Flux creep and flux jumping

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The flux jump instability of Bean's critical state in the flux-creep regime of type-II superconductors is considered. We find the flux-jump field B_j which determines stability criterion of the superconducting state. The dependence of B_j on the external magnetic-field ramp rate $\dot{B}_e$ is calculated. We demonstrate that under the conditions typical for most of the magnetization experiments the slope of the current-voltage curve in the flux-creep regime determines the stability of the Bean's critical state, i.e., the value of B_j. We show that a flux jump can be preceded by magnetothermal oscillations and find the frequency of these oscillations as a function of $\dot{B}_e$.

I. INTRODUCTION

Bean's critical state model[1] successfully describes the irreversible magnetization in type-II superconductors by introducing a critical current density $j_c(T,B)$, where T is the temperature and B is the magnetic field. In the framework of Bean's model the value of the slope of the stationary magnetic-field profile is less than or equal to $\mu_0 j_c(T,B)$. This nonuniform flux distribution does not correspond to an equilibrium state and under certain conditions flux jumps arise in the critical state. The flux-jumping process results in a flux redistribution towards the equilibrium state and is accompanied by a strong heating of the superconductor.

Flux jumping has been frequently studied in conventional and high-temperature superconductors (see the review papers,[2,3] references therein, and the recent experimental studies[4–7]). In the general case two types of flux jumps can be considered, namely, global and local flux jumps. A global flux jump involves vortices into motion in the entire volume of the sample. A local flux jump occurs in a small fraction of the sample volume. Depending on the initial perturbation and the driving parameters there are two qualitatively different types of global flux jumps, namely, complete and partial flux jumps. The first turns the superconductor to the normal state. The second self-terminates when the temperature is still less than the critical temperature.

We illustrate a global flux-jump origination in a superconducting slab with the thickness $2d$ subjected to an external magnetic field $\mathbf{B}_e$ parallel to the sample surface (yz plane). In the framework of Bean's critical state model the spatial distribution of flux obeys the equation

$$\frac{dB}{dx} = \pm \mu_0 j_c, \tag{1}$$

where the $\pm$ stays for $x>0$ and $x<0$, respectively. We show the dependence $B(x)$ in Fig. 1 for the case when the critical current density depends only on the temperature, i.e., $j_c = j_c(T)$.

Let us now suppose that the temperature of the sample T_0 is increased by a small perturbation δT_0 arising due to a certain initial heat release δQ_0. The critical current density $j_c(T)$ is a decreasing function of temperature. Thus, the density of the superconducting current screening of the external magnetic field at $T=T_0+\delta T_0$ is less than at $T=T_0$. This reduction of the screening current enhances the magnetic flux inside the superconductor as shown in Fig. 1. The motion of the magnetic flux into the sample, which occurs as a result of the temperature perturbation δT_0, induces an electric-field perturbation δE_0. The arise of δE_0 is accompanied by an additional heat release δQ_1, an additional temperature rise δT_1, and, therefore, an additional reduction of the superconducting screening current density j_c. Under certain conditions this results in an avalanche-type increase of the temperature and magnetic flux in the superconductor, i.e., in a global flux jump.

The relative effect of the flux and temperature redistribution dynamics on flux jumping depends on the ratio τ of the flux (t_m) and thermal (t_κ) diffusion time constants,[2] $\tau = t_m/t_\kappa$. The value of the dimensionless parameter τ is determined by the corresponding diffusion coefficients,

$$\tau = \mu_0 \frac{\lambda \sigma}{C}, \tag{2}$$

where λ is the heat conductivity, σ is the conductivity, and C is the heat capacity.

For $\tau \ll 1$ ($t_m \ll t_\kappa$), rapid propagation of flux is accompanied by an adiabatic heating of the superconductor, i.e., there is not enough time to redistribute and remove the heat re-

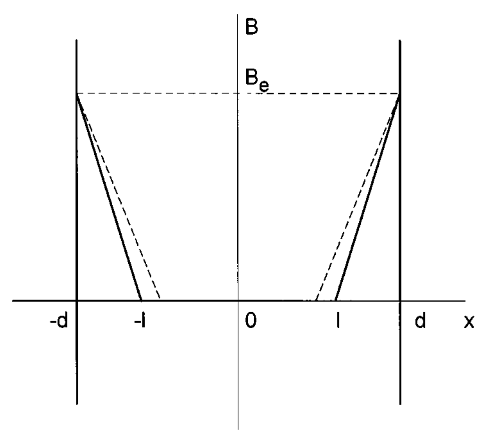

FIG. 1. Magnetic-field $B(x)$ distribution at different temperatures: $T=T_0$ (solid line), $T=T_0+\delta T$ (dashed line).

