PROPAGATING NORMAL DOMAINS IN LARGE COMPOSITE SUPERCONDUCTORS WITH TRANSITION LAYER

V.S. Kovner and R.G. Mints

School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Science
Tel-Aviv University
69978 Ramat-Aviv, Israel

ABSTRACT

We study nucleation and propagation of normal zone in large composite superconductors. We consider the case when a transition layer with high thermal and electrical resistance exists between the superconductor and the stabilizer. We treat the effect of this transition layer on the dynamics of normal zone. We show the existence of propagating normal domains for a wide range of parameters characterizing the composite.

INTRODUCTION

The normal zone of finite size (normal domains) in current-carrying superconductors has been a subject of continuous interest (see, for example, the review in Ref. 1, and references therein). In homogeneous superconductors normal domains are unstable, i.e., if a normal domain nucleates, it will either expand or shrink. Stable normal domains exist in the vicinity of different nonuniformities in the properties of the superconductor. These localized normal domains exist also in composite superconductors, if there is a transition layer with high thermal and electrical resistance between the superconductor and stabilizer. In this case normal domains are stable for a finite range of current. They shrink when the current is less than a certain value, while for higher currents a domain undergoes a periodic process of splitting. This splitting results in a periodic string of normal domains formed along the composite.[2,3]

Recently, superconducting composites with large stabilizer have been proposed for use in energy-storage devices. If a normal zone nucleates in the composite, the current redistributes into the stabilizer. This process leads to significant decrease of the Joule power thus to recovery of superconductivity. However, it was found experimentally[4] that normal domains of finite size propagate along a composite superconductor with a arge stabilizer despite the above stabilizing mechanism. Formation of these propagat-1g domains is a result of the relatively long process of current redistribution between 1e superconductor and the stabilizer. A number of theoretical studies were performed

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to investigate this effect. Huang et al.[5] performed numerical simulations treating diffusion of heat flux and redistribution of current in a large composite superconductor in the presence of a normal zone. Their simulations showed formation of a stable traveling normal domain. Dresner[6] proposed an analytical method to calculate the propagation velocity of a traveling normal domain, assuming the time dependence of the Joule power. He performed explicit calculations approximating the Joule power by an exponentially decaying term. Kupferman et al.[7] proposed an effective circuit model which allows investigation of the nucleation and propagation of normal zone in large composite superconductors both numerically and analytically.

We consider here the dynamics of normal domains in large composite superconductors with a transition layer between the superconductor and the stabilizer. This transition layer is changing the characteristic time and space scales for current redistribution between the superconductor and stabilizer. We use an effective circuit model for numerical simulations of the temperature field and the current density distribution in the composite.

THE MAIN EQUATIONS

In this section we review the effective circuit model and derive the equations describing the dynamics of the temperature and current density distributions in a composite superconductor in the presence of a normal zone.

Let us consider for simplicity a rectangular conductor consisting of three ribbons; namely, a superconductor (having a thickness of d_s), a transition layer (d_i) having high thermal and electrical resistance, and a normal metal (d_n). We suppose that $d_i << d_s$, d_n. The conductor carries a transport current I, and is in thermal contact with a heat reservoir of temperature T_0 (Fig. 1). To consider dynamics of the temperature and the current density distributions in the composite, one has to solve the heat diffusion equation for the temperature field coupled to the set of Maxwell equations for the current density distribution. These equations form a set of three-dimensional, time dependent, nonlinear equations, which is difficult for either analytical or numerical investigation. We simplify the problem by using the effective circuit model proposed by Kupferman et al.[7] In the framework of this model the temperature of the normal metal T_n, and of the superconductor T_s, as well as the current density in the normal metal j_n, and in the superconductor j_s, may be regarded as uniform in the plane transverse to the sample axis. The effective circuit model can be described by the electrical circuit

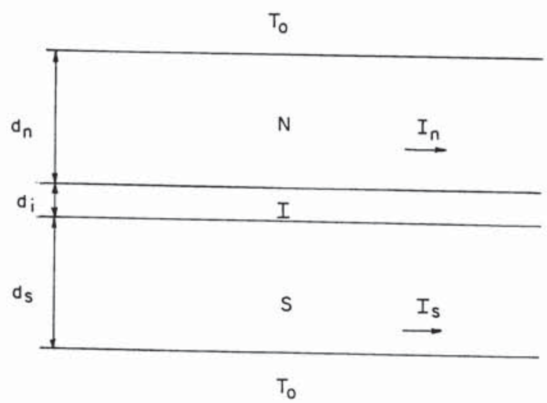

Figure 1 Schematic structure of the composite superconductor.

