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Josephson pancakes in layered superconductors

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We consider a pancake vortex in a layered superconductor with linear defects in the superconducting planes. We treat these defects as Josephson junctions. We show that the tunneling Josephson current through these junctions results in a pancake with a superconducting core. We find the mobility of a Josephson pancake.

1. Introduction

The most prominent high-temperature superconductors and, in particular, the Bi and Tl based compounds, consist of a periodic stack of the two-dimensional CuO layers (*ab* planes) were the superconductivity presumably resides. These materials are extremely anisotropic and, in particular, the density of the superconducting current in the direction perpendicular to the layers (*c* direction) is much less than in the *ab* planes. The discovery of the layered anisotropic high-T_c superconductors stimulated many theoretical studies of layered superconductors with weak interlayer Josephson coupling. In particular, the specific pointlike (or pancake) vortices were introduced and investigated [1–3].

A pointlike vortex is an elementary vortex existing in a layered superconductor. The normal core of each of these pointlike vortices is residing only in one of the superconducting layers. The self-energy of an isolated pointlike vortex is proportional to $\ln(L/\xi)$, where L is the characteristic size of the sample in the *ab* plane, and $\xi = \xi_{ab}$ is the coherence length in the *ab* plane. Thus, the self-energy of an isolated pointlike vortex diverges when $L/\xi \to \infty$ and it cannot exist in the bulk of a macroscopic sample.

Interaction with the sample surface affects the value of the self-energy of a pointlike vortex in the surface layer. This interaction consists of repulsion and attraction. The repulsion results from the interaction with the Meissner screening current. The attraction results from the increase of the superconducting current density of the pointlike vortex caused by the sample surface. The correlation between these two interactions is determined by the external magnetic field H. At a certain value of H the competition of attraction and repulsion leads to a stable state localized near the sample surface. The existence of this state results in a specific thermally activated mechanism of magnetization relaxation [4,5].

The rate of magnetization relaxation due to this mechanism is proportional to the mobility μ_p of a pointlike vortex. The value of μ_p is determined by the normal electrons in the core of the pointlike vortex. In the viscous flux-flow model the mobility μ_p is given by the formula [6]

$$\mu_p = \gamma \frac{c^2 \rho_n \xi^2}{\Phi_0^2 d}, \tag{1}$$

where γ is a numerical factor of the order of one, ρ_n is the resistivity in the normal state, Φ_0 is the flux quantum, and d is the distance between the superconducting layers. It follows from eq. (1) that the value of μ_p tends to a finite limit at $k_B T \ll \Delta$, where k_B is the Boltzmann constant, T is the temperature, and Δ is the energy gap. In particular, this limit determines the amplitude of the rate of the thermally activated mechanism of magnetization relaxation [4,5].

Magnetization relaxation rate is also strongly affected by the interaction of vortices and defects of the crystalline structure. A linear defect in a superconducting layer is a characteristic defect in a layered superconductor. In particular, grains boundaries and twins result in linear defects in the superconducting layers. Linear defect in a superconducting layer can be treated as a Josephson junction. Interaction with these Josephson junctions may result in localization of a pointlike vortex and thus, in particular, in a noticeable change of the rate of the thermally activated mechanism of magnetization relaxation.

In this paper we study a pointlike vortex localized by a linear defect in a superconducting layer. We consider the case of a very weak interlayer Josephson coupling. We treat the linear defects in the superconducting layers as Josephson junctions with a relatively high critical current. We show that the localized pointlike vortex has a superconducting core. We consider the mobility of a Josephson pancake. We find that at $k_B T \ll \Delta$ the mobility of a Josephson pancake is much higher than the mobility of a pointlike vortex.

2. Josephson vortex in a thin film

Let us consider a Josephson vortex in superconducting film with the thickness a and a Josephson junction along the y axis. We treat here a thin film limit, which means that $a \ll \delta$, where δ is the penetration depth. In this case the superconducting current density and magnetic field decrease with the characteristic space scale of the order of [7]

$$\delta_{\mathrm{eff}} = \frac{\delta^2}{a} \gg \delta \ . \tag{2}$$

As a result the contribution of the self-field to the current density j is negligible for $r \ll \delta_{\mathrm{eff}}$ and

$$j = \frac{c\Phi_0}{8\pi^2\delta^2} \nabla\theta \ . \tag{3}$$

We denote here the phase of the order parameter as θ and we consider a Josephson vortex located at the point $r = 0$, where $r = \sqrt{x^2 + y^2}$. It follows from eq. (3) and div $j = 0$ that the equation for $\theta(x, y)$ has the form