leased due to the flux motion. For $\tau \gg 1$ ($t_\kappa \ll t_m$), the spatial distribution of flux remains fixed during the stage of rapid heating. These adiabatic ($\tau \ll 1$) and dynamic ($\tau \gg 1$) approximations are the basis of the approach to the flux-jumping problem,[2] and the flux-jump scenario significantly depends on the relation between the values of the heat conductivity κ, heat capacity C, and conductivity σ that is defined as the slope of the j-E curve.

Let us now estimate the electric-field value typical for the magnetization experiments. In this case the external magnetic-field ramp rate $\dot{B}_e$ is usually in the interval $\dot{B}_e < 1$ T s^{-1}. The background electric field, E_b, induced by the magnetic-field variation is of the order of $E_b \sim \dot{B}_e(d-l)$, where $d-l$ is the width of the area occupied by the critical state (see, for example, Fig. 1). We estimate E_b as $E_b < 10^{-6}$ V cm^{-1} using the value $d-l < 10^{-4}$ m which is typical for the stability domain of Bean's critical state. This electric-field interval corresponds to the flux-creep regime. Therefore, for the magnetization experiments the background electric field E_b is from the flux-creep regime, where the relation between the current density j and the electric field E is strongly nonlinear. As a result, the value of σ, i.e., the slope of the j-E curve, strongly depends on the electric field and the flux jumping takes place on a background of a resistive state with a conductivity that strongly depends on the external magnetic-field ramp rate $\dot{B}_e$.

In order to calculate the conductivity in the flux-creep regime we use the dependence of j on E in the form

$$j = j_c + j_1 \ln\left(\frac{E}{E_0}\right), \tag{3}$$

where E_0 is the voltage criterion at which the critical current density j_c is defined, j_1 determines the slope of the j-E curve and $j_1 \ll j_c$. Note, that the actual choice of E_0 is critical. Indeed, by taking for the voltage criterion a certain value $\widetilde{E}_0$ instead of E_0 we change the critical current density from j_c to $\widetilde{j}_c = j_c - j_1 \ln(\widetilde{E}_0/E_0)$. The difference between $\widetilde{j}_c$ and j_c is small as $\ln(\widetilde{E}_0/E_0) \sim 1$ and $j_1 \ll j_c$. It is common to define the critical current value as the current density at $E_0 = 10^{-6}$ V cm^{-1}. Let us also note that a power law

$$j = j_c \left(\frac{E}{E_0}\right)^{1/n} \tag{4}$$

with $n \gg 1$ is often used to describe the j-E curve in the flux-creep regime. Expanding the dependence given by Eq. (4) in series in $1/n \ll 1$ and keeping the first two terms we find that if we take $n = j_c/j_1$, then Eqs. (3) and (4) coincide with the accuracy of $1/n^2 \ll 1$.

The relation (3) was derived in the framework of the Anderson-Kim model[8–10] considering the thermally activated uncorrelated hopping of bundles of vortices. The vortex-glass[11] and collective-creep[12,13] models result in more sophisticated dependences of j on E. However, these j-E curves coincide with the one given by Eq. (3) if $j - j_c \ll j_c$. The recently developed self-organized criticality approach to the critical state[14,15] also results in Eq. (3) if $j - j_c \ll j_c$. The logarithmic dependence of the current density j on the electric field E in the interval $j - j_c \ll j_c$ is in good agreement with numerous experimental data.[16] In this paper we use the j-E curve given by Eq. (3) to calculate the conductivity σ assuming that $j_1/j_c \ll 1$.