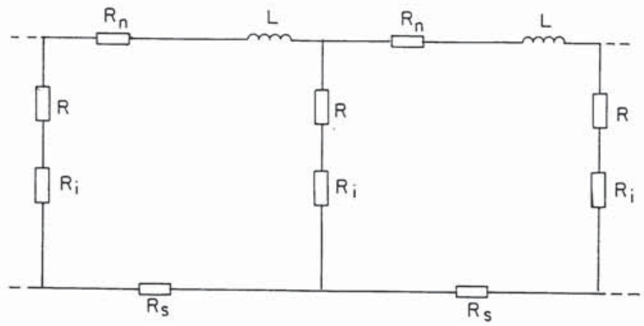

Figure 2 Effective electrical circuit describing the current distribution in the conductor.

sketched in Fig. 2. The upper chain of resistors represents the stabilizer, each resistor of resistance, $R_n = \rho_n \Delta x / d_n$, where ρ_n is the resistivity of the stabilizer and Δx is an arbitrary discretization length. Similarly, the lower chain of resistors represents the superconductor, each resistor of resistance, $R_s = \rho_s \Delta x / d_s$. Here ρ_s is the resistivity of the superconductor, which vanishes in the superconducting phase, and is finite in the normal phase. Both chains are linked through a chain of resistors $R = \gamma_R \rho_n d_n / \Delta x$, and $R_i = \rho_i d_i / \Delta x$. Here R is the transverse resistance of the stabilizer (γ_R is a numerical factor of the order of one, depending on the geometry of the conductor) and R_i is a resistance of the transition layer (ρ_i is the resistivity of transition layer, $\rho_i >> \rho_n$, ρ_s). Finally, the inclusion of a characteristic time scale in the electric current diffusion process is accomplished by taking into account the inductance of the stabilizer $\mathcal{L} = \gamma_l \mu_0 d_n \Delta x$. Here γ_l is another numerical factor. This model yields a set of three one–dimensional diffusion equations for the current density distributions in the stabilizer $j_n(x,t)$ and the superconductor $j_s(x,t)$ and for the temperature fields of the stabilizer, $T_n(x,t)$, and the superconductor, $T_s(x,t)$

$$\mathcal{L}\frac{\partial j_n}{\partial t} = (\gamma_R d_n \rho_n + d_i \rho_i)\frac{\partial^2 j_n}{\partial x^2} - \frac{\rho_n}{d_n} j_n + \frac{\rho_s}{d_n} j_s \tag{1}$$

$$C_n \frac{\partial T_n}{\partial t} = \frac{\partial}{\partial x}\Big(k_n \frac{\partial T_n}{\partial x}\Big) - \frac{W_0}{d_n}(T_n - T_0) - \frac{k_i}{d_i d_n}(T_n - T_s) + Q_n \tag{2}$$

$$C_s \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial x}\Big(k_s \frac{\partial T_s}{\partial x}\Big) - \frac{W_0}{d_s}(T_s - T_0) - \frac{k_i}{d_i d_s}(T_s - T_n) + Q_s. \tag{3}$$

