Normal domains in large multifilamentary composite superconductors

V. S. Kovner and R. G. Mints
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, 69978 Ramat-Aviv, Israel

(Received 1 December 1993; accepted for publication 27 January 1994)

We study the dynamics of normal domains in a multifilamentary composite superconductor with a large amount of stabilizer outside the multifilamentary area. We consider the case when there is a transition layer with high thermal and electrical resistance between the superconductor and the stabilizer. We show the existence of localized and propagating normal domains in these composites. We derive an implicit expression for the propagation velocity of a normal domain.

I. INTRODUCTION

The study of a normal zone of a finite size (normal domains) in current-carrying superconductors has been continuously a subject of interest in the field of applied superconductivity (see, for example, the review of Ref. 1 and references therein). It was shown that in homogeneous superconductors normal domains are always unstable, i.e., if a normal domain nucleates, it will either expand or shrink.[1] If inhomogeneities in the physical properties of the superconductor are present, stable normal domains can exist in their vicinity. These localized normal domains were also found in composite superconductors, in the presence of a high resistance transition layer between the superconductor and the stabilizer.[2] In this case normal domains are stable for a finite range of current. Namely, a normal domain shrinks when the current is less than a certain value I_*, while for currents higher than I^* ($I^* > I_*$), the initial domain undergoes a periodic process of splitting. This splitting results in a string of normal domains formed along the composite.[3]

Recently, it was found experimentally that stable normal domains can propagate along a multifilamentary composite superconductor with a large amount of stabilizer outside the multifilamentary area.[4] The existence of these propagating normal domains is a result of the high Joule power generated in the superconductor during the process of current redistribution between the superconductor and the stabilizer. A number of theoretical studies were performed to investigate this effect. Huang and Eyssa[5] performed numerical simulations for the diffusion of heat and the redistribution of current in the composite in the presence of a normal zone. Their simulations showed the formation of stable propagating normal domains. Dresner[6] proposed an analytical method to calculate the propagation velocity of a normal domain. He performed explicit calculations approximating the Joule power during the process of current redistribution by an exponentially decaying term. Kupferman *et al.*[7] proposed an effective circuit model to study the nucleation and propagation of normal domains in large composite superconductors. Using this model they performed numerical simulations which showed the existence of propagating domains in the cryostable regime.

In this article we consider normal domains in a multifilamentary composite superconductor with a large amount of stabilizer outside the multifilamentary zone. We consider a case when a transition layer with high thermal and electrical resistivity exists between the superconductor and the stabilizer. This transition layer considerably increases the characteristic space scale for current redistribution in the composite. We use an effective circuit model to simulate the process of current redistribution in the composite in the presence of a normal zone. This model accounts for the final values of characteristic time and length scales of the current redistribution process. Using this model we present a detailed study of normal domains dynamics. We show the existence of both localized and propagating normal domains in a large multifilamentary composite with a transition layer between the superconductor and the stabilizer.

This article is organized as follows. In Sec. II we review the effective circuit model,[7] which is used to simulate the behavior of a normal zone in a large multifilamentary composite with a transition layer. We obtain the main equations describing the temperature and the current-density fields in the presence of a normal zone. In Sec. III we present the results of numerical simulations of normal zone origination and propagation for different values of the parameters characterizing the composite. We calculate the propagation velocity of a normal domain both numerically and analytically. Finally, we discuss the results. A brief summary is given in Sec. IV.

II. MAIN EQUATIONS

In this section we review the effective circuit model. We derive the equations describing dynamics of the temperature and the current-density distributions in a large composite superconductor in the presence of a normal zone.

Let us consider for simplicity a rectangular conductor consisting of three ribbons, namely, a superconductor (having a thickness of d_s), a transition layer (d_i) having high thermal and electrical resistance, and a normal metal (d_n). We suppose that $d_i \ll d_s, d_n$. The conductor carries a transport current I, and is kept in thermal contact with a heat reservoir of temperature T_0. The geometry of the problem is shown in Fig. 1. The dynamics of normal zone in the conductor is determined by both the temperature and the current-density distributions. A complete treatment of the problem requires the solution of the heat diffusion equation, which defines the dynamics of the temperature field and the set of Maxwell equations, which define the dynamics of the current-density distribution. These equations form a set of three-dimensional, time-dependent, nonlinear equations,