It follows from Eq. (3) that for the flux-creep regime the conductivity σ is given by the formula

$$\sigma = \sigma(E) = \frac{dj}{dE} = \frac{j_1}{E}. \tag{5}$$

We estimate the value of σ as $\sigma > 10^{10}\ \Omega^{-1}$ cm^{-1} using the typical data $j_1 > 10^3$ A cm^{-2} and $E < 10^{-7}$ V cm^{-1}. It follows from this estimation that the conductivity σ determining the flux-jump dynamics for the magnetization experiments is very high. As a consequence the dimensionless ratio τ is also very high. Thus, the scenario of a flux jump for the magnetization experiments corresponds to the limiting case when $\tau \gg 1$ and the rapid heating stage takes place on the background of a ''frozen-in'' magnetic flux.

The nonlinear conductivity $\sigma(E)$ significantly affects the flux-jumping process. In particular, it results in the dependence of the flux-jump field B_j on the ramp rate $\dot{B}_e$. This dependence is known from experiments[2,3,5] but to our knowledge was not considered theoretically as originating from the logarithmic j-E curve.

Under certain conditions a flux jump is preceded by a series of magnetothermal oscillations.[2] These oscillations have been observed for both low-temperature[17,18] and high-temperature superconductors.[5] Theoretically, such magnetothermal oscillations were considered for a flux jump developing in the flux-flow regime.[19] In this case the j-E curve is linear and the value of the conductivity σ is electric-field independent. The high and electric-field-dependent conductivity $\sigma(E)$ significantly affects the flux dynamics and therefore the magnetothermal oscillations. In particular, it results in the dependence of the frequency of the magnetothermal oscillations on the magnetic-field ramp rate $\dot{B}_e$. To our knowledge, this effect of the logarithmic j-E curve on the magnetothermal oscillations was not treated theoretically.

In this paper we consider the flux-jump instability of Bean's critical state on the background of a nonuniform electric field determining the conductivity of the type-II superconductor in the flux-creep regime. We find the flux-jump field B_j that limits the critical state stability and its dependence on the external magnetic-field ramp rate $\dot{B}_e$. We show that a flux jump can be preceded by magnetothermal oscillations and find the frequency of these oscillations as a function of $\dot{B}_e$.

The paper is organized in the following way. In Sec. II, we consider the critical state stability qualitatively and obtain the stability criterion. In Sec. III, we derive the equations determining the development of the small temperature and electric-field perturbations and calculate the frequency of the magnetothermal oscillations. In Sec. IV, we summarize the overall conclusions.

II. QUALITATIVE CONSIDERATION

In this section we consider the critical state stability qualitatively assuming that the thermomagnetic instability develops much faster than the magnetic-flux diffusion. In other words, we treat the case when the heating accompanying the thermomagnetic instability takes place on the background of a "frozen-in" magnetic flux. In Sec. III we derive the exact criterion of applicability of the following qualitative reasoning.

Let us consider a superconducting slab with the thickness $2d$ subjected to a magnetic field parallel to the sample surface (see Fig. 1) and suppose that the temperature of the sample T_0 is increased by a small perturbation δT. To keep the critical state stable, i.e., to keep the screening current at the same level, an electric-field perturbation δE arises. The additional electric field δE causes an additional heat release $\delta Q \propto \delta E$, which is the "price" for keeping the total screening current density at the same level, i.e., the "price" for the "frozen-in" magnetic flux.

The critical state is stable if the additional heat release δQ can be removed to the coolant by the additional heat flux $\delta W \propto \delta T$ resulting from the temperature perturbation δT. Thus, the stability criterion for the critical state has the form

$$\delta W > \delta Q. \tag{6}$$

The additional heat release per unit length, δQ, is given by the integral of $j\,\delta E$ over the width of the superconducting slab

$$\delta Q = \int_{-d}^{d} j\,\delta E\,dx. \tag{7}$$

The additional heat flux δW is determined by the temperature perturbation δT at the sample surface, i.e.,

$$\delta W = h\,\delta T|_P, \tag{8}$$

where h is the heat transfer coefficient to the coolant with the temperature T_0 and P stays for the sample surface.

Using Eqs. (6), (7), and (8) we find the critical state stability criterion, namely, the inequality

$$\int_{-d}^{d} j\,\delta E\,dx < 2h\,\delta T|_P. \tag{9}$$

To derive the explicit form of this stability criterion we have to find the relation between δT and δE. To do it, we calculate the decrease of the current density δj_- resulting from the temperature perturbation δT and the increase of the current density δj_+ resulting from the electric-field perturbation δE. If the critical state is stable then the total screening current density stays constant. As a result, the relation between δE and δT is given by the equation

$$\delta j = \delta j_- + \delta j_+ = 0. \tag{10}$$

In the critical state, $j \approx j_c$, thus, the decrease of j due to the temperature perturbation δT is equal to

$$\delta j_- = -\left|\frac{\partial j_c}{\partial T}\right| \delta T \tag{11}$$

(note that $\partial j_c/\partial T < 0$).