Here we use the subscripts s, i and n to denote the physical characteristics of the superconductor, transition layer and normal metal. C is a heat capacity and k is a heat conductivity, W_0 is a coefficient of heat transfer to the coolant. The parameter $j \equiv I/d_s$ is the current density in the superconductor if all the current flows through it. The function $Q_s(T_s)$ is the rate of Joule heating in the superconductor per unit volume. It has the contributions from the Joule heating in the superconductor when it is in the normal state and from the perpendicular current in the transition layer ($j_\perp$). The function $Q_n(T_n)$ is the rate of Joule heating in stabilizer per unit volume having the contributions from the Joule heating in the stabilizer and from the perpendicular current in the transition layer and the stabilizer. As a result, $Q_s(T_s)$ and $Q_n(T_n)$ are given by

$$Q_s(T_s) = \rho_s j_s^2 + \frac{1}{2}\rho_i \frac{d_n^2 d_i}{d_s}\Big(\frac{\partial j_n}{\partial x}\Big)^2 \tag{4}$$

$$Q_n(T_n) = \rho_n j_n^2 + (\frac{1}{2}\rho_i d_i d_n + \gamma_R \rho_n d_n^2)\Big(\frac{\partial j_n}{\partial x}\Big)^2. \tag{5}$$

If the electrical resistance of the composite is dominated by the transition layer, then $j_\perp << j_n, j_s$. In this case the equation for j_n, j_s can be written as

$$\frac{d_n}{d_s} j_n + j_s = j. \tag{6}$$

For convenience, we use the following dimensionless variables, the temperature in the superconductor θ_s, the temperature in the stabilizer θ_n, and the current density in the stabilizer i_n

$$\theta_s \equiv \frac{T_s - T_0}{T_c - T_0}, \qquad \theta_n \equiv \frac{T_n - T_0}{T_c - T_0}, \qquad i_n \equiv \frac{d_n j_n}{d_s j_c} \tag{7}$$

where T_c is the critical temperature of the superconductor. We define L_{th}, the characteristic thermal length and τ_{th}, the characteristic thermal relaxation time of the superconductor

$$L_{th}{}^2 \equiv \frac{d_s k_s}{W}, \qquad \tau_{th} \equiv \frac{d_s C_s}{W} \tag{8}$$

were $W \equiv W_0 + k_i/d_i$. We define L_n, the characteristic thermal length and τ_n, the characteristic thermal relaxation time of the stabilizer

$$L_n{}^2 \equiv \frac{d_n k_n}{W}, \qquad \tau_n \equiv \frac{d_n C_n}{W}. \tag{9}$$

We define the characteristic length of the current redistribution L_m and the corresponding characteristic time τ_m

$$L_m{}^2 \equiv \gamma_R d_n{}^2, \qquad \tau_m \equiv \frac{\mathcal{L} d_n}{\rho_n}. \tag{10}$$

We assume here the "step model" for the resistivity of the superconductor[1]. In this case

$$\rho_s(j_s, T_s) = \rho_s \eta \big[j_s - j_c(T_s) \big], \tag{11}$$

where η is the Heaviside step function ($\eta = 0$ if $x < 0$ and $\eta = 1$ if $x > 0$), and $j_c(T)$ is the critical current density in the superconductor given by

$$j_c(T_s) = j_c \Big[1 - \frac{(T_s - T_0)}{(T_c - T_0)} \Big] = j_c(1 - \theta_s). \tag{12}$$

Finally, we introduce four dimensionless parameters

$$r^2 \equiv \frac{\rho_n d_s}{\rho_s d_n}, \qquad \alpha \equiv \frac{d_s^2 \rho_n j_c^2}{2 d_n W (T_c - T_0)}, \qquad h \equiv \frac{k_i}{d_i W}, \qquad L_i \equiv \frac{\rho_i}{\rho_n} d_n d_i \tag{13}$$

where r^2 is the ratio of the resistances of the stabilizer and the superconductor per unit length, α is the ratio of characteristic rates of Joule heating and the heat flux to the coolant (Stekly parameter), h characterizes the coupling between the temperatures in the superconductor and the stabilizer. L_i is a transition length, where the current flows over from the normal metal to the superconductor. Using Eqs.(4)-(13) we obtain Eqs.(1)-(3) in the dimensionless form

$$\frac{\partial \theta_s}{\partial t} = \frac{\partial^2 \theta_s}{\partial x^2} - \theta_s + h\theta_n + \frac{2\alpha}{r^2}(i - i_n)^2 \eta(i - i_n + \theta_s - 1) + \alpha L_2^2 \Big(\frac{\partial i_n}{\partial x}\Big)^2 \tag{14}$$