0021-8979/94/75(11)/7538/7/$6.00

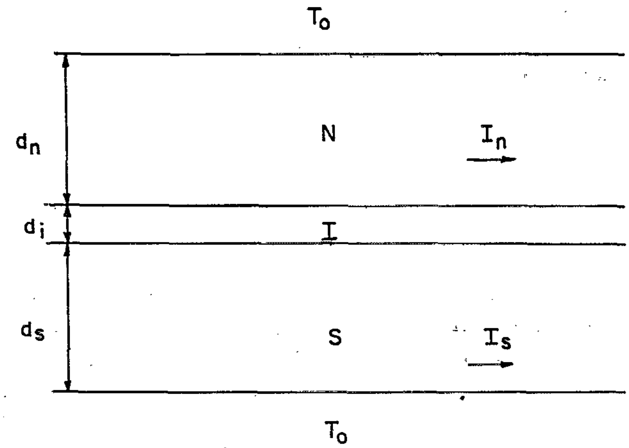

FIG. 1. Schematic structure of the considered composite superconductor.

which are difficult for either analytical or numerical investigation. We simplify the problem by using the effective circuit model proposed by Kupferman *et al.*[7] In the framework of this model the temperature of the normal metal T_n and of the superconductor T_s, as well as the current density in the normal metal j_n and in the superconductor j_s, may be regarded as almost uniform in the plane transverse to the sample axis. In this case we can consider their average values $T_s(x)$, $T_n(x)$, $j_s(x)$, $j_n(x)$ which are functions only of the coordinate along the conductor.

The process of current redistribution in the conductor is modeled by the effective electrical circuit sketched in Fig. 2. In this model each component is described by a discrete chain of resistors. The upper chain of resistors represents the stabilizer, each resistor of resistance, $R_n=\rho_n\Delta x/d_n$, where ρ_n is the resistivity of the stabilizer and Δx is an arbitrary discretization length. Similarly, the lower chain of resistors represents the superconductor, each resistor of resistance, $R_s=\rho_s(T_s)\Delta x/d_s$. Here ρ_s is the resistivity of the superconductor, which depends on both the local temperature and the current density in the superconductor. It vanishes in the superconducting phase, and it is finite above the normal transition. Both chains are linked through a chain of resistors $R=\gamma_R\rho_n d_n/\Delta x$, and $R_i=\rho_i d_i/\Delta x$. Here R is the transverse resistance of the stabilizer (γ_R is a numerical factor of the order of one, depending on the geometry of the conductor) and R_i is a resistance of the transition layer (ρ_i is the resistivity of transition layer, $\rho_i\gg\rho_n,\rho_s$). Finally, the inclusion of a characteristic time scale in the electric current diffusion process is accomplished by taking into account the inductance of the stabilizer $\mathscr{L}=\gamma_l\mu_0 d_n\Delta x$. Here γ_l is another numerical factor. Applying Kirchhow's laws on this circuit we obtain the following equation for the current-density distributions in the superconductor $j_s(x,t)$ and in the normal metal $j_n(x,t)$:

$$\mathscr{L}\frac{\partial j_n}{\partial t}=(\gamma_R d_n\rho_n+d_i\rho_i)\frac{\partial^2 j_n}{\partial x^2}-\frac{\rho_n}{d_n}j_n+\frac{\rho_s}{d_n}j_s. \tag{1}$$

One-dimensional heat equations for the averaged temperatures $T_s(x,t)$, $T_n(x,t)$ are

$$C_n\frac{\partial T_n}{\partial t}=\frac{\partial}{\partial x}\left(k_n\frac{\partial T_n}{\partial x}\right)-\frac{W_0}{d_n}(T_n-T_0)-\frac{k_i}{d_i d_n}(T_n-T_s)+Q_n, \tag{2}$$

$$C_s\frac{\partial T_s}{\partial t}=\frac{\partial}{\partial x}\left(k_s\frac{\partial T_s}{\partial x}\right)-\frac{W_0}{d_s}(T_s-T_0)-\frac{k_i}{d_i d_s}(T_s-T_n)+Q_s. \tag{3}$$

We use here the subscripts s, i, and n to denote the physical characteristics of the superconductor, transition layer, and normal metal, C is the heat capacity, and k is the heat conductivity, W_0 is the heat-transfer coefficient to the coolant, $j\equiv I/d_s$ is the maximum current density in the superconductor. The function $Q_s(T_s)$ is the effective rate of Joule heating in the superconductor per unit volume. It has contributions coming from the superconductor when it is in the normal state and from the transition layer. The function $Q_n(T_n)$ is the effective rate of Joule heating in the stabilizer per unit volume. It has contributions coming from the stabilizer and from the transition layer. As a result, $Q_s(T_s)$ and $Q_n(T_n)$ are given by the following formulas:

$$Q_s(T_s)=\rho_s j_s^2+\frac{1}{2}\rho_i\frac{d_n^2 d_i}{d_s}\left(\frac{\partial j_n}{\partial x}\right)^2, \tag{4}$$

$$Q_n(T_n)=\rho_n j_n^2+\left(\frac{1}{2}\rho_i d_i d_n+\gamma_R\rho_n d_n^2\right)\left(\frac{\partial j_n}{\partial x}\right)^2. \tag{5}$$

