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Point–like vortices and magnetization relaxation in layered superconductors

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The energy of a point–like vortex is calculated for a layered superconductor with weak interlayer Josephson coupling. An energy barrier existing near the sample surface is found. The effect of the interlayer Josephson coupling on this barrier is considered. Magnetization relaxation due to thermally activated penetration of point–like vortices is considered. An initial avalanche–type decay of magnetization is predicted.

1. INTRODUCTION

The discovery of Bi and Tl based high–T_c superconductors stimulated theoretical studies of layered superconductors with interlayer Josephson coupling. In particular, the specific point–like (or pancake) vortices were introduced and investigated [1-3]. An isolated point–like vortex can not exist in the bulk of a macroscopic sample as its self–energy logarithmically diverges, when $L/\xi \to \infty$, where L is the characteristic size of the sample, and ξ is the coherence length.

Interaction of a point–like vortex with the sample surface consists of repulsion and attraction. The repulsion results from the interaction with the Meissner screening currents. The attraction results from the increase of the superconducting current density of the point–like vortex caused by the sample surface. The correlation between these two interactions is determined by the external magnetic field H. At a certain value of H the competition of attraction and repulsion can lead to a stable state localized near the surface. The existence of this stable state effects the flux penetration process and, in particular, the magnetization relaxation.

In this paper we study the penetration of point–like vortices into a layered superconductor. We show that the energy of a point–like vortex G_v has a minimum G_m detached by an energy barrier G_g from the surface. We show that G_m is negative if the external magnetic field H is higher than a certain value H_1, which is of the order of the lower critical field H_{c1}. We estimate the effect of the interlayer Josephson coupling on the energy barrier G_g. We present a scenario of magnetization relaxation due to thermally activated penetration of point–like vortices inside the sample. We calculate the time dependence of the magnetization relaxation. We consider the case, when $\xi \ll \lambda$ and $d \ll \lambda$, where λ is the London penetration depth, and d is the distance between the superconducting layers.

2. SURFACE BARRIER

To calculate the magnetization relaxation we find first the energy of a point–like vortex residing in one of the superconducting layers in the vicinity of the sample surface. We use for calculations the Lawrence–Doniach model [4], where we neglect the interlayer Josephson coupling.

Consider a semi–infinite layered superconductor subjected to a magnetic field $\mathbf{H}$ parallel to the surface and perpendicular to the layers. Suppose the z–axis is parallel to $\mathbf{H}$ and the x–axis is perpendicular to the surface. As we consider the case $\xi \ll \lambda$, the energy of a point–like vortex $G_v(x)$ is determined mostly by the magnetic field $\mathbf{B}$ distribution into the sample.

For an isolated point–like vortex the magnetic field distribution $\mathbf{B}(\mathbf{r})$ may be written as

$$\mathbf{B}(\mathbf{r}) = \mathbf{H}\exp(-\frac{x}{\lambda}) + \mathbf{b}(\mathbf{r}) \qquad (1)$$

Here the first term represents the magnetic field penetration into the sample in the absence of vortices. The magnetic field $\mathbf{b}(\mathbf{r})$ results from the point-like vortex, and can be calculated by means of the method of images It means that to the point-like vortex located at (x, y) we add an image point-like antivortex located at $(-x, y)$ and take for $\mathbf{b}(\mathbf{r})$ the sum of the field due to the vortex and antivortex Thus the field $\mathbf{b}(\mathbf{r})$ automatically vanishes on the sample surface and the boundary condition $\mathbf{B}(0, y, z) = \mathbf{H}$ is satisfied. Using the magnetic field distribution, we calculate the isolated point-like vortex energy

$$G_v = G_{va} + G_{vh}, \qquad x > \xi, \tag{2}$$

where

$$G_{va} = \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 d\ \ln(\frac{2x}{\xi}), \qquad x > \xi \tag{3}$$

$$G_{vh} = \frac{\Phi_0}{4\pi} Hd\left[\exp(-\frac{x}{\lambda}) - 1\right], \qquad x > \xi \tag{4}$$

The term G_{va} represents the attraction between the point-like vortex and its image (point-like antivortex) It is minimal near the surface and increases monotonically with increase of x The term G_{vh} represents the repulsion of the point-like vortex from the surface due to the external magnetic field and the associated screening currents It is maximal at $x = 0$ and decreases monotonically with the increase of x The dependence of G_v on x increase monotonically if $H < H_0$, where

$$H_0 = e\frac{\Phi_0}{4\pi\lambda^2} \tag{5}$$

For $H > H_0$ the curve $G_v(x)$ has a maximum G_g at $x = x_g$ and a minimum G_m at $x = x_m$ The explicit formulae for G_g, x_g, G_m, and x_m can be derived by means of Eqs (2)–(4) in the case, when $H \gg H_0$ and $\ln(\lambda/\xi) \gg 1$

$$G_g \approx \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 d\ \ln\left[\frac{\Phi_0}{2\pi}\frac{1}{\lambda\xi H}\right], \tag{6}$$

$$x_g \approx \frac{\Phi_0}{4\pi\lambda}\frac{1}{H} < \lambda, \tag{7}$$

$$G_m \approx \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 d\ln\left[2e\frac{\lambda}{\xi}\ln(\frac{4\pi\lambda^2}{\Phi_0}H)\right] - \frac{\Phi_0}{4\pi}dH \tag{8}$$

$$x_m \approx \lambda\ln(\frac{4\pi\lambda^2}{\Phi_0}H) > \lambda \tag{9}$$

It follows from Eq (8) that the value of the minimum energy G_m becomes negative when $H > H_1$, where

$$H_1 \approx \frac{\Phi_0}{4\pi\lambda^2}\ln\left[2e\frac{\lambda}{\xi}\ln\ln(\frac{\lambda}{\xi})\right] \tag{10}$$