$$\tau_1 \frac{\partial \theta_n}{\partial t} = L_1^2 \frac{\partial^2 \theta_n}{\partial x^2} - \theta_n + h\theta_s + 2\alpha i_n^2 + \alpha(L_2^2 + 2L_3^2)\Big(\frac{\partial i_n}{\partial x}\Big)^2 \tag{15}$$

$$\tau_2 \frac{\partial i_n}{\partial t} = (L_2^2 + L_3^2)\frac{\partial^2 i_n}{\partial x^2} - i_n + \frac{1}{r^2}(i - i_n)\eta(i - i_n + \theta_s - 1) \tag{16}$$

where time is measured in units of τ_{th} and length in units of L_{th}. The dimensionless parameters i, τ_1, τ_2, L_1, L_2 and L_3 are

$$i \equiv \frac{j}{j_c}, \qquad \tau_1 \equiv \frac{\tau_n}{\tau_{th}}, \qquad \tau_2 \equiv \frac{\tau_m}{\tau_{th}}, \qquad L_1 \equiv \frac{L_n}{L_{th}}, \qquad L_2 \equiv \frac{L_i}{L_{th}}, \qquad L_3 \equiv \frac{L_m}{L_{th}}$$

It should be emphasized that all the properties of the system depend on the eight dimensionless parameters, defined above, as the final equations (14)-(16) include only these parameters.

RESULTS

To study the dynamics of a normal zone in the case of final time of current redistribution in the composite, we performed numerical simulations of Eqs. (14)-(16) for different values of the parameters i and τ_2. Note that τ_2 is a dimensionless characteristic time of current redistribution between the superconductor and stabilizer. The values of the other dimensionless parameters were evaluated using experimental data.[4] These values are: $\tau_1 = 1$, $L_1 = 10$, $L_2 = 20$, $L_3 = 1$, $\alpha = 2$, $r = 0.03$, $h = 0.1$. We treated the temperature and the current density evolution over time. The composite is initially in the superconducting state, except for a normal nucleus of the length L_{th}, located in the superconductor, in which the temperature is equal to the critical value $\theta_s = 1$.

The results of numerical simulations can be described using the diagram shown in Fig. 3. This diagram consists of five regions, marked with the numbers 1 to 5. Each region corresponds to a different type of behaviour of the initial normal seed. Region 1 in the diagram corresponds to the cryostable regime. In this case the initial normal nucleus decays, and superconductivity recovers in the sample.

In region 2 a stationary resistive domain exists.[2] The process of its formation is shown in Fig. 4. Note, that the initial normal nucleus (see Fig. 4a) reaches a steady state after a time interval of the order of τ_{th} (see Fig. 4c). The final temperature distribution in the domain depends on the introduced dimensionless parameters and is independent on the temperature of the initial nucleus.

In region 3 a pair of stationary resistive domains exists. The process of a pair domains formation is shown in Fig. 5. The initial normal nucleus starts to expand during the diffusion of current out into the stabilizer. After the normal domain reaches

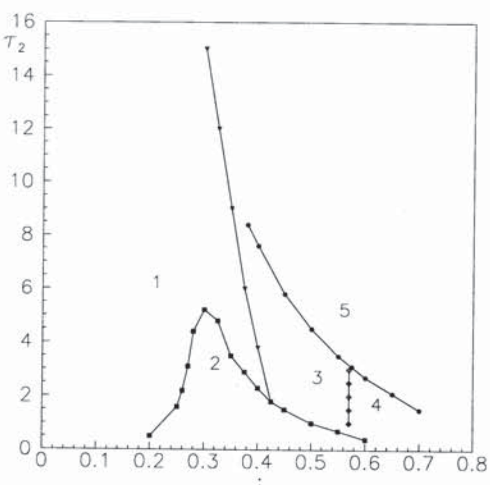

Figure 3 Diagram of a normal domain dynamics. Points represent the results of numerical simulations.