If the electrical resistance of the composite is dominated by the transition layer, then the perpendicular current density in the transition layer is small, i.e., $j_\perp\ll j_n,j_s$. In this case the relation between j_n,j_s can be written as

$$\frac{d_n}{d_s}j_n+j_s=j. \tag{6}$$

For convenience, we use the following dimensionless variables, the temperature in the superconductor θ_s, the temperature in the stabilizer θ_n, and the current density in the stabilizer i_n:

$$\theta_s\equiv\frac{T_s-T_0}{T_c-T_0},\quad \theta_n\equiv\frac{T_n-T_0}{T_c-T_0},\quad i_n\equiv\frac{d_n j_n}{d_s j_c}, \tag{7}$$

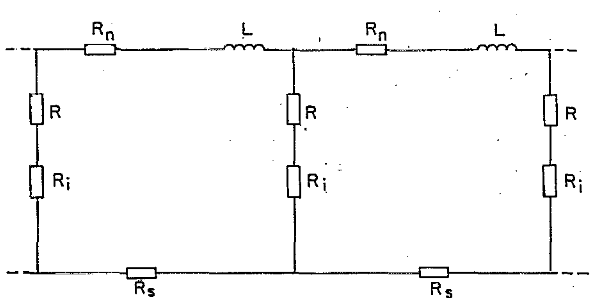

FIG. 2. Effective electrical circuit describing the current distribution in the composite superconductor.

where T_c is the critical temperature of the superconductor. We define $L_{\rm th}$ the characteristic thermal length and $\tau_{\rm th}$, the characteristic thermal relaxation time for the superconductor

$$L_{\rm th}^2 \equiv \frac{d_s k_s}{W}, \quad \tau_{\rm th} \equiv \frac{d_s C_s}{W}, \tag{8}$$

where $W \equiv W_0 + k_i/d_i$. We define l_n the characteristic thermal length and t_n the characteristic thermal relaxation time for the stabilizer

$$l_n^2 \equiv \frac{d_n k_n}{W}, \quad t_n \equiv \frac{d_n C_n}{W}. \tag{9}$$

We define l_m the characteristic length and t_m the corresponding characteristic time for the current redistribution

$$l_m^2 \equiv \gamma_R d_n^2, \quad t_m \equiv \frac{\mathscr{L} d_n}{\rho_n}. \tag{10}$$

We consider here the "step model" for the resistivity of the superconductor,[1] assuming that

$$\rho_s(j_s, T_s) = \rho_s \eta [j_s - j_c(T_s)], \tag{11}$$

where η is the Heaviside step function (η=0 if $x<0$ and η=1 if $x>0$), and $j_c(T)$ is the critical current density in the superconductor given by

$$j_c(T_s) = j_c \left[1 - \frac{(T_s - T_0)}{(T_c - T_0)} \right] = j_c(1 - \theta_s). \tag{12}$$

Finally, we introduce four dimensionless parameters

$$\xi \equiv \frac{\rho_s d_n}{\rho_n d_s}, \quad \alpha \equiv \frac{d_s^2 \rho_n j_c^2}{2 d_n W (T_c - T_0)},$$
$$h \equiv \frac{k_i}{d_i W}, \quad l_i \equiv \frac{\rho_i}{\rho_n} d_n d_i, \tag{13}$$

where ξ is the ratio of the resistances of the superconductor and the stabilizer per unit length, α is the ratio of the characteristic rates of Joule heating and the heat flux to the coolant, h characterizes the thermal coupling between the superconductor and the stabilizer, l_i is the transition length of current redistribution between the superconductor and normal metal. Finally, the dimensionless Eqs. (1)–(3) take the form

$$\frac{\partial \theta_s}{\partial t} = \frac{\partial^2 \theta_s}{\partial x^2} - \theta_s + h\theta_n + 2\alpha\xi(i - i_n)^2 \eta(i - i_n + \theta_s - 1) + \alpha L_i^2 \left(\frac{\partial i_n}{\partial x} \right)^2, \tag{14}$$

$$\tau_n \frac{\partial \theta_n}{\partial t} = L_n^2 \frac{\partial^2 \theta_n}{\partial x^2} - \theta_n + h\theta_s + 2\alpha i_n^2 + \alpha (L_i^2 + 2L_m^2) \left(\frac{\partial i_n}{\partial x} \right)^2, \tag{15}$$