The increase of the current density due to the electric-field perturbation δE can be written as

$$\delta j_+ = \frac{dj}{dE}\,\delta E = \sigma\,\delta E. \tag{12}$$

Note that the conductivity σ is the differential conductivity, i.e., it is determined by the slope of the j-E curve.

Combining Eqs. (5) and (12), we find the relation between δj_+ and δE in the form

$$\delta j_+ = \frac{j_1}{E_b}\,\delta E = \frac{j_c}{nE_b}\,\delta E, \tag{13}$$

where $n = j_c/j_1 \gg 1$.

It follows from Eqs. (10), (11), and (13) that

$$\delta E = \frac{1}{\sigma}\left|\frac{\partial j_c}{\partial T}\right| \delta T = \frac{nE_b}{j_c}\left|\frac{\partial j_c}{\partial T}\right| \delta T. \tag{14}$$

Equations (5) and (14) allow us to understand the effect of the background electric field E_b on the critical state stability. It follows from Eq. (5) that a low electric field E_b results in a high differential conductivity ($\sigma \propto 1/E_b$). In its turn a high conductivity σ leads to a low electric-field perturbation [indeed, it follows from Eq. (14) that $\delta E \propto 1/\sigma \propto E_b$]. The smaller the δE, the less "costly" it is to remove the additional heat release. As a result the lower the background electric field E_b, the more stable the superconducting state.

Substituting Eq. (14) into Eq. (9) we find the critical state stability criterion in the form

$$\int_{-d}^{d} nE_b\left|\frac{\partial j_c}{\partial T}\right| \delta T\,dx < 2h\,\delta T|_P. \tag{15}$$

We have to treat the temperature perturbation δT in more detail to derive the final form of Eq. (15). The variation of the function $\delta T(x)$ on the interval $-d \leqslant x \leqslant d$ depends on the value of the Biot number

$$Bi = \frac{dh}{\kappa}, \tag{16}$$

where κ is the heat conductivity of the superconductor. Let us assume that the value of the heat transfer coefficient h is relatively low. As a result, $Bi \ll 1$ and the temperature perturbation $\delta T(x)$ is almost uniform over the width of the superconducting slab. It means that δT cancels in both sides of Eq. (15) and the stability criterion takes the following final form:

$$\mathscr{J} = \frac{n}{2h}\int_{-d}^{d} E_b\left|\frac{\partial j_c}{\partial T}\right| dx < 1. \tag{17}$$

Let us note, that this criterion was first derived in order to calculate the maximum value of a superconducting current under conditions typical for the critical current measurements, i.e., for a superconducting wire carrying a current that is increased with a given ramp rate.[20]

In addition, we assume for simplicity that the value of n is temperature and magnetic-field independent if $T < T_c$ and $B < B_{c2}$, where T_c is the critical temperature and B_{c2} is the upper critical field. This assumption is in a good agreement with numerous experimental data[16] as well as with the self-organized criticality approach to Bean's critical state.[14,15]

Using Eq. (1) we can rewrite the criterion given by Eq. (17) in the following form, which is convenient for the further analysis:

$$\mathscr{T}=\frac{n}{h}\int_0^d E_b\left|\frac{\partial j_c}{\partial T}\right|dx=\frac{n}{\mu_0 h}\int_{B^*}^{B_e}\frac{E_b}{j_c}\left|\frac{\partial j_c}{\partial T}\right|dB<1. \quad (18)$$

Here $B^*=B(0)$ is the magnetic field in the middle plane of the superconducting slab.

