Dynamics of Josephson pancakes in layered superconductors

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We consider a pointlike vortex in a layered superconductor with linear defects in the superconducting layers. We treat these defects as Josephson junctions with high critical current density. We consider the electrodynamics of these junctions within the framework of nonlocal Josephson electrodynamics. We show that Josephson current through a linear defect in a superconducting layer results in a pointlike vortex with a superconducting core residing in this layer (Josephson pancake). We find the mobility of a Josephson pancake. We consider a small amplitude wave in a Josephson junction with nonlocal electrodynamics. We treat a bending wave for an infinite stack of Josephson pancakes. We find the dispersion relation for these waves. We show that their velocities tend to a certain finite limit when the wavelength tends to infinity.

I. INTRODUCTION

The most prominent high-temperature superconductors and, in particular, the Bi- and Tl-based compounds, consist of a periodic stack of the two-dimensional CuO layers (*ab* planes) where the superconductivity presumably resides. These materials are extremely anisotropic and, in particular, the density of the superconducting current in the direction perpendicular to the layers (*c* direction) is much less than in the *ab* planes. The discovery of the layered anisotropic high-T_c superconductors stimulated many theoretical studies of layered superconductors with weak interlayer Josephson coupling. In particular, the specific pointlike (or pancake) vortices were introduced and investigated.[1-3]

A pointlike vortex is an elementary vortex existing in a layered superconductor. The normal core of each of these pointlike vortices resides only in one of the superconducting layers. An Abrikosov vortex in a layered superconductor can be treated as an infinite stack of pointlike vortices. In the case of extremely weak interlayer Josephson coupling, these vortices interact only via the magnetic field existing between the superconducting layers. The self-energy of an isolated pointlike vortex is proportional to $\ln(L/\xi)$, where L is the characteristic size of the sample in the *ab* plane, and $\xi=\xi_{ab}$ is the coherence length in the *ab* plane. Thus, the self-energy of an isolated pointlike vortex diverges when $L/\xi\to\infty$ and it cannot exist in the bulk of a macroscopic sample with $L\gg\xi$.

Interaction with the sample surface affects the value of the self-energy of a pointlike vortex in the surface layer. This interaction consists of repulsion and attraction. The repulsion results from the interaction with the Meissner screening current. The attraction results from the increase of the superconducting current density of the pointlike vortex caused by the sample surface. The correlation between these two interactions is determined by the external magnetic field H. At a certain value of H the competition of attraction and repulsion leads to a stable state localized near the sample surface. The existence of this state results, in particular, in a specific thermally activated mechanism of magnetization relaxation.[4,5]

A two-dimensional (2D) pointlike vortex can be strongly affected by an interaction with a defect of the crystalline structure. This effect can be especially significant when a defect exists in the plane where the normal core of the pointlike vortex resides. In particular, interaction with a linear defect in a superconducting layer can lead to localization of a pointlike vortex in the direction perpendicular to the defect. In this case free motion of a pointlike vortex is possible only along the defect. This effect can be important for different transport phenomena as linear defects in superconducting layers are characteristic for layered superconductors. In particular, grains boundaries and twins result in linear defects in the superconducting layers.

A linear defect in a superconducting layer can be treated as a Josephson junction with a relatively high value of the critical current density j_c. It results in a relatively small value of the characteristic space scale of variation of the tunneling current j_φ along the junction. In case of a linear defect in a superconducting layer this length l_J can become less or even much less than the penetration depth λ. The value of l_J is given by the formula $l_J=\lambda_J^2/\lambda$ if $l_J<\lambda$,[6] where λ_J is the Josephson length.[7] In the case when $l_J<\lambda$ the relation between the tunneling current j_φ and the superconducting current j_s is nonlocal and the electrodynamics of such Josephson junction is the nonlocal Josephson electrodynamics.[6,8]

In this paper we study the dynamics of a pointlike vortex localized by a linear defect in a superconducting layer. We show that this vortex (a Josephson pancake) has a superconducting core resulting in a high mobility at low temperatures. We use here for calculations the Lawrence-Doniach model[9] in the limit of a very weak interlayer Josephson coupling. We treat the linear defect in the superconducting layer as a Josephson junction with a relatively high critical current density and we consider it within the framework of nonlocal Josephson electrodynamics.