$$\tau_m \frac{\partial i_n}{\partial t} = (L_i^2 + L_m^2) \frac{\partial^2 i_n}{\partial x^2} - i_n + \xi(i - i_n) \times \eta(i - i_n + \theta_s - 1), \tag{16}$$

where time is measured in units of $\tau_{\rm th}$ and length in units of $L_{\rm th}$. The dimensionless parameters i, τ_n, τ_m, L_n, L_i, and L_m are defined by

$$i \equiv \frac{j}{j_c}, \quad \tau_n \equiv \frac{t_n}{\tau_{\rm th}}, \quad \tau_m \equiv \frac{t_m}{\tau_{\rm th}},$$
$$L_n \equiv \frac{l_n}{L_{\rm th}}, \quad L_i \equiv \frac{l_i}{L_{\rm th}}, \quad L_m \equiv \frac{l_m}{L_{\rm th}}.$$

To complete the presentation of the model, we identify the characteristic time and length scales of the system. These scales are defined by Eq. (16). If the system is in its superconducting state (η=0), current diffuses from the stabilizer to the superconductor with the characteristic length $\lambda \equiv (L_2^2 + L_3^2)^{1/2}$ and the relaxation time τ_m. If the system is in its normal state (η=1), current redistributes into the stabilizer with a characteristic length $\lambda/(1+\xi)$ and a relaxation time $\tau_m/(1+\xi)$. Using the experimental data from Ref. 1 we estimated the values of L_i and L_m as L_i=10–100 and L_m=0.1–1. Thus the characteristic length scale of current redistribution is determined by the properties of the transition layer and is relatively large, and the characteristic time scale of current redistribution τ_m is determined by properties of the stabilizer. Using the experimental data[1] we estimate that τ_n=1, L_n=10, α=2, ξ=1100, h=0.1.

III. RESULTS

A. Numerical simulations

We study the dynamics of a normal zone in a multifilamentary composite superconductor with a large amount of stabilizer outside the multifilamentary area. We consider the case when a transition layer of high thermal and electrical resistance exists between the superconductor and the stabilizer. In order to study the normal zone dynamics in these systems we perform numerical simulations of Eqs. (14)–(16). We observe how the temperature and current-density distributions evolve in time. The composite is initially in the superconducting state, except a nucleus of the length $L_{\rm th}$. Inside this nucleus the temperature equals to the critical value θ_s=1.

The results of the numerical simulations can be described using the i,τ_m diagram shown in Fig. 3. This diagram consists of five regions, labeled with numbers from 1 to 5. Each region corresponds to a different type of evolution in time of the initial normal seed.

Region 1 in the i,τ_m diagram corresponds to the cryostable regime. In this case the initial normal nucleus decays after a time interval of the order of $\tau_{\rm th}$, and superconductivity recovers in the whole sample.

In region 2 in the i,τ_m diagram a stationary resistive domain exists.[2] The formation process of this domain is shown in Fig. 4. (Due to the symmetry of temperature distributions we show only the left half of a sample in this and following pictures.) The initial normal nucleus [see Fig. 4(a)] reaches the steady state after a time interval of the order of $\tau_{\rm th}$ [see Fig. 4(c)]. The final temperature distribution depends on the dimensionless parameters and is independent on the temperature of the initial nucleus.

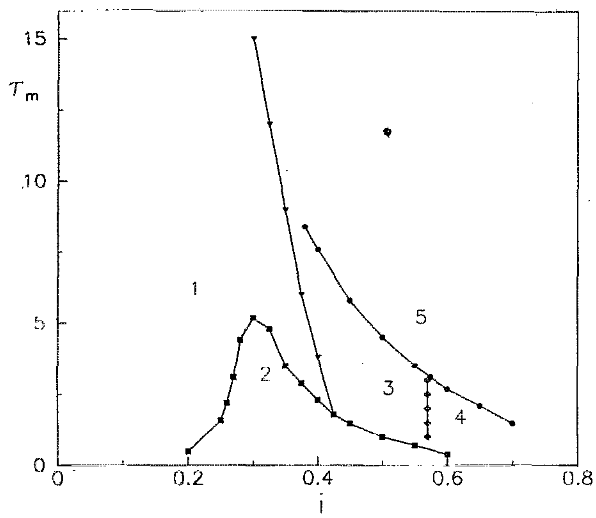

FIG. 3. i, τ_m diagram representing the results of numerical simulations.

