experiments the temperature θ is practically uniform over the cross-section of the sample. Therefore, to solve eq. (2) we consider θ to be constant. In a saturated slab the background electric field $E_\mathrm{i} = \dot{B}_\mathrm{e} x$, where $x = 0$ corresponds to the middle plain. In this case eq. (2) takes the form

$$\frac{n\dot{B}_\mathrm{e}}{\mu_0 \gamma j_\mathrm{c}}\, x\, \epsilon'' - \epsilon = -\frac{n\dot{B}_\mathrm{e}\theta}{j_\mathrm{c}} \left|\frac{\partial j_\mathrm{c}}{\partial T}\right| x \tag{4}$$

with the boundary conditions $\epsilon'(\pm b_i) = 0$. The magneto-thermal oscillations exist for low values of $\dot{B}_\mathrm{e}$, where $l = n\dot{B}_\mathrm{e}/\mu_0\gamma j_\mathrm{c} \ll b_i$ and the appropriate solution of eq. (4) reads

$$\epsilon(x) = \frac{n\dot{B}_\mathrm{e}\theta}{j_\mathrm{c}} \left|\frac{\partial j_\mathrm{c}}{\partial T}\right| \left[x - \sqrt{b_i l}\, \exp\left[-\frac{b_i - |x|}{\sqrt{b_i l}}\right]\right]. \tag{5}$$

We now use the heat balance equation to find the frequency $\omega = \mathrm{Im}\,\gamma$. In order to do it, we integrate eq. (3) over the width of the sample with the spatial distribution of the electric field $\epsilon(x)$ given by eq. (5). Assuming that j_c is a linear function of T and $\delta Q_\mathrm{u} \ll \delta Q_\mathrm{s}$, we end up with the frequency of the magneto-thermal oscillations in the form

$$\omega \approx \frac{n\dot{B}_\mathrm{e}}{\sqrt{\mu_0 C(T_0)(T_\mathrm{c} - T_0)}}, \qquad \text{if} \quad \dot{B}_\mathrm{e} > \dot{B}_0, \tag{6}$$

where T_c is the critical temperature and $\dot{B}_0$ is the minimum ramp rate below which the critical state is stable against the flux jumps.

Thus, the frequency of the magneto-thermal oscillations in a granular superconductor is proportional to the magnetic-field sweep rate $\dot{B}_\mathrm{e}$ if $\dot{B}_0 < \dot{B}_\mathrm{e}$. It also follows from eq. (6) that for $\dot{B}_0 < \dot{B}_\mathrm{e}$ the ratio $\mu = \omega\sqrt{C(T_0)(T_\mathrm{c} - T_0)}/\dot{B}_\mathrm{e}$ is independent of T_0 and $\dot{B}_\mathrm{e}$, *i.e.* it is a constant characterizing the properties of the superconductor and its granular structure.

We perform an experimental study of the magneto-thermal oscillations in a textured $YBa_2Cu_3O_{7-\delta}$ superconductor grown from the melt and heat-treated after the preparation [14]. First, we do the measurements using the original sample (S0) with the size of $8 \times 9 \times 5.5\ \mathrm{mm}^3$. Next, we cut this sample in two approximately equal parts (S1 and S2) and we measure the magneto-thermal oscillations in S1 and S2. In this way we check the independence of the oscillations frequency from the size of the sample.

We perform the magnetic characterization by DC and AC measurements in a Quantum Design SQUID magnetometer using the sample S3 with a size of $(0.3 \times 0.5 \times 0.8)\ \mathrm{mm}^3$ cut from the sample S2. We find the onset temperature at zero magnetic field $T_\mathrm{c} \approx$ 88 K and the AC susceptibility transition width $\Delta T \approx$ 5 K.

We have measured one quarter of the DC magnetization loops for the sample S3 for several temperatures in the magnetic-field range $0 < B_\mathrm{e} < 5$ T. We also find that in parts of the sample S0 smaller than S3 the Bean field remains the same as for S3. It means that the space scale of the screening current loops is smaller than the dimensions of the sub-samples, *i.e.* the sample consists of small superconducting grains. We show in the inset in fig. 1 the temperature variation of the saturation value of the magnetization which is proportional to the critical current. As can be seen, the critical current density decreases linearly in the range $2\,\mathrm{K} < T < 6\,\mathrm{K}$.