We consider a small amplitude wave in a Josephson junction with nonlocal electrodynamics and calculate the dispersion relation. We find that the velocity of this wave tends to a certain nonzero limit when the wavelength tends to infinity. We consider a small amplitude bending wave propagating along an infinite stack of Josephson pancakes. We show that when the wavelength tends to infinity the frequency of this wave is proportional to the wave vector.

The paper is organized in the following way. In Sec. II, we consider a Josephson junction with nonlocal electrodynamics in a thin film. We calculate the current distribution and the mass for a Josephson vortex. We find the dispersion relation for a small amplitude wave. In Sec. III, we apply the results obtained in the previous section to an infinite stack of superconducting layers, i.e., to a layered superconductor. We consider the Josephson pancake and calculate its mobility. We treat a small amplitude bending wave propagating along an infinite stack of Josephson pancakes and find the dispersion relation for this wave. In Sec. IV, we summarize the overall conclusions.

II. JOSEPHSON JUNCTION WITH NONLOCAL ELECTRODYNAMICS

Let us consider a superconducting film with the thickness a and a Josephson junction along the y axis (see Fig. 1). We treat here the thin film limit, which means that $a \ll \delta$, where δ is the penetration depth. In this case the superconducting current density decreases with the characteristic space scale of the order of[10]

$$\delta_{\mathrm{eff}}=\frac{\delta^2}{a}\gg\delta \; . \tag{1}$$

As a result the contribution of the vector potential $\mathbf{A}$ to the current density $\mathbf{j}$ can be neglected and

$$\mathbf{j}=\frac{c\Phi_0}{8\pi^2\delta^2}\nabla\theta, \quad r\ll\delta_{\mathrm{eff}} \tag{2}$$

where θ is the phase of the order parameter. It follows from Eq. (2) and the continuity equation $\operatorname{div}\mathbf{j}=0$ that the equation for $\theta(x,y)$ has the form

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=0 \; . \tag{3}$$

The boundary conditions for Eq. (3) are given at $x=\pm 0$ by the Josephson relation

$$j_x(\pm 0,y)=\frac{c\Phi_0}{8\pi^2\delta^2}\frac{\partial\theta}{\partial x}(\pm 0,y)=-j_c\sin\varphi(y) \; . \tag{4}$$

Here j_c is the Josephson current critical density, and $\varphi(y)$

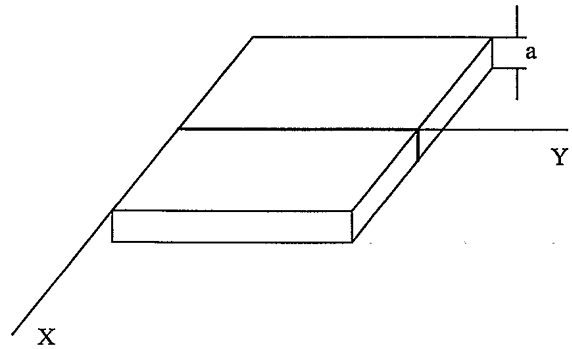

FIG. 1. A superconducting film with a Josephson junction (thick line).

is the phase difference defined as

$$\varphi(y)=\theta(+0,y)-\theta(-0,y) \; . \tag{5}$$

To find the space distributions of $\theta(x,y)$ and $\varphi(y)$ we apply the Fourier cosine transformation to Eq. (3) introducing

$$\theta_k(y)=2\int_0^\infty dx\,\cos(kx)\theta(x,y) \; . \tag{6}$$

It leads to the following equation for $\theta_k(y)$:

$$\frac{d^2\theta_k}{dy^2}-k^2\theta_k=-\frac{16\pi^2\delta^2 j_c}{c\Phi_0}\sin\varphi \; . \tag{7}$$

The solution of Eq. (7) is given by the formula,

$$\theta_k(y)=\frac{8\pi^2\delta^2 j_c}{c\Phi_0 k}\int_{-\infty}^{\infty}dy'\,\sin\varphi(y')\exp(-k|y-y'|) \; . \tag{8}$$

