

SUPERCONDUCTING STATE STABILITY

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ABSTRACT

We present a review of new results concerning the superconducting state stability under nonstationary conditions. We consider the effect of small perturbations of the temperature δT and the electric field $\delta \mathbf{E}$. We present a general approach to the problem of superconducting state stability in a multifilamentary composite wire carrying a time-dependent transport current $I(t)$. We derive an equation determining the current-carrying capacity of this wire, *i.e.*, the maximum superconducting current, I_m . We show that I_m depends on the physical properties of the multifilamentary superconducting composite, the geometry of the multifilamentary area, the cooling conditions, and the transport current ramp rate \dot{I} . We consider a quench propagation in a multifilamentary composite wire carrying a time-dependent transport current $I(t)$. We concentrate on the effect of current ramp rate \dot{I} and the superconducting state instability on the normal zone propagation velocity v . We present an equation determining v in a multifilamentary composite wire as a function of \dot{I} .

GENERAL CONSIDERATION

Let us consider the superconducting state stability in a multifilamentary composite superconductor against small perturbations of temperature δT and electric field $\delta \mathbf{E}$. We start with a general approach to the problem. We treat the superconducting state stability in a wire carrying the transport current $I(t)$ in the transverse magnetic field $\mathbf{B}_a(t)$. Time-dependent $I(t)$ and $\mathbf{B}_a(t)$ cause a background electric field $\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$ and a background temperature $T(\mathbf{r}, t) > T_0$, where T_0 is the coolant temperature. We concentrate on the effect of $\mathbf{E}(\mathbf{r}, t)$ on the stability criterion, *i.e.*, on the current-carrying capacity of the wire.

Physics of superconducting state stability criterion can be understood qualitatively from the diagram shown in Fig. 1.

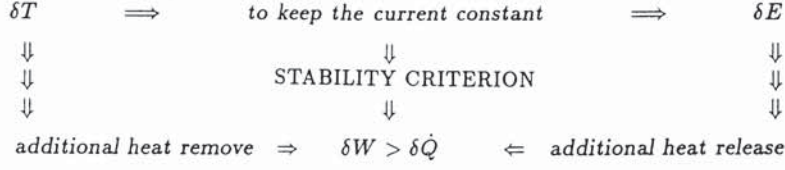


Figure 1. Stability diagram showing main stabilizing and destabilizing processes.

Let us suppose that a temperature perturbation $\delta T > 0$ arises. A temperature increase δT causes a decrease of superconducting current. To keep the total current at the same level, an electric field perturbation δE arises. The additional electric field δE causes an additional heat release $\delta \dot{Q} \propto \delta E$, which is the "price" for keeping the total current at the same level. The superconducting state is stable if the additional heat release $\delta \dot{Q}$ can be removed to the coolant by the additional heat flux $\delta W \propto \delta T$ resulting from the temperature perturbation δT . Thus the superconducting state stability criterion has the form

$$\delta W > \delta \dot{Q}. \quad (1)$$

The additional heat release per unit length $\delta \dot{Q}$ is given by the integral of $\mathbf{j} \delta \mathbf{E}$ over the cross-sectional area A of the wire

$$\delta \dot{Q} = \int_A \mathbf{j} \delta \mathbf{E} dA, \quad (2)$$

where \mathbf{j} is the current density. The additional heat flux per unit length, δW , is given by the integral of $h \delta T$ over the cooling perimeter P of the wire:

$$\delta W = \int_P h \delta T dP, \quad (3)$$

where h is the heat transfer coefficient to the coolant. Using Eqs. (1), (2) and (3) we find superconducting state stability criterion in the form¹⁻³

$$\int_A \mathbf{j} \delta \mathbf{E} dA < \int_P h \delta T dP, \quad (4)$$

where δE is longitudinal (parallel to the filaments) electric field perturbation.