In region 3 in the i, τ_m diagram a pair of stationary resistive domains exists. The formation process of this pair is shown in Fig. 5. The initial normal nucleus starts to expand during the diffusion of current out into the stabilizer. When the normal domain reaches a certain length, the center of the normal zone starts to cool down, while its outer parts continue to expand [Fig. 5(a)]. As a result, superconductivity recovers in the center of the normal zone and two normal domains traveling in opposite directions appear [Fig. 5(b)]. When this splitting process is completed, the domains slow down and finally stop at a certain distance from each other [see Figs. 5(c) and 5(d)]. The distance between these domains increases when the values of the parameters i and τ_m increase.

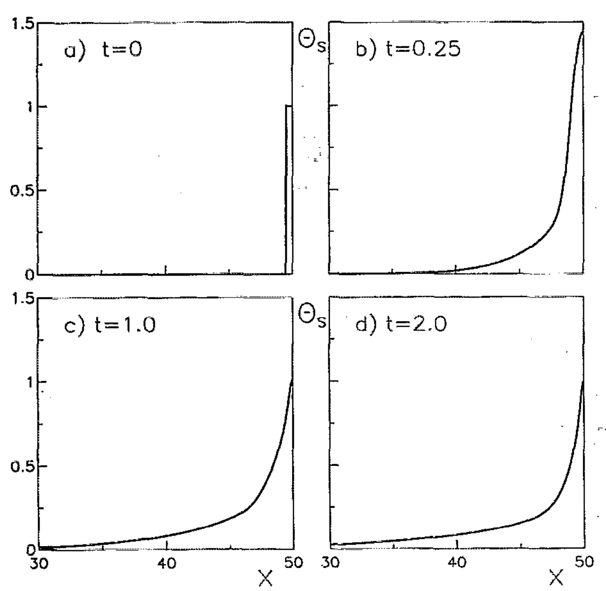

FIG. 4. The dynamics of temperature distribution in the superconductor, θ_s, in the case of a resistive domain. The parameters are $\lambda=20$, $\tau_m=3$, $i=0.3$.

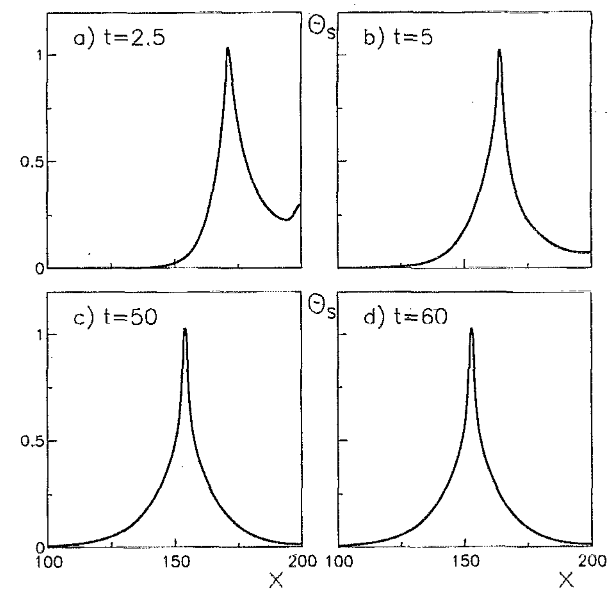

FIG. 5. The dynamics of temperature distribution in the superconductor, θ_s, in the case of a pair of resistive domains. The parameters are $\lambda=20$, $\tau_m=3$, $i=0.5$.

In region 4 in the i, τ_m diagram a string of resistive normal domains exists.[3] The formation process of this string is shown in Fig. 6. Its first stage is similar to the one described in the previous paragraph. The initial nucleus expands and its center cools down. Two domains appear, recede, and grow in size. When the distance between them is of the order of L_i each of them splits again into two domains [Fig. 6(a)]. Further propagation of the normal zone proceeds as follows. The outer resistive domains continue to move away from the middle of the sample. Their length increases and each of

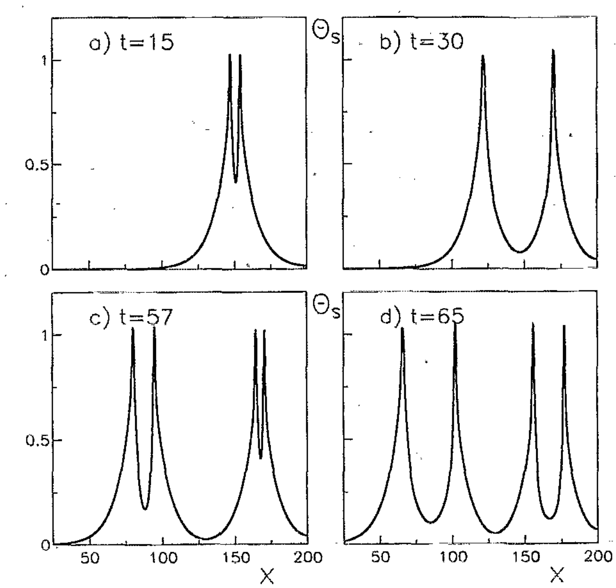

FIG. 6. The dynamics of temperature distribution in the superconductor, θ_s, in the case of a string of resistive domains. The parameters are $\lambda=20$, $\tau_m=2$, $i=0.6$.