The inverse Fourier transformation leads then to the relations

$$\frac{\partial\theta}{\partial y}(\pm 0,y)=\pm\frac{8\pi\delta^2 j_c}{c\Phi_0}\int_{-\infty}^{\infty}dy'\frac{\sin\varphi(y')}{y-y'} \; . \tag{9}$$

Using Eqs. (5) and (9) we find for the phase difference $\varphi(y)$ the nonlinear integro-differential equation,

$$\frac{\partial\varphi}{\partial y}=\frac{16\pi\delta^2 j_c}{c\Phi_0}\int_{-\infty}^{\infty}dy'\frac{\sin\varphi(y')}{y-y'} \; . \tag{10}$$

It follows from Eq. (10) that the value of the characteristic space scale for $\varphi(y)$ is given by the length l_J, where

$$l_J=\frac{c\Phi_0}{16\pi^2\delta^2 j_c} \; . \tag{11}$$

Using the Hilbert transformation we can rewrite Eq. (10) as

$$\sin\varphi=\frac{l_J}{\pi}\int_{-\infty}^{\infty}\frac{dy'}{y'-y}\frac{\partial\varphi}{\partial y'} \; . \tag{12}$$

The above consideration is valid if $l_J\ll\delta_{\mathrm{eff}}$. It means that the critical current density of the Josephson junction j_c has to be high enough, i.e., $j_c\gg ac\Phi_0/16\pi^2\delta^4$.

The space distribution of the phase difference $\varphi(y)$ for a Josephson vortex is given by the following exact solution of Eq. (10):

$$\varphi(y)=2\arctan\left[\frac{y}{l_J}\right]+\pi\ . \tag{13}$$

Note that this solution was obtained in Ref. 6 while treating a Josephson junction with the nonlocal electrodynamics.

Using Eq. (9) and the inverse Fourier transformation we find that for $r \gg l_J$ the current lines of a Josephson vortex are circles and the current density $\mathbf{j}$ is given by the formula

$$\mathbf{j}=\frac{c\Phi_0}{8\pi^2\delta^2}\frac{\mathbf{n}\times\mathbf{r}}{r^2}\ , \tag{14}$$

where $\mathbf{n}$ is the unit vector along the z axis. The current density distribution given by Eq. (14) is similar to the current density distribution of a Pearl vortex.[10] The only difference is that the core of a Pearl vortex is normal and the core of a Josephson vortex is superconducting. The size of this superconducting core is of the order of l_J.

Let us now consider a nonstationary case and derive the equation determining the phase difference distribution $\varphi(y,t)$ for a Josephson junction without damping. As the value of φ depends on time, the displacement current arises in the junction. It means that the current flowing through the Josephson junction $j_x(+0,y)$ is a sum of the Josephson tunneling current $j_c\sin\varphi$ and the displacement current

$$j_d=\frac{\hbar C}{2e}\frac{\partial^2\varphi}{\partial t^2}\ , \tag{15}$$

where C is the capacity of the Josephson junction.

The same derivation as above leads then to the following nonlinear integro-differential equation determining the phase difference distribution $\varphi(y,t)$ in the nonstationary case:

$$\sin\varphi+\frac{1}{\omega_p^2}\frac{\partial^2\varphi}{\partial t^2}=\frac{l_J}{\pi}\int_{-\infty}^{\infty}\frac{dy'}{y'-y}\frac{\partial\varphi}{\partial y'}\ . \tag{16}$$

Here we introduced the characteristic plasma frequency[7] as

$$\omega_p=\left[\frac{2ej_c}{C\hbar}\right]^{1/2}\ . \tag{17}$$

Motion of a Josephson vortex along the junction results in an electrical field localized inside the Josephson junction. The potential difference $U(y)$ appearing due to this electrical field across the junction is given by the formula

$$U(y)=\frac{\hbar}{2e}\frac{\partial\varphi}{\partial t}\ . \tag{18}$$

Let us suppose that the velocity v of Josephson vortex is small, i.e., $v \ll \omega_p l_J$. In this case the solution of Eq. (16) is given by Eq. (13), where instead of y we have to substitute $y-vt$. The energy of the electrical field in the Josephson junction is equal to

$$W_{\mathrm{el}}=\frac{1}{2}C\left[\frac{\hbar}{2e}\right]^2\int_{-\infty}^{\infty}dy\left[\frac{\partial\varphi}{\partial t}\right]^2=\frac{v^2C\hbar^2}{8e^2}\int_{-\infty}^{\infty}dy\left[\frac{\partial\varphi(y)}{\partial y}\right]^2\ . \tag{19}$$