To derive the explicit form of the stability criterion we have to find the relation between δT and δE . To do it, we follow the idea illustrated by the diagram shown in Fig. 1. We calculate the decrease of the current density δj_- resulting from the temperature perturbation δT and the increase of the current density δj_+ resulting from the electric field perturbation δE . If the superconducting state is stable, then

$$\delta j = \delta j_- + \delta j_+ = 0. \quad (5)$$

In the superconducting state, $j \approx j_c$, where j_c is the critical current density. Thus, the decrease of j due to the temperature perturbation δT is given by

$$\delta j_- = - \left| \frac{\partial j_c}{\partial T} \right| \delta T. \quad (6)$$

The increase of current density due to the electric field perturbation δE can be written as

$$\delta j_+ = \frac{\partial j}{\partial E} \delta E = \sigma_d \delta E, \quad (7)$$

where

$$\sigma_d = \frac{\partial j}{\partial E} \quad (8)$$

is the differential conductivity determined by the slope of the current–voltage characteristics and E is the longitudinal (parallel to the filaments) background electric field. The dependence of j on E is strongly nonlinear for superconductors with high critical current density and thus $\sigma_d = \sigma_d(E)$. To find the explicit relation between δE and δT we have to know $\sigma_d(E)$, *i.e.*, we have to consider the current–voltage characteristics of a multifilamentary superconductor.

CURRENT–VOLTAGE CHARACTERISTICS

It was shown experimentally⁴ that for $NbTi$ based multifilamentary composite superconductors at electric field E below $E_m \approx 10^{-3} \text{ V/m}$, the dependence of j on E can be presented as

$$j = j_c + j_1 \ln \left(\frac{E}{E_c} \right) + \sigma_{\parallel} E, \quad (9)$$

where $j_c(T, B)$ is the critical current density, $j_1 = j_1(T, B)$ and $E_c = \text{const}$ are parameters of the current–voltage characteristics, and σ_{\parallel} is the longitudinal (parallel to the filaments) conductivity of the composite resulting from the normal matrix.

The value of j_c is usually given for $E = E_c = 10^{-4} \text{ V/m}$. The dependence of the ratio j_1/j_c on T and B is assumed to be weak. It was shown experimentally⁴ that for $NbTi$ based superconductors $j_1/j_c \approx 0.03$, *i.e.* the ratio j_1/j_c is small.

Let us now compare the current density j_1 and the maximum value of $\sigma_{\parallel} E$. We use here for estimations the data for $NbTi$ based composites with Cu matrix, *i.e.*, $\sigma_{\parallel} \approx 10^9 \Omega^{-1} \text{ m}^{-1}$, $j_c \approx 10^9 \text{ A/m}^2$ and $j_1 \approx 0.03 j_c \approx 3 \cdot 10^7 \text{ A/m}^2$. The current–voltage characteristics given by Eq. (9) is valid up to $E_m \approx 10^{-3} \text{ V/m}$. The maximum value of $\sigma_{\parallel} E$ corresponding to $E = E_m$ is equal to $\sigma_{\parallel} E_m \approx 10^6 \text{ A/m}^2$. It is much less than j_1 . Thus, for all cases of practical interest $\sigma_{\parallel} E \ll j_1 \ll j_c$.

Let us note that the current–voltage characteristics of a multifilamentary superconductor can be also presented as

$$j = j_c \left(\frac{E}{E_0} \right)^{1/n} + \sigma_{\parallel} E, \quad (10)$$

where $n \gg 1$, and the dependence of n on T and B is assumed to be weak. In case of $n \gg 1$ Eqs. (9) and (10) are equivalent. To show it we rewrite the first term in Eq. (10) in the following way

$$j_c \left(\frac{E}{E_0} \right)^{1/n} = j_c \exp \left[\frac{1}{n} \ln \left(\frac{E}{E_0} \right) \right] \quad (11)$$

and expand it in series keeping the first two terms

$$j_c \left(\frac{E}{E_0} \right)^{1/n} \approx j_c + \frac{j_c}{n} \ln \left(\frac{E}{E_0} \right). \quad (12)$$

Comparing Eqs. (9), (10) and (12) we see that both presentations of the current-voltage characteristics are equivalent and the relation between j_1 and n has the form

$$j_1 = \frac{j_c}{n}. \quad (13)$$

Thus, it follows from Eqs. (9) and (10), that the value of the differential conductivity σ_d of a multifilamentary composite superconductor at $E < E_m$ is given by the expression

$$\sigma_d = \frac{\partial j}{\partial E} = \frac{j_1}{E} \propto \frac{1}{E}. \quad (14)$$

The accuracy of Eq. (13) is of the order of $\sigma_{\parallel} E / j_1 < 0.03 \ll 1$. Note, that with the same accuracy $\sigma_d \gg \sigma_{\parallel}$.