The background electric field E_b is induced by the varying external magnetic field $B_e(t)$ and thus the spatial distribution of E_b is given by the Maxwell equation

$$\frac{dE_b}{dx}=\frac{dB}{dt}. \quad (19)$$

Combining Eqs. (1) and (19) and taking into account that $j\approx j_c$ we find that

$$\frac{dE_b}{dB}=\pm\frac{\dot{B}}{\mu_0 j_c(B)}, \quad (20)$$

where the $\pm$ stands for $x>0$ and $x<0$ correspondingly. At the same time Eq. (19) results in the relation

$$\frac{\dot{B}_e}{j_c(B_e)}=\frac{\dot{B}}{j_c(B)}. \quad (21)$$

It follows from Eqs. (20) and (21) that the dependence of the background electric field E_b,B is given by

$$E_b=\pm\frac{\dot{B}_e(B-B^*)}{\mu_0 j_c(B_e)}, \quad (22)$$

where the $\pm$ stays for $x>0$ and $x<0$ correspondingly.

Let us now apply the criterion (18) to calculate the flux-jump field B_j assuming that initially there is no flux inside the superconducting slab, i.e., we calculate now the magnetic field of the first flux jump. Using Eqs. (18) and (22) we find the stability criterion in the form

$$\mathscr{T}=\frac{n\dot{B}_e}{\mu_0^2 h j_c(B_e)}\int_{B^*}^{B_e}\frac{B-B^*}{j_c(B)}\left|\frac{\partial j_c}{\partial T}\right|dB<1. \quad (23)$$

The value of the magnetic field B^* is given by the following system of equations:

$$B^*=0, \quad \text{if } B_e<B_p, \quad (24)$$

$$\int_{B^*}^{B_e}\frac{dB}{j_c(B)}=\mu_0 d, \quad \text{if } B_e>B_p, \quad (25)$$

where the penetration field for the magnetic flux, B_p, is determined by

$$\int_0^{B_p}\frac{dB}{j_c(B)}=\mu_0 d. \quad (26)$$

It follows from Eqs. (23) and (25) that $\mathscr{T}$ is an increasing function of the external magnetic field B_e if $B_e<B_p$ and $\mathscr{T}$ is a decreasing function of B_e if $B_e>B_p$. In other words, if for a given value of $\dot{B}_e$ the superconducting state is stable in the region $0<B_e<B_p$ then it is stable for any magnetic field. Thus, if a flux jump occurs it occurs only if $B_e<B_p$. Therefore, we consider now a superconducting slab that is wide enough meaning that $B_j<B_p$.

We have the criterion $\mathscr{T}(B_j)=1$ to find the flux-jump field B_j in the case when $B_e<B_p$. Thus, it follows from Eq. (23) that the dependence $B_j(\dot{B}_e)$ is given by the equation

$$\mathscr{T}(B_j)=\frac{n\dot{B}_e}{\mu_0^2 h j_c(B_j)}\int_0^{B_j}\frac{B}{j_c(B)}\left|\frac{\partial j_c}{\partial T}\right|dB=1. \quad (27)$$

Let us approximate the value of $|\partial j_c/\partial T|$ as

$$\left|\frac{\partial j_c}{\partial T}\right|\approx\frac{j_c(B)}{T_c(B)-T_0}. \quad (28)$$

Using Eq. (28) we rewrite Eq. (27) in the following form:

$$\frac{n\dot{B}_e}{\mu_0^2 h j_c(B_j)}\int_0^{B_j}\frac{B}{T_c(B)-T_0}dB=1. \quad (29)$$

We treat now the case when $T_0\ll T_c(B_j)$ or in other words $B_j\ll B_{c2}(T_0)$. It means that $T_c(B)\approx T_c$, where T_c is the critical temperature at zero magnetic field. It follows finally from Eq. (29) that the stability criterion determining the dependence $B_j(\dot{B}_e)$ is given by

$$\frac{B_j^2}{j_c(B_j)}=\frac{2\mu_0^2 h(T_c-T_0)}{n\dot{B}_e}. \quad (30)$$

Let us now consider the particular case of Bean's critical state model, namely, let us assume that the critical current density is magnetic-field independent, i.e.,

$$j_c=j_c(T). \quad (31)$$

Using Eq. (30) we find the following formula for the first flux-jump field B_j:

$$B_j=\sqrt{\frac{2\mu_0^2 j_c(T_0)h(T_c-T_0)}{n\dot{B}_e}}\propto\frac{1}{\dot{B}_e^{1/2}}. \quad (32)$$

It follows from Eq. (32) that the value of B_j is inversely proportional to the square root of the magnetic-field ramp rate $\dot{B}_e$ and is, therefore, decreasing with the increase of $\dot{B}_e$. The physics of this effect is related to the decrease of the conductivity $\sigma(E)$ in the flux-creep regime with the increase of the background electric field E_b, i.e., with the increase of $\dot{B}_e$.