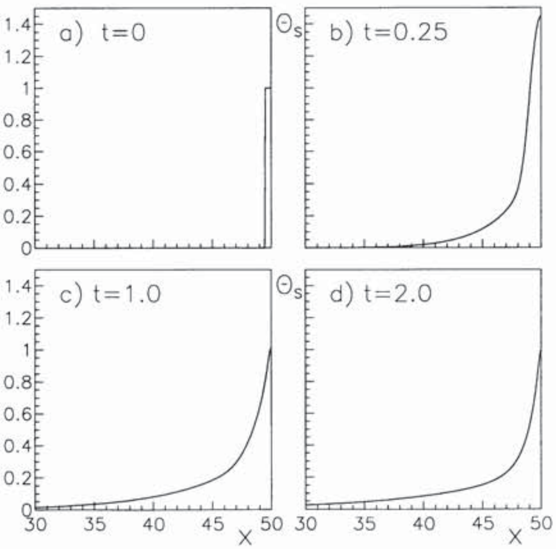

Figure 4 Formation of a stable resistive domain.

a certain length, the center of the normal zone starts to cool down, while its outer parts continue to expand (Fig. 5a). As a result, superconductivity recovers in the center of the normal zone and two normal domains traveling in opposite directions appear(Fig. 5b). When this separation process is completed, domains slow down and finally stop (see Fig. 5c-5d). Thus, we find a steady state with two localized normal domains. The distance between these domains depends on the temperature of the initial nucleus (as well as on the values of the parameters i and τ_2).

In region 4 a string of resistive normal domains exists.[3] The process of this string formation is shown in Fig. 6. Initially the process is the same as described above. Initial nucleus expands and its center cools down. The two daughter domains separate, recede

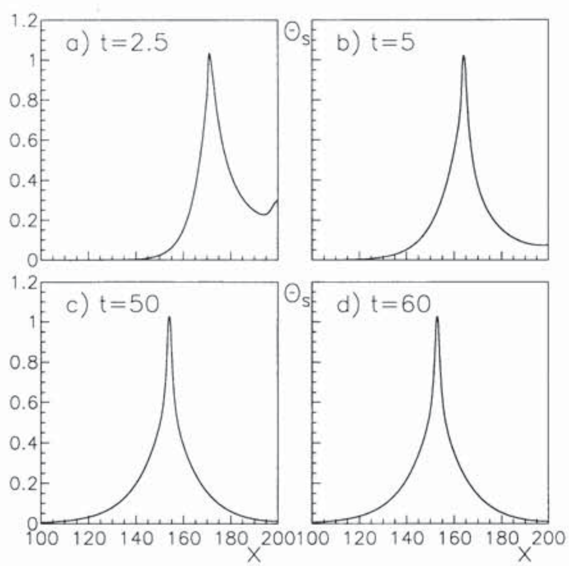

Figure 5 Formation of a pair of stable resistive domains.

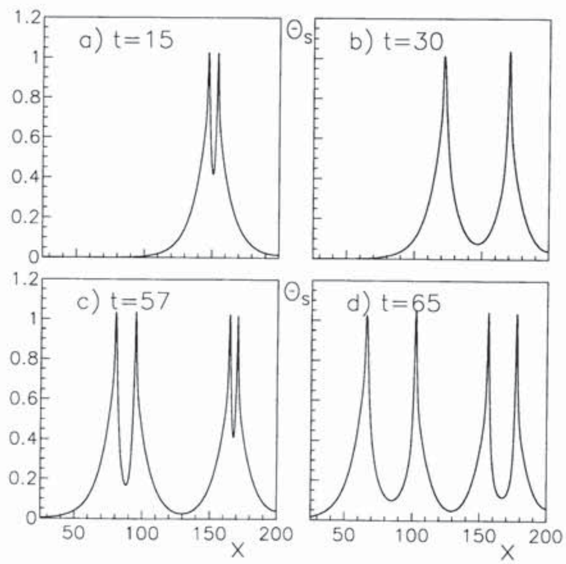

Figure 6 Splitting of normal domains.

and grow in size. When the distance between them is of the order of L_i each of them again splits in two (Fig. 6a). Further propagation of the normal zone proceeds as follows. The outer resistive domains continue to move away from the middle of the sample. Their length increases and each of them splits in two at a distance of the order of L_i from the place of the previous splitting (Fig. 6c). Note, that inner domains also split (Fig. 6c, right-hand domain). Finally, a steady string of resistive domains appears.