STABILITY CRITERION

Combining Eqs. (7) and (14), we find

$$\delta j_+ = \frac{j_1}{E} \delta E. \quad (15)$$

It follows from Eqs. (5), (6) and (15) that

$$\delta E = \frac{1}{\sigma_d} \left| \frac{\partial j_c}{\partial T} \right| \delta T = \frac{E}{j_1} \left| \frac{\partial j_c}{\partial T} \right| \delta T. \quad (16)$$

Equations (14) and (16) allow to understand the effect of the background electric field E on the superconducting state stability. It follows from Eq. (14) that low electric field E results in high differential conductivity $\sigma_d \propto 1/E$. High conductivity σ_d leads to low electric field perturbation $\delta E \propto 1/\sigma_d \propto E$. The smaller is δE , the less "costly" it is to remove the additional heat release. Thus, the lower the background electric field is the more stable is the superconducting state.

Substituting Eq. (16) into Eq. (4) we find the superconducting state stability criterion in the form¹⁻³

$$\int_A E \frac{j_c}{j_1} \left| \frac{\partial j_c}{\partial T} \right| \delta T dA < \int_P h \delta T dP. \quad (17)$$

Let us now assume that the physical properties of the wire are uniform. It means that we can take out from the integrals in Eq. (17) the values of j_c , j_1 , $|\partial j_c / \partial T|$ and h . The nonuniformity (over the wire cross-sectional area) of temperature perturbation δT is determined by the Biot parameter

$$Bi = \frac{A}{P} \frac{h}{\kappa_{\perp}}, \quad (18)$$

where κ_{\perp} is the transverse (perpendicular to the filaments) heat conductivity. Using for estimations $A/P \approx 2 \cdot 10^{-4} m$ and $\kappa_{\perp} \approx 10^2 W/mK$, we get $Bi \approx 2 \cdot 10^{-6} h$ (h is given here in W/m^2K). The value of heat transfer coefficient h strongly depends on the cooling conditions and the electrical insulation of the wire. For most cases of practical interest we can evaluate $h < 10^3 W/m^2K$ and therefore $Bi < 2 \cdot 10^{-3} \ll 1$. The temperature perturbation δT is almost uniform over the cross-sectional area of the wire if $Bi \ll 1$. It means that we can take δT out of the integrals in Eq. (17).

Finally we find by means of Eq. (17) the superconducting state stability criterion in the form¹⁻³

$$\langle E \rangle = \frac{1}{A} \int E dA < E_{\max} = \frac{Ph}{A} \frac{j_1}{j_c} \left| \frac{\partial j_c}{\partial T} \right|^{-1}, \quad (19)$$

This criterion means that the longitudinal background electric field E averaged over the cross-sectional area of the wire has to be less than a certain critical value E_{\max} . The electric field E depends on the transport current $I(t)$, the transport current ramp rate $\dot{I}(t)$, the magnetic field ramp rate $\dot{B}_a(t)$ and the geometry of the multifilamentary area in the wire. At the same time, the critical field E_{\max} is a function of the physical properties, geometry and cooling conditions of the wire. Thus, the maximum value of the superconducting current I_m depends on all these parameters.

We demonstrate now how to use the superconducting state stability criterion given by Eq. (19) in order to solve problems of practical interest. We consider current-carrying capacity, *i.e.*, the maximum superconducting current, I_m , of a superconducting multifilamentary wire carrying a time-dependent transport current $I(t)$. We suppose that the mechanism controlling the value of I_m is the instability of the superconducting state against small perturbations of the electrical field $\delta \mathbf{E}$ and the temperature δT . In this case to find the current I_m we have first to calculate the longitudinal background electric field E averaged over the cross-sectional area of the wire.