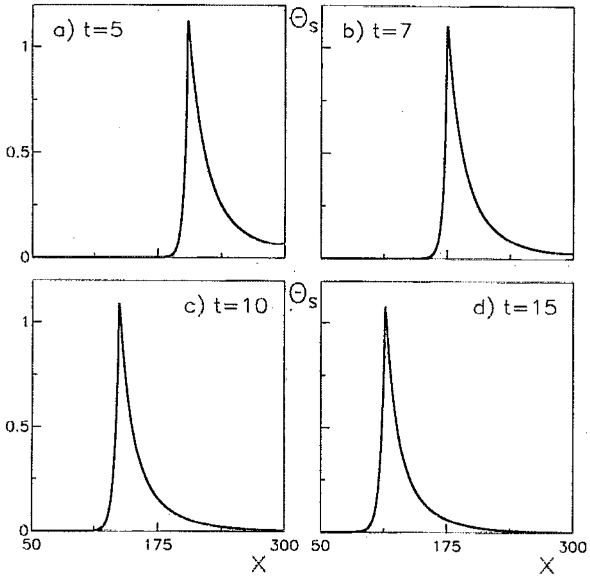

FIG. 7. The dynamics of temperature distribution in the superconductor, θ_s, in the case of propagating normal domains. The parameters are $\lambda=20$, $\tau_m=5$, $i=0.6$.

them splits in two at a distance of the order of L_2 from the place of the previous splitting [Fig. 6(c)]. Finally, a steady periodic string of resistive domains appears.

In region 5 in the i,τ_m diagram a steady state of propagating normal domains exists. The initial nucleus splits in two normal domains propagating with constant velocity in opposite directions, while superconductivity recovers behind them (see Fig. 7). The temperature field at the front of the propagating domain reaches a steady shape after a time interval of the order of τ_{th}. Note that the tail of the temperature profile reaches its steady state only after a relatively long time interval, which is of the order of the current distribution relaxation time τ_m. The velocity of propagation attains its final value much faster than the time required to obtain the steady profile.

B. Normal domains propagation velocity

Let us consider an analytical solution of Eqs. (14)–(16) in the regime of propagating normal domains (region 5 in the i,τ_m diagram). In a reference frame moving along the conductor with an arbitrary velocity v to be determined Eqs. (14)–(16) take the form

$$\frac{\partial^2\theta_s}{\partial x^2}-v\frac{\partial\theta_s}{\partial x}+\theta_s+h\theta_n+2\xi\alpha(i-i_n)^2\eta(i-i_n+\theta_s-1)+\alpha\lambda^2\left(\frac{\partial i_n}{\partial x}\right)^2=0, \tag{14}$$

$$L_n^2\frac{\partial^2\theta_n}{\partial x^2}-\tau_n v\frac{\partial\theta_n}{\partial x}-\theta_n+h\theta_s+2\alpha i_n^2+\alpha\lambda^2\left(\frac{\partial i_n}{\partial x}\right)^2=0, \tag{15}$$

$$\lambda^2\frac{\partial^2 i_n}{\partial x^2}-v\tau_m\frac{\partial i_n}{\partial x}-i_n+\xi(i-i_n)\eta(i-i_n+\theta_s-1)=0. \tag{16}$$

We define $x=0$ to be the point where the normal transition occurs and $x=m$ to be the point where superconductivity recovers. One can solve Eq. (16) in three regions, namely, $x<0(\eta=0)$, $0<x<m(\eta=1)$, and $m<x(\eta=0)$. In each of these regions Eq. (16) becomes a linear equation with constant coefficients. In order to simplify the problem we use the fact that the recovery of superconductivity occurs far behind the propagating front and hence does not affect significantly the value of the propagation velocity.[7] Thus, we can ignore the recovery of superconductivity and solve Eq. (16) in two regions. Namely, in the superconducting region, $x<0(\eta=0)$, and in the normal region, $0<x(\eta=1)$. The analytical solution of Eq. (16) in these two regions is given by