In region 5 a steady state of propagating normal domains exists. The initial nucleus results in two normal domains propagating with constant velocity, while superconductivity recovers behind them (see Fig.7). The temperature field at the front of the propagating domain reaches a steady shape after a time interval of the order τ_{th}. Note

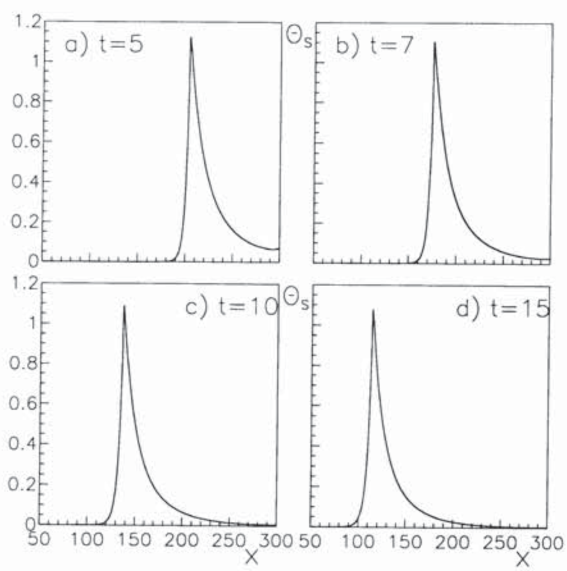

Figure 7 Steady state of propagating normal domains.

that the tail of the temperature profile reaches its steady state only after a relatively long time interval, which is of the order of the current distribution relaxation time τ_m. The velocity of propagation attains its final value much faster than the time required to obtain the steady profile.

DISCUSSION

Let us now discuss the dynamics of a normal zone in large composite superconductors with transition layer. It was found previously[3] that in these systems normal zone can propagate via multiply splitting of resistive domains (see Fig. 7). This process starts if a transport current in the composite, I, is larger than a certain threshold value, I_f. For smaller currents, $I < I_f$, it was shown[2] that a stationary resistive domain exists.

Our simulations show that for a finite values of the current redistribution time between the superconductor and the stabilizer, τ_2, a different solutions also exist. Namely, in the region 3 (Fig. 3) a pair of steady resistive domains exists, and in the region 5 steady propagating normal domains exist.

The existence of the propagating normal domains in large composite superconductors with transition layer can be explained by the following arguments[7]. As the redistribution of current requires a finite duration, the stabilizing mechanism suffers an effective delay time $\tau_m r^2$ (since this is a characteristic duration of current diffusion to the stabilizer). The Joule power in the normal zone during this time interval is consequently high as in the case of an unstabilized superconductor. This heat release leads to the expansion of normal zone boundary with constant velocity (as in the case of unstabilized superconductor). After the redistribution of current is complete, the conductor cools down during a time interval of the order of the thermal relaxation time τ_{th}. When the temperature crosses the critical temperature, the superconducting state is recovered, and the current rediffuses back to the superconductor during a time interval of the order of τ_m. The normal zone expansion, accompanied with the local recovery of superconductivity at each point, is the origin of the existence of traveling normal domains of finite size. As it is seen in Fig.7 the temperature distribution has three characteristic regions. The length of each region is determined by the product of the velocity of propagation of the normal zone v and the characteristic time relaxation time of the current redistribution in this region. The first part is a region of length $v\tau_m r^2$ behind the transition point, where current is diffusing into the stabilizer. The second part is a region of length $v\tau_{th}$, where the current flows mostly through the stabilizer and the temperature is decreasing toward the transition point. Behind the normal zone, there is a region of length $v\tau_m$, which is in the superconducting state and where current diffuses back into the superconductor.

CONCLUSION

We consider the dynamics of a normal zone in a large composite superconductor with a transition layer. The effective circuit model is used to obtain the equations describing the temperature and the current density distributions in the composite. Our simulations show the existence of two new types of steady state solutions. Namely a pair of localized resistive domains and propagating normal domains. We find tha the existence of these solutions is a result of a finite time of the current redistributio in the composite.

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