BACKGROUND ELECTRIC FIELD

Let us consider a superconducting multifilamentary composite wire carrying an increasing transport current $I(t)$. In this case, the transport current progressively penetrates the wire from outside in and the region with $j_{\parallel} \approx j_c$ has the form of a ring. The outer radius of this ring coincides with the radius of the wire R . The inner radius of this ring, R_i , depends on the current and is given by

$$R_i = R \sqrt{1-i}, \quad (20)$$

where the dimensionless current i is defined as

$$i = \frac{I}{\pi R^2 j_c} = \frac{I}{I_c}. \quad (21)$$

The longitudinal background electric field E induced by the varying transport current $I(t)$ in the region $R_i < r < R$ can be calculated by means of Maxwell equations

$$\begin{cases} r \frac{dB_{\varphi}}{dr} + B_{\varphi} = \mu_0 j_c r, & R_i < r < R, \\ \frac{dE}{dr} = \dot{B}_{\varphi}, & R_i < r < R, \end{cases} \quad (22)$$

with the boundary conditions

$$\begin{cases} B_\varphi = 0, & \text{for } r = R_i; \\ E = 0, & \text{for } r = R_i. \end{cases} \quad (23)$$

Here B_φ is the φ -component of the magnetic field induced by the current flowing in the saturated region. It follows from Eqs. (22) and (23) that

$$E = \frac{\mu_0 \dot{I}}{2\pi} \ln\left(\frac{r}{R\sqrt{1-i}}\right), \quad R\sqrt{1-i} < r < R. \quad (24)$$

Thus the time-dependent transport current $I(t)$ causes a longitudinal background electric field $E \propto \mu_0 \dot{I}$. Using Eq. (24) we find for the value of E averaged over the cross-sectional area of the wire the following expression

$$\langle E \rangle = -\frac{\mu_0 \dot{I}}{4\pi} \left[i + \ln(1-i) \right]. \quad (25)$$

CURRENT-CARRYING CAPACITY

We use now Eqs. (19) and (25) to find the current-carrying capacity of a superconducting multifilamentary composite wire. Substituting Eq. (25) into Eq. (19) we get the following equation determining the maximum superconducting current I_m

$$-\frac{\mu_0 \dot{I}}{4\pi} \left[i_m + \ln(1-i_m) \right] = \frac{2hj_1}{Rj_c} \left| \frac{\partial j_c}{\partial T} \right|^{-1}. \quad (26)$$

Let us rewrite Eq. (26) in the form

$$\left[i_m + \ln(1-i_m) \right] + \frac{\dot{I}_q}{\dot{I}} = 0. \quad (27)$$

where

$$\dot{I}_q = \frac{8\pi h}{\mu_0 R} \frac{j_1}{j_c} \frac{1}{\left| \frac{\partial j_c}{\partial T} \right|}. \quad (28)$$

Approximating the derivative $|\partial j_c / \partial T|$ as

$$\left| \frac{\partial j_c}{\partial T} \right| \approx \frac{j_c}{T_c - T_0}, \quad (29)$$

we find for \dot{I}_q the expression

$$\dot{I}_q = \frac{8\pi h(T_c - T_0)}{\mu_0 R j_c} \frac{j_1}{j_c}. \quad (30)$$

Thus it follows from Eq. (29) that the maximum superconducting current I_m is a function of one dimensionless parameter \dot{I}/\dot{I}_q .

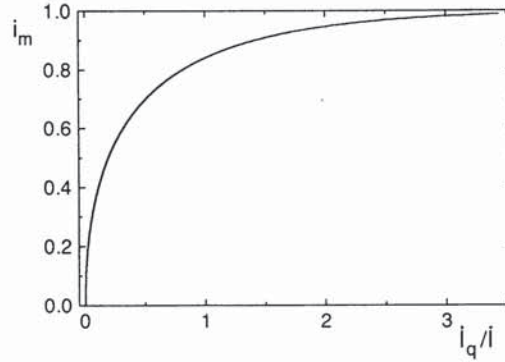


Figure 2. The dependence of the dimensionless maximum superconducting current i_m on the ratio \dot{I}/\dot{I}_q .

Let us now estimate the value of \dot{I}_q . Using for j_c and j_1 the same data as above, and assuming that $T_c - T_0 \approx 3K$ we get

$$\dot{I}_q \approx 5h, \quad (31)$$

where \dot{I}_q is given in A/s , and h is given in W/m^2K .