We derive the expression for B_j assuming that the rapid heating stage of a flux jump takes place on the background of a ''frozen-in'' magnetic flux. This approach is valid if $\tau\gg 1$ which is the same as

$$\dot{B}_e\ll\frac{1}{n}\frac{B_p}{t_\kappa}, \quad (33)$$

where we introduce the typical thermal diffusion time constant t_κ as

$$t_\kappa=\frac{d^2 C}{\kappa}. \quad (34)$$

Let us now compare the values of B_j and B_a, where B_a determines the flux-jump field for the adiabatic stability criterion.[21] This well-known criterion is based on the suggestion that the heating accompanying a flux jump is an adiabatic process, i.e., it is assumed that there is no heat redistribution during a flux jump. Therefore, the adiabatic stability criterion corresponds to the limiting case of $\tau \ll 1$ that is not the case typical for the magnetization experiments.

The value of B_a is given by the formula[21,2]

$$B_a = \frac{\pi}{2}\sqrt{\mu_0 C(T_0)(T_c - T_0)}. \tag{35}$$

It follows from the comparison of Eqs. (32) and (35) that $B_a < B_j$ if

$$\dot{B}_e < \frac{8}{\pi^2}\frac{\mu_0 j_1 h}{C} = \frac{8}{\pi^2}\frac{Bi}{n}\frac{B_p}{t_\kappa}. \tag{36}$$

Note that the inequality given by Eq. (36) is stronger than the one given by Eq. (33) as we assume that $Bi \ll 1$.

The critical current density is decreasing with the increase of the magnetic field. Let us now consider the effect of this dependence on the critical state stability assuming that the value of B_j is relatively high. To do it we use the Kim-Anderson model[10] to describe the function $j_c(B)$. In the case of a high magnetic field this model postulates the relation

$$j_c = \frac{\alpha(T)}{B}. \tag{37}$$

Using Eqs. (37) and (30) we find for the flux-jump field B_j the formula

$$B_j = \left(\frac{2\mu_0^2 \alpha(T_0) h (T_c - T_0)}{n \dot{B}_e}\right)^{1/3} \propto \frac{1}{\dot{B}_e^{1/3}}. \tag{38}$$

The comparison of Eqs. (38) and (32) shows that the magnetic-field dependence of the critical current density slows down the decrease of B_j with the increase of $\dot{B}_e$.

III. QUANTITATIVE CONSIDERATION

We treat now the critical state stability in more detail and, in particular, we take into consideration the magnetothermal oscillations. We consider a superconducting slab with thickness $2d$ subjected to an external magnetic field parallel to the z axis (see Fig. 1).

We use for calculation Bean's critical state model assuming that the critical current density is magnetic-field independent, i.e., $j_c = j_c(T)$. We suppose also that $B_e \ll B_p = \mu_0 j_c d$. The background electric field E_b is then given by the formulas

$$E_b(x) = \begin{cases} \dot{B}_e(x - l), & \text{if } l < x < d, \\ 0, & \text{if } -l < x < l, \\ \dot{B}_e(x + l), & \text{if } -d < x < -l, \end{cases} \tag{39}$$

where the magnetic-field penetration depth l is equal to

$$l = d - \frac{B_e}{\mu_0 j_c}. \tag{40}$$

We consider now the stability of the stationary electric field and temperature distributions corresponding to the Bean's critical state against small perturbations of electric field and temperature. To do this we present the electric field $E(x,t)$ and the temperature $T(x,t)$ in the following forms:

$$E(x,t) = E_b(x) + \delta E(x) = E_b(x) + \epsilon(x)\exp(\gamma t), \tag{41}$$

$$T(x,t) = \widetilde{T}_0 + \delta T(x) = \widetilde{T}_0 + \theta(x)\exp(\gamma t), \tag{42}$$