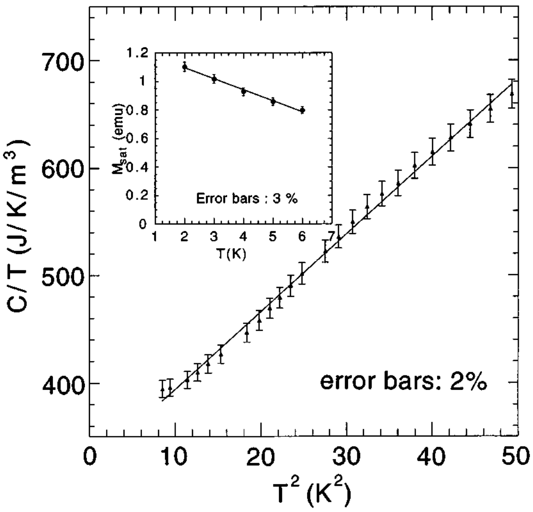

Fig. 1.

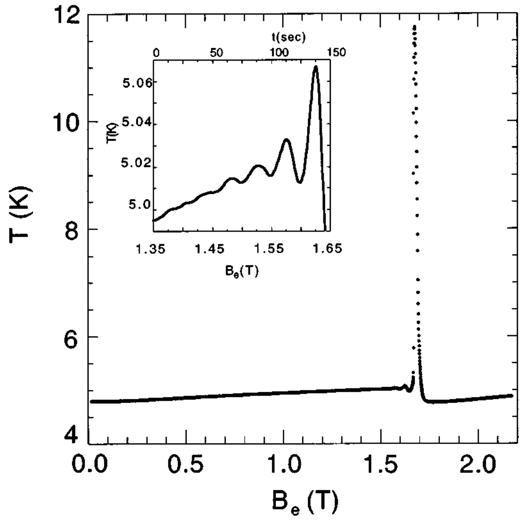

Fig. 2.

Fig. 1. – The specific-heat dependence on the temperature in the range $3\,\mathrm{K} < T < 7\,\mathrm{K}$: the experimental data (filled circles), the fit given by eq. (7) (solid line). The inset shows the temperature dependence of the saturation magnetization.

Fig. 2. – The temperature of the sample S1 as a function of the magnetic field B_e for the field sweep rate $\dot{B}_\mathrm{e} = 20.4$ G/s and $T_0 = 5.0$ K. The inset shows the magneto-thermal oscillations.

We show in fig. 1 the heat capacity measured for the sample S1 in the range $3\ \mathrm{K} < T < 7\ \mathrm{K}$. We find that within the accuracy of 2% the dependence $C(T)$ is given by

$$C(T) = 322\,T + 7.22\,T^3\ (\mathrm{JK^{-1}m^{-3}}) \tag{7}$$

in this range of T which corresponds to our experiments.

The experimental set-up for the magneto-thermal oscillations measurements consists essentially of a sample holder with a thin hollow powdered graphite column with low thermal conductance on which the YBCO sample is maintained. A small-size (0.2 mm thick, 2 mm^2 surface) carbon thermometer is glued to the sample. The entire sample holder is maintained under a very low pressure of He gas (2×10^{-6} Torr) in order to reduce the heat leak to the coolant. The sample is first cooled down, in zero magnetic field, to the starting temperature T_s, which is close to T_0. Then the field is established at a controlled rate and the temperature variation of the sample is measured.

We measure the magneto-thermal oscillations at temperatures in the range $3\,\mathrm{K} < T_0 < 7\,\mathrm{K}$ and the field sweep rates within the interval $10\,\mathrm{G/s} < \dot{B}_\mathrm{e} < 45\,\mathrm{G/s}$. We show in fig. 2 a typical sample temperature time-dependence for $T_0 = 5$ K and $\dot{B}_\mathrm{e} = 20.4$ G/s. At low values of the magnetic field $B_\mathrm{e}(t)$ there is a slow temperature increase due to the small vortices avalanches establishing the critical state. Above a certain magnetic-field value, the sample temperature oscillations appear with a period τ in the range $10\,\mathrm{s} < \tau < 70\,\mathrm{s}$ and an amplitude increasing in time. We mainly explore and analyse these magneto-thermal oscillations. A flux jump occurs close to the Bean field accompanied by a temperature rise up to about 12 K with a characteristic time of the order of 1 s. Then the sample temperature relaxes to the coolant temperature with a rate depending on the heat leak.