The dependence of i_m on \dot{I}/\dot{I}_q is shown in Fig. 2. It is seen from Fig. 2 that the value of I_m is close to I_c if $\dot{I} \leq \dot{I}_q$ and $I_m \ll I_c$ if $\dot{I} \gg \dot{I}_q$.

It follows from Eq. (27) that for $\dot{I} \gg \dot{I}_q$, the dependence of i_m on \dot{I}/\dot{I}_q has the form

$$i_m \approx \sqrt{\frac{2\dot{I}_q}{\dot{I}}}, \quad \dot{I} \gg \dot{I}_q. \quad (32)$$

It thus appears that a serious degradation of the critical current only occurs if $\dot{I} \gg \dot{I}_q$.

On the other hand for $\dot{I} \ll \dot{I}_q$, the dependence of i_m on \dot{I}/\dot{I}_q has the form

$$i_m \approx 1 - \exp\left(-1 - \frac{\dot{I}_q}{\dot{I}}\right), \quad \dot{I} \ll \dot{I}_q. \quad (33)$$

It follows from Eq. (33) that the difference between I_m and I_c is less than 1% if $\dot{I} < 0.28 \dot{I}_q$.

QUENCH PROPAGATION

Let us now consider a quench propagation in a multifilamentary composite wire carrying a time-dependent transport current $I(t)$ ⁵. Physics of the effect of the current ramp rate \dot{I} on normal zone propagation velocity v can be understood from the following qualitative consideration.

Time-dependent transport current $I(t)$ causes a background electric field $E \propto \dot{I}$. First, this electric field determines the current-carrying capacity of the wire: a stable superconducting state exists if $0 < I < I_m$. An increase of \dot{I} decreases I_m thus making any given value of the transport current I closer to the stability threshold. The closer is the current to the stability threshold the higher is the quench propagation velocity⁶⁻⁸. Second, the background electric field causes preheating of the supercon-

ductor. The higher is the temperature of the superconducting state the higher is the quench propagation velocity⁶⁻⁸. Both these mechanisms⁵ determine the increase of v with an increase of \dot{I} and thus the effect of \dot{I} on v .

Let us consider a multifilamentary composite wire carrying a transport current $I(t)$. The transition from the superconducting to the resistive state is determined by the temperature distribution $T(\mathbf{r}, t)$. As we mentioned before, in most cases of practical interest the Biot parameter $Bi \ll 1$ and the function $T(\mathbf{r}, t)$ is almost uniform over the cross-sectional area of the wire. The temperature distribution $T(\mathbf{r}, t)$ determining the quench propagation is thus one-dimensional and $T(\mathbf{r}, t) = T(z - vt)$, where the z axis is along the wire. The function $T(z - vt)$ is given by an appropriate solution of the heat diffusion equation

$$\frac{d}{d\xi} \kappa_{\parallel} \frac{dT}{d\xi} + vC \frac{dT}{d\xi} + \dot{Q} - W = 0, \quad (34)$$

where $\xi = z - vt$, κ_{\parallel} and C are the longitudinal heat conductivity and the heat capacity per unit volume averaged over the wire cross-section, $\dot{Q} = jE$ and $W = hP(T - T_0)/A$ are the power of Joule heat release and the heat flux to the coolant per unit volume.

The temperatures of the superconducting T_s and normal T_n steady states are the appropriate boundary conditions to Eq. (34), *i.e.* $T(\pm\infty) = T_{s,n}$. The values of T_s and T_n are given by the roots of the heat balance equation

$$\dot{Q}(T_{s,n}) = W(T_{s,n}). \quad (35)$$

During the transition from the superconducting to the resistive state, the current is redistributed from the superconductor to the stabilizer. The current flowing in the stabilizer determines the quench-driving heat release, *i.e.*, the Joule heat release in the vicinity of the transition front. The increase of superconductor temperature decreases the critical current $I_c(T)$. The process of transport current sharing between the superconductor and the stabilizer starts at a the transition temperature T_r . The value of T_r is determined by the equation

$$I_c(T_r) = I. \quad (36)$$

We consider here the case when the Joule heating power \dot{Q} is approximated by a step function of the temperature T , *i.e.*, we use for \dot{Q} the expression

$$\dot{Q} = \begin{cases} \dot{q}_s, & T < T_r, \\ \dot{q}_n, & T > T_r, \end{cases} \quad (37)$$

The value of \dot{q}_n is determined by the Joule heating in the normal state and thus

$$\dot{q}_n = \rho_{\parallel} j^2, \quad (38)$$

where ρ_{\parallel} is the longitudinal resistivity of the wire.