where

$$\widetilde{T}_0 = T_0 + \frac{\dot{B}_e B_e^2}{2h\mu_0^2 j_c}, \tag{43}$$

$\epsilon(x) \ll E_b(x)$, $\theta(x) \ll T_0$, $(\mathrm{Re}\gamma)^{-1}$ is the characteristic time of the increase of the magnetothermal instability, and $\mathrm{Im}\gamma$ is the frequency of the magnetothermal oscillations. The stationary temperature $\widetilde{T}_0$ is different from T_0 due to the Joule heating power $j_c E_b$ produced by the background electric field E_b. Let us note that the difference between $\widetilde{T}_0$ and T_0 is small, i.e., $\widetilde{T}_0 - T_0 \ll T_c - T_0$. Indeed, using Eq. (30) we estimate the value of $\widetilde{T}_0 - T_0$ as $\widetilde{T}_0 - T_0 \approx (T_c - T_0)/n \ll T_c - T_0$.

The small perturbations $\delta T(x) = \theta(x)\exp(\gamma t)$ and $\delta E = \epsilon(x)\exp(\gamma t)$ decay if $\mathrm{Re}\gamma < 0$. Therefore, the stability margin of Bean's critical state is determined by the condition $\mathrm{Re}\gamma < 0$.

Substituting Eqs. (41) and (42) into the heat diffusion equation and the Maxwell equation

$$\kappa \frac{\partial^2 T}{\partial x^2} + j_c E = C \frac{\partial T}{\partial t}, \tag{44}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \frac{\partial j}{\partial t}, \tag{45}$$

we find a system of equations describing $\epsilon(x)$ and $\theta(x)$,

$$\theta'' - \frac{\gamma C}{\kappa}\theta = -\frac{j_c}{\kappa}\epsilon, \tag{46}$$

$$\epsilon'' - \frac{\mu_0 \gamma j_c}{n E_b(x)}\epsilon = -\frac{\mu_0 \gamma j_c}{T_c - T_0}\theta. \tag{47}$$

The prime denotes the derivative with respect to x, and we have used Eq. (39) and the relation

$$\frac{\partial j}{\partial t} = \frac{\partial j}{\partial E}\frac{\partial E}{\partial t} - \left|\frac{\partial j_c}{\partial T}\right|\frac{\partial T}{\partial t} = \frac{\gamma j_c}{n E_b}\epsilon - \frac{\gamma j_c}{T_c - T_0}\theta. \tag{48}$$

We assume that the superconducting slab is in thermal contact with a coolant with temperature T_0 and that the external magnetic-field ramp rate is given, i.e., $E'(\pm d) = \dot{B}_e$. In addition, the electric field $E(x)$ is equal to zero in the inner region of the superconducting slab ($|x| \le l$). As a result, the boundary conditions for Eqs. (46) and (47) are

$$\theta'(\pm d) = \mp \frac{h}{\kappa}\theta(\pm d), \tag{49}$$

$$\epsilon(\pm l) = 0, \quad \epsilon'(\pm d) = 0. \tag{50}$$

Let us present the solution for $\epsilon(x)$ in the form

$$\epsilon = \frac{n\theta}{T_c - T_0} E_b(x) + \epsilon_1(x), \tag{51}$$

where the first term corresponds to the approximation of a ''frozen-in'' magnetic flux [see Eq. (14)] and the second term describes the deviation from this approximation. It follows from Eqs. (47), (50), and (51) that with the accuracy of $Bi \ll 1$ the equation for $\epsilon_1(x)$ takes the form

$$\epsilon_1'' - \frac{\mu_0 \gamma j_c}{n\dot{B}_e(|x| - l)}\epsilon_1 = 0 \tag{52}$$

with the boundary conditions

$$\epsilon_1(\pm l) = 0, \quad \epsilon_1'(\pm d) = -\frac{n\dot{B}_e\theta}{T_c - T_0}. \tag{53}$$

We consider here the case of $\tau \gg 1$, i.e., the case when to the first approximation in $\tau^{-1} \ll 1$ the magnetic flux is frozen-in in the bulk of the superconducting slab. This means that during the stage of the rapid heating the magnetic-flux redistribution takes place only in a thin surface layer with the thickness $\delta_s \ll d - l$. In other words, the function $\epsilon_1(x)$ decays inside the superconducting slab and differs from zero only if $d - |x|$ is less or of the order of the skin depth δ_s. In the region $d - |x| \ll d - l$ Eq. (52) takes the form

$$\epsilon_1'' - \frac{\gamma B_p^2}{n\dot{B}_e B_e d^2}\epsilon_1 = 0. \tag{54}$$