$$i_n(x)=\begin{cases}A\exp(k_+x), & \text{at } x<0\\ B\exp(-k_-x)+i\xi/(\xi+1), & \text{at } x>0,\end{cases} \tag{17}$$

where

$$k_+=\frac{v\tau_2+\sqrt{(v\tau_2)^2+4\lambda^2}}{2\lambda^2},$$

$$k_-=\frac{-v\tau_2+\sqrt{(v\tau_2)^2+4\lambda^2(1+\xi)}}{2\lambda^2}.$$

In the case of a high thermal resistance of the transition layer the coupling constant h is small, i.e., $h\ll1$. It allows one to neglect the term $h\theta_n$ in Eq. (14). The explicit expression for $i_n(x)$ can be then substituted into Eq. (14) yielding a linear equation for $\theta_s(x)$. We solve this equation in two regions, namely, $x<0$ and $x>0$. The matching conditions and the requirement of self-consistency at the transition point $i-i_n(0)=1-\theta_s(0)$ yield an implicit equation for the velocity v:

$$(1-i)\sqrt{v^2+4}=-\frac{(v+\sqrt{v^2+4}-4k_+)}{2(k_++k_-)^2}\times\frac{k_-^2k_+^2}{(4k_+^2-2k_+v-1)}\alpha\lambda^2i^2-\frac{k_-\sqrt{v^2+4}}{(k_-+k_+)}i. \tag{18}$$

Figure 8 shows the comparison of the velocity calculated by the numerical simulations with the velocity obtained from Eq. (18). We found that the velocity calculated by Eq. (18) is very close to the exact value, deviating from it less than 5%.

C. Discussion

Our results show the existence of both localized and propagating normal domains in a multifilamentary composite superconductor with a large amount of stabilizer outside the multifilamentary area. The formation of localized normal domains is a result of the relatively large characteristic length of current redistribution in the composite.[2,3] The value of this characteristic length is determined by the transition layer existing between the superconductor and the stabilizer. The stable propagating normal domains arise as a result of the relatively long process of current redistribution in the composite.[7] The characteristic time of current redistribution is determined by the properties of the stabilizer.

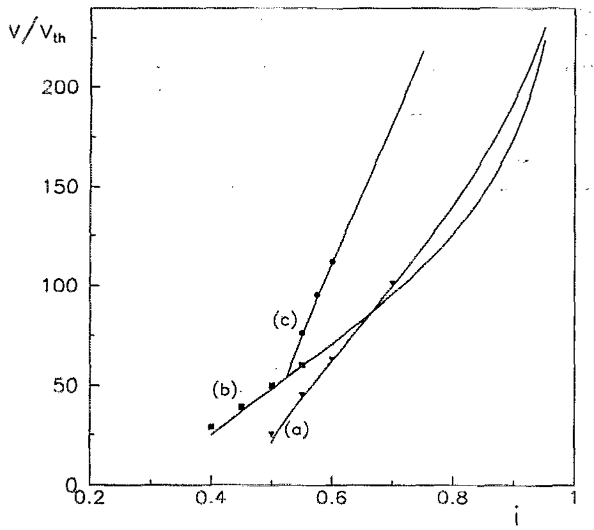

FIG. 8. The velocity of a propagating normal domain in units of $v_{th}=L_{th}/\tau_{th}$ as a function of transport current. The parameters are $\alpha=2$, $\xi=1111$, (a) $\tau_m=5$ and $\lambda=20$, (b) $\tau_m=10$ and $\lambda=20$, (c) $\tau_m=5$ and $\lambda=40$.

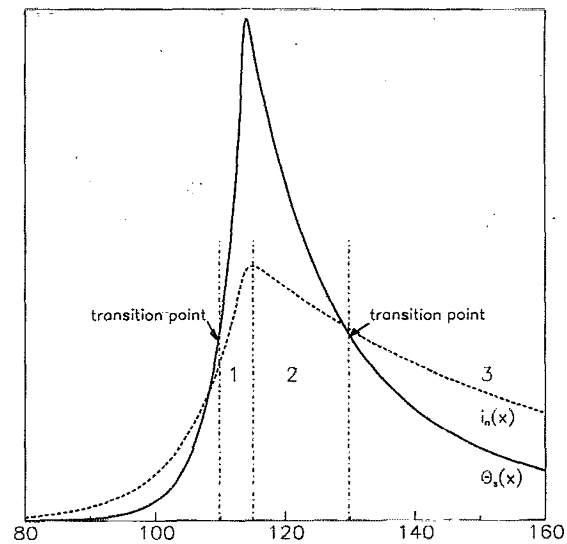

FIG. 9. Temperature distribution in the superconductor, $\theta_s(x)$, (solid line) and current-density distribution in the stabilizer, $i_n(x)$, (dashed line) in a propagating normal domain.