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0 \ . \tag{4}$$

The solution of eq. (4) describing the phase distribution for a Josephson vortex $\theta_v(x, y)$ is an odd function of x, i.e., $\theta_v(x, y) = -\theta_v(-x, y)$. This solution has to satisfy to the boundary condition at $x = 0$ which is given by the Josephson relation

$$j_x(+0, y) = \frac{c\Phi_0}{8\pi^2\delta^2} \frac{\partial\theta_{\mathrm{v}}}{\partial x}(+0, y) = -j_{\mathrm{c}} \sin\varphi(y) \ . \tag{5}$$

Here j_{c} is the Josephson current critical density, and $\varphi(y)$ is the phase difference defined as

$$\varphi(y) = \theta_{\mathrm{v}}(+0, y) - \theta_{\mathrm{v}}(-0, y) \ . \tag{6}$$

To obtain the space distributions of $\theta_{\mathrm{v}}(x, y)$ and $\varphi(y)$ we apply the Fourier cosine transformation to eq. (4). It leads to the following equation:

$$\frac{\mathrm{d}^2\theta_k}{\mathrm{d}y^2} - k^2\theta_k = \frac{16\pi^2\delta^2 j_{\mathrm{c}}}{c\Phi_0} \sin\varphi \ , \tag{7}$$

where

$$\theta_k(y) = 2\int_0^\infty \mathrm{d}x \cos(kx)\, \theta_{\mathrm{v}}(x, y) \ . \tag{8}$$

The solution of eq. (7) is given by the formula

$$\theta_k(y) = -\frac{8\pi^2\delta^2 j_c}{c\Phi_0 k} \int_{-\infty}^{\infty} dy' \sin\varphi(y') \exp(-k|y-y'|) . \tag{9}$$

The inverse Fourier transformation leads then to the relation

$$\frac{\partial\theta}{\partial y}(+0, y) = \frac{8\pi\delta^2 j_c}{c\Phi_0} \int_{-\infty}^{\infty} dy' \frac{\sin\varphi(y')}{y-y'} . \tag{10}$$

Taking into account that $\varphi(y) = 2\theta(+0, y)$ we find for the phase difference $\varphi(y)$ the nonlinear integro-differential equation

$$\frac{\partial\varphi}{\partial y} = \frac{16\pi\delta^2 j_c}{c\Phi_0} \int_{-\infty}^{\infty} dy' \frac{\sin\varphi(y')}{y-y'} . \tag{11}$$

It follows from eq. (11) that the value of the characteristic space scale for $\varphi(y)$ is given by

$$\lambda_J = \frac{c\Phi_0}{16\pi\delta^2 j_c} . \tag{12}$$

Thus, the above consideration is valid if $\lambda_J \ll \delta_{eff}$. It means that the critical current density of the Josephson junction j_c has to be high enough, i.e.,

$$j_c \gg \frac{ac\Phi_0}{16\pi\delta^4} . \tag{13}$$

The space distribution of the phase difference $\varphi(y)$ for a Josephson vortex is given by the following exact solution of eq. (11):

$$\varphi(y) = 2\arctan\left(\frac{\pi y}{\lambda_J}\right) + \pi . \tag{14}$$

Note that this solution was obtained in [8] while considering the nonlocal Josephson electrodynamics.

Using eq. (9) and the inverse Fourier transformation it is easy to show that for $r \gg \lambda_J$ the current lines are circles and

$$\boldsymbol{j} = \frac{c\Phi_0}{8\pi^2\delta^2} \frac{\boldsymbol{n}\times\boldsymbol{r}}{r^2} , \tag{15}$$

where $\boldsymbol{n}$ is the unit vector along the z axis.

Thus, we show that a Josephson vortex in a thin film is similar to a Pearl vortex [7], but with a superconducting core. The size of this superconducting core is of the order of λ_J.

3. Josephson pancake in a layered superconductor

Let us consider now a Josephson pointlike vortex in a layered superconductor, i.e., the Josephson pancake. We use here for calculations the Lawrence–Doniach model [9] in the limit of very weak interlayer Josephson coupling, i.e., we neglect the superconducting current in the c-direction. Thus we treat the layered superconductor as a periodic stack of electromagnetically coupled superconducting layers with the distance d between them. The free energy functional F in this case has the form [9]

$$F=\frac{d}{2\pi}\left(\frac{\Phi_0}{4\pi\lambda}\right)^2\sum_n\int \mathrm{d}^2\boldsymbol{r}\left(\boldsymbol{\nabla}\theta_n-\frac{2e}{c}\boldsymbol{A}\right)^2+\frac{1}{8\pi}\int \mathrm{d}^3\boldsymbol{r}\,(\boldsymbol{H}-\boldsymbol{B})^2\,, \tag{16}$$

where n is the number of the layer, θ_n is the phase of the order parameter in the nth layer, $\boldsymbol{A}=(A_x, A_y)$ is the vector potential, $\boldsymbol{H}$ is the external magnetic field, $\boldsymbol{B}=\boldsymbol{\nabla}\times\boldsymbol{A}$ is the magnetic field, and $\boldsymbol{\nabla}=(\nabla_x, \nabla_y)$ is the gradient in the layer plane.