It follows from Eq. (19) that $W_{\mathrm{el}} \propto v^2$ and we can thus introduce the Josephson vortex mass M presenting W_{el} as

$$W_{\mathrm{el}}=\frac{Mv^2}{2}\ , \tag{20}$$

where

$$M=\frac{\pi\hbar^2C}{2e^2l_J}\ . \tag{21}$$

Let us now consider a small amplitude wave propagating in a Josephson junction with nonlocal electrodynamics. In this case the solution of Eq. (17) has the form

$$\varphi(y,t)=\varphi_0\exp(iky-i\omega t)\ , \tag{22}$$

where the amplitude $\varphi_0 \ll 1$. We neglect the contribution of the vector potential $\mathbf{A}$ to the superconducting current density while deriving Eqs. (12) and (16). As mentioned above it can be done if $r \ll \delta_{\mathrm{eff}}$ or $k\delta_{\mathrm{eff}} \gg 1$.

To determine the dependence of the frequency ω on the wave vector k we substitute $\varphi(y,t)$ given by Eq. (22) in Eq. (16). The integral in the right-hand side of Eq. (16) is then equal to

$$\int_{-\infty}^{\infty}\frac{dy'}{y'-y}\frac{\partial\varphi}{\partial y'}=ik\varphi\int_{-\infty}^{\infty}\frac{dz\,\exp ikz}{z}=-\pi\varphi|k|\ . \tag{23}$$

Using Eqs. (16), (22), and (23) we find that the dispersion relation $\omega(k)$ has the form

$$\omega=\omega_p(1+|k|l_J)^{1/2},\quad k\delta_{\mathrm{eff}} \gg 1\ . \tag{24}$$

In particular, it follows from Eq. (24) that for $1/\delta_{\mathrm{eff}} \ll k \ll 1/l_J$ the velocity of the wave tends to a certain value s, where

$$s=\frac{\omega_p l_J}{2}\ . \tag{25}$$

Note that in case of a Josephson junction with the local electrodynamics the analogous dispersion relation $\omega(k)$ is given by the equation[7]

$$\omega=\omega_p(1+k^2\lambda_J^2)^{1/2}\ . \tag{26}$$

III. JOSEPHSON PANCAKE IN A LAYERED SUPERCONDUCTOR

Let us consider a Josephson pointlike vortex in a layered superconductor, i.e., a Josephson pancake. We use for calculations the Lawrence-Doniach model.[9] We consider here the limit of very weak interlayer Josephson coupling, neglecting the superconducting current in the direction perpendicular to the superconducting layers. We treat thus a layered superconductor as a periodic stack of electromagnetically coupled superconducting layers with the distance d between them.

The free energy functional F in this case has the form[9]

$$F=\frac{d}{2\pi}\left[\frac{\Phi_0}{4\pi\lambda}\right]^2\sum_n\int d^2\mathbf{r}\left[\nabla\theta_n-\frac{2e}{c}\mathbf{A}\right]^2+\frac{1}{8\pi}\int d^3\mathbf{r}(\mathbf{H}-\mathbf{B})^2\;, \tag{27}$$

where n is the number of the layer, θ_n is the phase of the order parameter in the nth layer, $\mathbf{A}=(A_x,A_y)$ is the vector potential, $\mathbf{H}$ is the external magnetic field, $\mathbf{B}=\nabla\times\mathbf{A}$ is the magnetic field, and $\nabla=(\nabla_x,\nabla_y)$ is the gradient in the layers plane.

Let us suppose that in each of the superconducting layers there is a linear defect (see Fig. 2). We treat these defects as Josephson junctions coinciding with the lines $x=0$, $z=nd$ ($n=0,\pm1,\pm2,\ldots$). As we take into account only the electromagnetic coupling between the superconducting layers the space scale for the magnetic field variation inside the sample is the penetration depth λ.