The solution of Eq. (54) matching the boundary conditions (53) reads

$$\epsilon_1(x) = -\frac{n\dot{B}_e\delta_s\theta}{T_c - T_0}\exp\left(\frac{|x| - d}{\delta_s}\right), \tag{55}$$

where we introduce the value of the skin depth δ_s as

$$\delta_s = d\sqrt{\frac{nB_e\dot{B}_e}{\gamma B_p^2}}. \tag{56}$$

To find the values of $\mathrm{Re}\gamma$ and $\mathrm{Im}\gamma$ we integrate Eq. (46) over x from $-d$ to d. Using Eqs. (49), (51), and (55) we find the equation determining γ in the form

$$h - \frac{n\dot{B}_e B_e^2}{2\mu_0^2 j_c(T_c - T_0)} = -\gamma C d - \frac{B_e n^2 \dot{B}_e^2}{\gamma\mu_0^2 j_c(T_c - T_0)}. \tag{57}$$

We show schematically the dependences of $\mathrm{Re}\gamma$ and $\mathrm{Im}\gamma$ on B_e in Fig. 2, where the field B_i is determined by the equation

$$\frac{B_i^2}{B_j^2} - 1 = \sqrt{\frac{8Cd}{h}\frac{n\dot{B}_e B_i}{B_j^2}}. \tag{58}$$

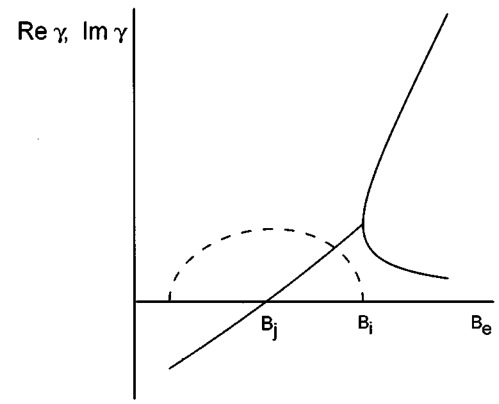

FIG. 2. The dependences of $\mathrm{Re}\gamma$ (solid line) and $\mathrm{Im}\gamma$ (dashed line) on B_e.

The difference between B_i and B_j is small in the case when the ramp rate $\dot{B}_e$ is low, i.e., $B_i - B_j \ll B_j$ if

$$\dot{B}_e < \frac{Bi}{2\pi^{2/3}n}\frac{B_p}{t_\kappa}\left(\frac{B_a}{B_p}\right)^{2/3}. \tag{59}$$

It follows from Eq. (57) that $\mathrm{Re}\gamma = 0$ if $B_e = B_j$, i.e., Bean's critical state is stable if $B_e < B_j$, where the flux-jump field B_j is given by the Eq. (32). We find also that at the stability threshold (for $B_e = B_j$) the value of γ is imaginary, i.e., $\gamma = i\omega$. Thus the magnetothermal instability is preceded by magnetothermal oscillations with the frequency ω given by the formula

$$\omega = \left(\frac{2n^3\dot{B}_e^3 h}{\mu_0^2 j_c d^2 C^2(T_c - T_0)}\right)^{1/4} \propto \dot{B}_e^{3/4}. \tag{60}$$

The approximation of the frozen-in magnetic flux is valid if the surface layer where $\epsilon_1(x) \neq 0$ is thin, i.e., if $\delta_s \ll d - l$. Using Eqs. (35), (40), (56), and (60) we find the applicability criterion of the above approach in the form

$$B_j \gg B_a\left(\frac{B_p}{\pi^2 B_a}\right)^{1/3}. \tag{61}$$

IV. SUMMARY

To summarize, the flux-jump instability of Bean's critical state in type-II superconductors is considered. We show that under the conditions typical for most of the magnetization experiments this instability arises in the flux-creep regime. The flux-jump field B_j that determines the critical state stability criterion is found. We show that Bean's critical state stability is determined by the slope of the current-voltage curve. The dependence of B_j on the external magnetic-field ramp rate $\dot{B}_e$ is calculated. We find the frequency of the magnetothermal oscillations preceding a flux jump as a function on the external magnetic field ramp rate $\dot{B}_e$.

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