The Joule heat release in the superconducting state is presented in Eq. (37) by the term \dot{q}_s , where $\dot{q}_s \ll \dot{q}_n$. To determine the value of the power \dot{q}_s in a self-consistent way we use the superconducting state stability criterion. It implies that the steady superconducting state vanishes at $I = I_m$, *i.e.*, the solution of Eq. (35) for T_s vanishes at $I = I_m$. Using Eqs. (35) and (37), and considering the critical current I_c to be

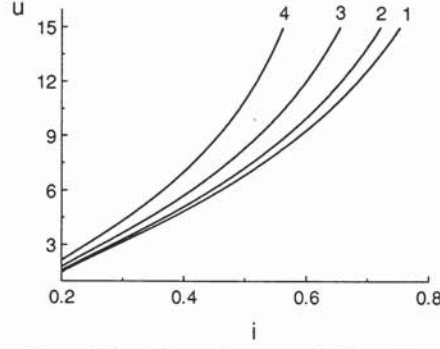


Figure 3. The dependence of the dimensionless quench propagation velocity u on the dimensionless current i at different values of the ratio \dot{I}/\dot{I}_q : $\dot{I}/\dot{I}_q = 0(1)$, $0.5(2)$, $1(3)$, $2(4)$.

linear in temperature we get

$$\dot{q}_s = \frac{hP(T_c - T_0)}{A} (1 - i_m). \quad (39)$$

We assume here for the sake of simplicity that the heat capacity C , the longitudinal heat conductivity κ_{\parallel} , the heat transfer coefficient to the coolant h , the longitudinal resistivity ρ_{\parallel} and thus the coefficients in Eq. (34) are constants. Solving Eq. (34) with the boundary conditions given by Eq. (35) we find then the dependence $v(i)$ in the following form

$$v = v_h \frac{\alpha i^2 + 2i - i_m - 1}{\sqrt{(\alpha i^2 + i - 1)(i_m - i)}}, \quad (40)$$

where

$$v_h = \frac{1}{C} \sqrt{\frac{\kappa_{\parallel} h P}{A}}, \quad (41)$$

is the characteristic normal zone propagation velocity, and

$$\alpha = \frac{\rho_{\parallel} j_c^2 A}{h(T_c - T_0)P}, \quad (42)$$

is the Stekly parameter.

Let us note that if the current ramp rate \dot{I} is low ($\dot{I} \ll \dot{I}_q$) the maximum superconducting current I_m equals to I_c , i.e., $i_m = 1$. In this limiting case Eq. (42) coincides with a well known equation determining the quench propagation velocity as a function of the transport current⁶⁻⁸.

The dependence of the dimensionless velocity $u = v/v_h$ on the dimensionless current i is given in Fig. 3 for different values of the ratio \dot{I}/\dot{I}_q and $\alpha = 100$. The effect of \dot{I} on quench propagation velocity is seen from the comparison of curves 1 corresponding to $\dot{I} = 0$ and curves 2-4 corresponding to $\dot{I} \neq 0$.

Note, that the quench propagation velocity v is finite for all values of the current if the heat release \dot{Q} is a smooth function of the temperature T ^{6,9,10}. In this case the value of $v(i_m)$ can be estimated⁵ as $v \approx \sqrt{2\alpha i_m} v_h$ for $\alpha \gg 1$.

SUMMARY

To conclude we review new results concerning the superconducting state stability under nonstationary conditions. We treat the case of a multifilamentary composite wire carrying a time-dependent transport current $I(t)$. We present equations determining the maximum superconducting current I_m and normal zone propagation velocity v as functions of current ramp rate \dot{I} . We show that the effect of time-dependent transport current has to be taken into account when calculating the current-carrying capacity and quench propagation velocity in multifilamentary composite superconductors.

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