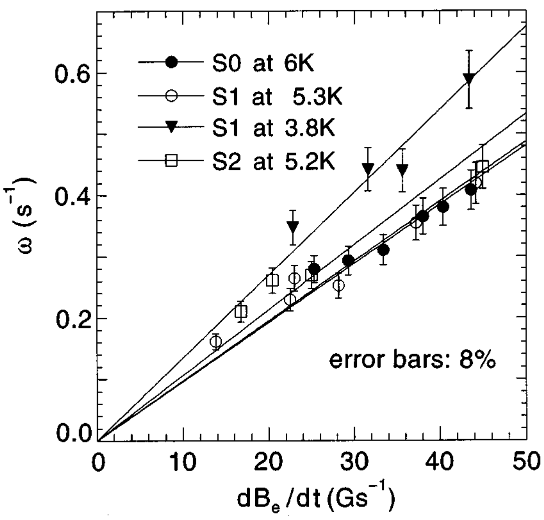

Fig. 3.

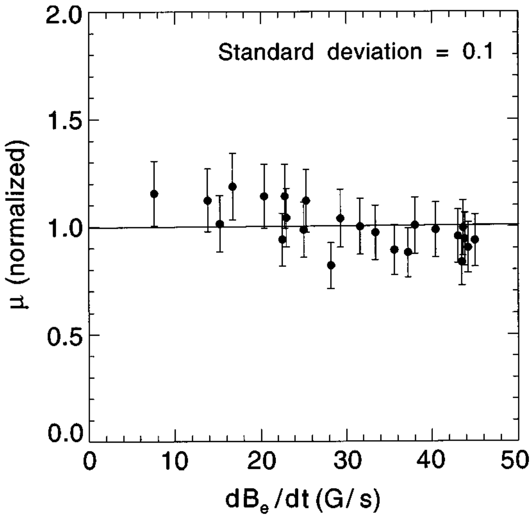

Fig. 4.

Fig. 3. – The dependences $\omega(\dot{B}_\mathrm{e})$ for the samples: S0 at $T_0 = 6\,\mathrm{K}$ (filled circles), S1 at $T_0 = 3.8\,\mathrm{K}$ (filled triangles) and at $T_0 = 5.3\,\mathrm{K}$ (open circles), and sample S2 at $T_0 = 5.2\,\mathrm{K}$ (open squares). The error bars are 8%.

Fig. 4. – The ratio μ for the samples S0, S1 and S2 as a function of the sweep rate for different temperatures T_0 from the range $3\,\mathrm{K} < T_0 < 7\,\mathrm{K}$. The values of μ are normalized by the mean value of their distribution. The error bars are 13%.

We show in fig. 3 the magneto-thermal oscillations frequency dependence on the field sweep rate, for several temperatures and different samples. As can be seen, the value of ω increases proportionally to $\dot{B}_\mathrm{e}$ within the accuracy of our measurements.

We show in fig. 4 the ratio $\mu = \omega\sqrt{C(T_0)(T_\mathrm{c} - T_0)}/\dot{B}_\mathrm{e}$ as a function of the magnetic-field sweep rate $\dot{B}_\mathrm{e}$ for the samples S0, S1, and S2 for different temperatures T_0 from the interval $3\,\mathrm{K} < T_0 < 7\,\mathrm{K}$. We use the relation given by eq. (7) to calculate the heat capacity and we normalize the values of μ by the mean value $\bar{\mu}$ of their distribution. We see in fig. 4 that the ratio μ is a constant within the accuracy of our experiments as predicted by eq. (6). The minor slope that can be seen in fig. 4 is related to the fact that eq. (6) is valid only for high values of $\dot{B}_\mathrm{e}$. Using the value of $\bar{\mu}$ we estimate the value of n as $n \approx 14.5$ which is in good agreement with the known experimental data [13].

To verify the uniformity of the sample temperature we estimate the characteristic heat redistribution time $t_\mathrm{h} = Cd^2/\kappa$, where d is the sample size. We find that $t_\mathrm{h} \approx 0.15$ s using the data $T \approx 5\,\mathrm{K}$, $C \approx 2500$ $\mathrm{JK^{-1}m^{-3}}$, $\kappa \approx 1$ $\mathrm{WK^{-1}m^{-1}}$ [15], and $d \approx 1$ cm. This time constant is almost two orders of magnitude less than both the period of the magneto-thermal oscillations and the temperature relaxation time. Thus, the temperature of the sample is uniform as suggested above.

In conclusion, we show theoretically that the self-organized criticality of Bean's state in each of the grains of a granular superconductor results in magneto-thermal oscillations. We study these oscillations experimentally in a granular YBCO sample at temperatures $3\,\mathrm{K} < T < 7\,\mathrm{K}$ and field sweep rates $10\,\mathrm{G/s} < \dot{B}_\mathrm{e} < 45\,\mathrm{G/s}$. We find both experimentally and theoretically that the frequency of the magneto-thermal oscillations is a linear function of the magnetic-field sweep rate and is inversely proportional to the square root of the heat capacity.

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