Let us consider a Josephson pancake located in the layer $n=0$ and suppose that $l_J \ll \lambda$. In this case the core of the Josephson pancake can be described as it was done in the previous section. The only difference is in substituting λ instead of δ. It means that, in particular, the electrodynamics of the Josephson junction is nonlocal if

$$l_J=\frac{c\Phi_0}{16\pi^2\lambda^2 j_c}\ll\lambda\;, \tag{28}$$

and the current density in the core of the Josephson pancake is given by the equation

$$\mathbf{j}=\frac{c\Phi_0}{8\pi^2\lambda^2}\frac{\mathbf{n}\times\mathbf{r}}{r^2},\quad r>l_J\;. \tag{29}$$

Variation of Eq. (27) with respect to the vector potential $\mathbf{A}$ results in the generalized London equation. Inside the superconductor it has the following form:[1,9]

$$\mathbf{A}-\lambda^2\nabla^2\mathbf{A}=\frac{\Phi_0}{2\pi}\frac{\mathbf{n}\times\mathbf{r}}{r^2}\delta(z),\quad r>l_J\;. \tag{30}$$

Solution of Eq. (30) leads to the magnetic-field $\mathbf{B}$ distribution which is the same as for a pointlike vortex.[1-3] The only difference is that the core of a Josephson pancake is superconducting. The size of this superconducting core is of the order of l_J instead of ξ for the normal core of a pointlike vortex.

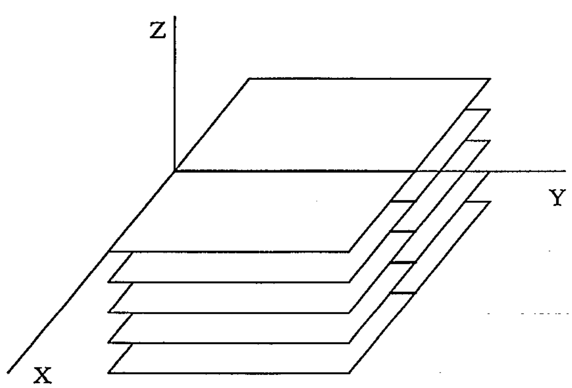

FIG. 2. A stack of superconducting layers with linear defects (thick lines).

We calculate now the mobility of a Josephson pancake. The motion of a Josephson pancake results in a voltage $U(y)$ applied to the Josephson junction. This voltage leads to energy dissipation due to the resistive component of the current flowing through the Josephson junction. Using Eq. (18) we find that the rate of this dissipation $\dot{Q}$ is equal to

$$\dot{Q}=\frac{d}{R}\int_{-\infty}^{\infty}U^2dy=\frac{d\hbar^2}{4Re^2}\int_{-\infty}^{\infty}\left[\frac{\partial\varphi}{\partial t}\right]^2dy=\frac{v^2}{\mu_j}\;, \tag{31}$$

where R is the resistance per unit area of the Josephson junction, v is the velocity, and μ_j is the mobility of the Josephson pancake. In the limit $v\to0$ the phase difference $\varphi(y,t)$ is given by Eq. (13), where y is substituted by $y-vt$. It follows then from Eq. (31) that

$$\mu_j=\frac{2\pi^2c^2l_JR}{\Phi_0^2 d}\;. \tag{32}$$

The resistance R of the SIS-type Josephson junction results from the normal electrons. The number of these electrons is exponentially small when $k_BT\ll\Delta$.

The mobility μ_p of a pointlike vortex is determined by the normal electrons in the core of the vortex. In the viscous flux-flow model the value of μ_p is given by the equation[11]

$$\mu_p=\gamma\frac{c^2\rho_n\xi^2}{\Phi_0^2 d}\;, \tag{33}$$

where γ is a numerical factor of the order of unity, and ρ_n is the resistivity in the normal state. It follows from comparison of Eqs. (32) and (33) that at low temperatures the mobility of a Josephson pancake becomes much higher than the mobility of a pointlike vortex.