Let us suppose that in each of the superconducting layers there is a linear defect. We treat these defects coinciding with the lines $x=0$, $z=nd$ ($n=0, \pm1, \pm2, \ldots$) as Josephson junctions.

As we take into account only the electromagnetic coupling between the superconducting layers the space scale for the magnetic field decrease is the penetration depth λ. We consider now a Josephson pancake located in the layer $n=0$ and suppose that $\lambda_J<\lambda$. In this case the core of the Josephson pancake can be described as was done in the previous section. The only difference is in substituting λ instead of δ. Thus it means that, in particular, we have

$$\lambda_J=\frac{c\Phi_0}{16\pi\lambda^2 j_c}\ll\lambda\,, \tag{17}$$

and the current distribution in the core of the Josephson pancake is given by

$$\boldsymbol{j}=\frac{c\Phi_0}{8\pi^2\lambda^2}\frac{\boldsymbol{n}\times\boldsymbol{r}}{r^2}\,,\qquad r>\lambda_J\,. \tag{18}$$

Variation of the eq. (16) with respect to the vector potential $\boldsymbol{A}$ results in the generalized London equation. Inside the superconductor it has the following form [1,9]:

$$A - \lambda^2 \nabla^2 A = \frac{\Phi_0}{2\pi} \frac{n \times r}{r^2} \delta(z) , \quad r > \lambda_J . \tag{19}$$

The solution of eq. (19) leads to the magnetic field $\boldsymbol{B}$ distribution which is the same as for the pointlike vortex [1–3]. The only difference is that the core of a Josephson pancake is superconducting and its size is of the order of λ_J instead of ξ.

We calculate now the mobility of a Josephson pancake. The motion of a Josephson pancake results in a voltage $U(y)$ applied to the Josephson junction. The value of $U(y)$ is given by the formula [10]

$$U(y) = \frac{\hbar}{2e} \frac{\partial \varphi}{\partial t} . \tag{20}$$

This voltage leads to energy dissipation due to the resistive component of the current flowing through the Josephson junction. The rate of this dissipation $\dot{Q}$ is equal to

$$\dot{Q} = \frac{d}{R} \int_{-\infty}^{\infty} U^2 \, \mathrm{d}y = \frac{d\hbar^2}{4Re^2} \int_{-\infty}^{\infty} \left(\frac{\partial \varphi}{\partial t} \right)^2 \mathrm{d}y = \frac{v^2}{\mu_J} , \tag{21}$$

where R is the resistance per unit area of the Josephson junction, $\boldsymbol{v}$ is the velocity, and μ_J is the mobility of the Josephson pancake. In the limit $v \to 0$ the phase difference $\varphi(y, t)$ is given by eq. (14), where y is substituted by $y - vt$. It follows then from eq. (21) that

$$\mu_J = \frac{2\pi c^2 \lambda_J R}{\Phi_0^2 d} . \tag{22}$$

The resistance R of the SIS-type Josephson junction results from the normal electrons. The number of them is exponentially small when $k_B T \ll \Delta$. It follows then from eqs. (1) and (22) that at low temperatures the mobility of a Josephson pancake becomes much higher than the mobility of a pointlike vortex.

4. Summary

To summarize, we have shown that in a layered superconductor with linear defects in the superconducting layers exists a new type of vortices localized in one of the superconducting layers. These vortices, i.e., the Josephson pancakes, have a superconducting core. The size of this core is of the order of the Josephson characteristic length λ_J. At low temperatures the mobility of a Josephson pancake is much higher than the mobility of a pointlike vortex.

References

[1] M.E. Feigelman, V.B. Geshkenbein and A.I. Larkin, Physica C 167 (1990) 177.
[2] K.B. Efetov, Sov. Phys. JETP 49 (1979) 905.
[3] J.R. Clem, Phys. Rev. B 43 (1991) 7837.
[4] R.G. Mints and I.B. Snapiro, Europhys. Lett. 21 (1993) 611.
[5] R.G. Mints and I.B. Snapiro, Phys. Rev. B 47 (1993) 3273.
[6] L.P. Gorkov and N.B. Kopnin, Usp. Fiz. Nauk 116 (1975) 413; Sov. Phys. Uspekhi 18 (1975) 496.
[7] J. Pearl, Appl. Phys. Lett. 5 (1964) 65.
[8] A. Gurevich, Phys. Rev. B 46 (1992) 3187.
[9] W.E. Lawrence and S. Doniach, in: Proc. 12th Int. Conf. on Low Temperature Physics, Kyoto (1970), E. Kanda, ed. (Keigaku, Tokyo, 1971) p. 361.
[10] A. Barone and G. Paterno, Physics and Applications of the Josephson Effect (Wiley, New York, 1982).