Let us now discuss the mechanism of a normal domain propagation in a large composite superconductor with a high resistance transition layer. Imagine that an initial normal seed originates in the superconductor. As a result the current starts to redistribute between the superconductor and the stabilizer. This process has a characteristic time scale τ_m/ξ and a characteristic length scale λ/ξ. After the redistribution of the current is complete, the conductor cools down during a certain time interval which is of the order of the thermal relaxation time τ_{th}. When the temperature becomes less than the critical temperature, the superconducting state starts to recover and the current diffuses back to the superconductor. This process takes a time interval which is of the order of τ_m and takes place on the characteristic length λ.

As the redistribution of current between the superconductor and the stabilizer requires a finite duration, the stabilizing mechanism suffers an effective delay time τ_m/ξ. During this time the current is confined in the superconductor and Joule power is high as in the case of an unstabilized superconductor. The effective Stekly parameter determining the characteristic rate of Joule heating in this temporarily unstabilized superconductor is given by $\alpha_{\text{eff}}=\alpha\xi$.[7] Note that in most cases of practical interest $\alpha_{\text{eff}}\gg 1$. The presence of a temporarily unstabilized superconductor in the front of a normal zone is the reason for the propagating domain's existence.

Let us now consider the temperature, $\theta_s(x)$, and current density, $i_s(x)$, distributions in a propagating normal domain. These dependences are shown in Fig. 9. It is seen that the temperature and the current density distributions in a normal domain have two characteristic parts. The first part (region 1 in Fig. 9) is a region behind the transition point ($\theta_s>1-i$) where the current redistributes into the stabilizer. This is a region of a temporarily unstabilized superconductor where the heat generation power is high. Using the analytical solution of Eq. (16) for $x>0$ ($\eta=1$), we find that the characteristic length of this region l_1 is given by

$$l_1=\frac{2\lambda^2}{-v\tau_m+\sqrt{(v\tau_m)^2+4\xi\lambda^2}}. \tag{19}$$

The second part (region 2 in Fig. 9) is the region where the temperature of the superconductor decreases towards the transition point. The current density, $i_n(x)$, in this interval is a slowly varying function of x and the current flows mostly through the stabilizer. The length of this region, l_2, is determined by the product of the propagation velocity of the normal domain and the characteristic thermal relaxation time

$$l_2=v\tau_{th}. \tag{20}$$

Behind the normal zone there is a superconducting region where current diffuses back to the superconductor (region 3 in Fig. 9). The characteristic length of this region, l_3, can be obtained from the analytical solution of the Eq. (16) for $x>0$ $\eta=0$ and is given by

$$l_3=\frac{2\lambda^2}{-v\tau_2+\sqrt{(v\tau_2)^2+4\lambda^2}}. \tag{21}$$

Using the implicit expression (18) and Eqs. (19)–(21) we calculate l_1, l_2, and l_3. The results of these calculations are in a reasonable agreement with the results of numerical simulations.

IV. SUMMARY

To summarize, we studied the dynamics of normal domains in a multifilamentary composite superconductors. We considered the case where a transition layer exists between the superconductor and the stabilizer. The effective circuit model was used to account for a relatively large time and for length scales of this system. We found the existence of both localized and propagating normal domains. In the regime of propagating domains we found an analytical expression for the velocity of propagation.

[1] A. Vl. Gurevich and R. G. Mints, Rev. Mod. Phys. **59**, 941 (1987).
[2] A. A. Akhmetov and R. G. Mints, J. Phys. D **16**, 2505 (1983).
[3] A. A. Akhmetov and R. G. Mints, J. Phys. D **18**, 925 (1985).
[4] J. M. Pfotenhauer, M. K. Abdelsalam, F. Bodker, D. Huttelstone, Z. Jiang, O. D. Lokken, D. Scherbarth, B. Tao, and D. Yu, IEEE Trans. Magn. **MAG-27**, 1704 (1991).
[5] X. Huang and Y. M. Eyssa, Cryogenics **32**, No. 1, 28 (1992).
[6] L. Dresner, in *Proceedings of the Eleventh Conference on Magnetic Technology (MT-11)*, Tsukuba, Japan, August 28–September 1, 1989, edited by T. Sekiguchi and S. Shimamoto (Elsevier Applied Science, New York, 1990), p. 1084.
[7] R. Kupferman, R. G. Mints, and E. Ben-Jacob, J. Appl. Phys. **70**, 7484 (1991).