Let us now consider small oscillations of an infinite stack of Josephson pancakes, i.e., of an Abrikosov vortex localized by linear defects existing in each of the superconducting layers (see Fig. 2). The current density and magnetic-field distributions of a Josephson pancake coincide with analogous distributions for a pointlike vortex if the distance from the core ρ is much larger than λ, i.e., $\rho\gg\lambda$. Thus the interaction energy for an infinite stack of Josephson pancakes may be written in the form[2]

$$E=\frac{d^2\Phi_0^2}{8\pi}\int\frac{d^2q\,dk}{(2\pi)^3}\frac{q^2+k^2}{q^2[1+\lambda^2(q^2+k^2)]}\times\sum_{m,n}\exp[ik(z_m-z_n)+i\mathbf{q}(\vec{\rho}_n-\vec{\rho}_m)]\;, \tag{34}$$

where $\vec{\rho}_n=(x_n,y_n)$, $z_n=nd$ ($n=0,\pm1,\pm2,\ldots$) are the coordinates of the Josephson pancakes of the stack. We consider here the case when in the stationary state $\vec{\rho}_n=0$. In case of small amplitude oscillations the energy E can be expanded in series of $|\mathbf{q}(\vec{\rho}_n-\vec{\rho}_m)|\ll1$. The energy difference ΔE for small deviations from the stationary state is then equal to

$$\Delta E = -\frac{d^2\Phi_0^2}{16\pi}\int \frac{d^2q\,dk}{(2\pi)^3}\frac{q^2+k^2}{q^2[1+\lambda^2(q^2+k^2)]} \times \sum_{m,n} e^{ikd(m-n)} q_y^2 (y_n - y_m)^2 . \quad (35)$$

We obtain the equations of Josephson pancakes motion equating $M\ddot{y}_n$ and the forces

$$F_n = -\frac{\partial \Delta E}{\partial y_n} . \quad (36)$$

It results in the following set of equations:

$$M\ddot{y}_n = \frac{d^2\Phi_0^2}{8\pi}\int \frac{d^2q\,dk}{(2\pi)^3}\frac{q^2+k^2}{q^2[1+\lambda^2(q^2+k^2)]} \times \sum_{n,m} e^{ikd(m-n)} q_y^2 (y_n - y_m) . \quad (37)$$

We take the solution of Eq. (37) in the form of a planar wave

$$y_n = e^{-i\omega t + ipnd} , \quad (38)$$

where p is the wave vector $(-\pi/d < p < \pi/d)$. To find the dependence of the frequency ω on the wave vector p we substitute Eq. (38) into Eq. (37). It leads to the following equation:

$$\omega^2 = \frac{d\Phi_0^2}{64\pi^2\lambda^4 M}\sum_n \ln\frac{d^2+\lambda^2(2\pi n - pd)^2}{d^2+(2\pi n\lambda)^2} . \quad (39)$$

Using the Poisson summation formula and taking into account that $d \ll 2\pi\lambda$ we obtain the dispersion relation $\omega(p)$ in the final form

$$\omega^2 = \frac{dl_J c^2}{32\pi\lambda^4 C}\ln\left[1+\frac{4\lambda^2}{d^2}\sin^2\frac{dp}{2}\right] . \quad (40)$$

The dependence $\omega(p)$ is linear for small wave vectors, i.e., for $dp \ll 1$ we find

$$\omega \approx \frac{c}{4\lambda}\sqrt{dl_J/2\pi C}\,|p| \propto |p| . \quad (41)$$

Note that at low temperatures these bending oscillations of a localized Abrikosov vortex result in a contribution to the specific heat $C_v \propto T$ at low temperatures.

IV. SUMMARY

To summarize, we have shown that in a layered superconductor with a linear defect in a superconducting layer Josephson pancakes exist. A Josephson pancake is a pointlike vortex localized by a linear crystalline structure defect. It has a superconducting core with a size of the order of the Josephson length l_J. At low temperatures the mobility of a Josephson pancake is much higher than the mobility of a pointlike vortex. We have calculated the dispersion relation for a small amplitude bending wave propagating along an infinite stack of Josephson pancakes. The velocity of this wave tends to a finite limit when its wavelength tends to infinity. At low temperatures these bending oscillations result in a contribution to the specific heat which is proportional to the temperature.

We have shown that a small amplitude wave can propagate in a Josephson junction with nonlocal electrodynamics.

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