Note, that the magnetic field H_1 is higher than the lower critical field H_{c1}

Let us now estimate the effect of the interlayer Josepson coupling on the energy barrier G_g and the magnetic field H_1 The interaction energy between the point-like vortex and antivortex G_{va} including the interlayer Josepson coupling was considered in [5] It was shown that the value of G_{va} is determined by the interaction via magnetic field for $r \ll \lambda_J$ and is determined by the interlayer Josepson coupling for $r \gg \lambda_J$ Here r is the distance between the point-like vortex and antivortex, and λ_J is the Josephson length. Applying these results to the interaction of the point-like vortex and its image we find that the value of G_{va} for $x \ll \lambda_J$ is given by Eq (3) and for $x \gg \lambda_J$ by the formula [5]

$$G_{va} = 2\sqrt{2}\pi\left(\frac{\Phi_0}{4\pi\lambda}\right)^2 d\frac{x}{\lambda_J}, \qquad x \gg \lambda_J \tag{11}$$

The space scale of G_{vh} is the penetration depth λ Thus, two characteristic limiting cases can be considered, i e , $\lambda \ll \lambda_J$ and $\lambda \gg \lambda_J$ The first one corresponds to the Tl-based high-T_c superconductors, and the second one corresponds to the Bi-based high-T_c superconductors [6] The above results concerning H_0, H_1 and G_g are valid if $\lambda \ll \lambda_J$ Let us now consider the case, when $\lambda \gg \lambda_J$ It follows then from Eqs (2)–(4) that the change of the field H_0 is considerable and H_0 is of the order of

$$H_0 \sim \frac{\Phi_0}{4\pi\lambda\lambda_J} \tag{12}$$

Using Eqs (2), (4) and (11) it can be shown that for $H \gg H_0$ the value of $x_g \ll \lambda_J$ and thus the energy barrier G_g is given by Eq (6) The change of the field H_1 is also considerable, i e.,

$$H_1 = \frac{1}{2\sqrt{2}}\frac{\Phi_0}{\lambda\lambda_J}\ln\left(2\sqrt{2}\frac{\lambda\lambda_J H}{\Phi_0}\right) \tag{13}$$

Thus, the interlayer Josepson coupling in case, when $\lambda \gg \lambda_J$ results mainly in increasing of the characteristic magnetic fields H_0 and H_1 and doesn't change considerably the value of G_g

3. MAGNETIZATION RELAXATION

We consider now the magnetization relaxation for magnetic field $H_1 < H < H^*$ In this case, the energy of an Abrikosov vortex is negative in the bulk However, the Bean–Livingstone barrier [7] for a nucleous of an Abrikosov vortex (vortex loop) is of the order of $\lambda G_g/d \gg G_g$ [8] On the other hand, for the same magnetic field interval, the minimum energy of a point-like vortex G_m is negative, and the energy barrier is equal to G_g At non-zero temperature it can lead to a thermally activated penetration of the point-like vortices into a sample and thus to a specific mechanism of magnetization relaxation in layered superconductors This mechanism is especially effective if the external magnetic field is low, i e , when $H_1 < H < H^*$

We consider here, as an illustration, the magnetization relaxation for the following problem A semi-infinite layered superconductor is cooled down to a certain temperature T below the critical temperature in zero magnetic field Then, a magnetic field $\mathbf{H}$ parallel to the sample surface is instantaneously turned on We suppose that the superconducting layers are perpendicular to $\mathbf{H}$ and the field is from the interval $H_1 < H < H^*$ We treat here the following scenario of magnetization relaxation

The initial magnetization $\mathbf{M}_0$ is equal to

$$\mathbf{M}_0 = -\frac{1}{4\pi}\mathbf{H} \tag{14}$$

The thermally activated penetration of point-like vortices into the sample lead to the decay of the magnetization $\mathbf{M}$ The rate of this process is determined mostly by the energy barrier for the point-like vortices, and it depends exponentially on this barrier After penetration into the sample the point-like vortices reside randomly in the superconducting layers in the vicinity of the plane $x = x_m$ The interaction of the incoming vortex with these vortices changes the energy barrier Thus, to find the magnetization relaxation rate we have to find this energy barrier shift δG_g.

The value of δG_g is determined by the repulsion of the point-like vortices residing in the same layer and the attraction of the point-like vortices residing in different layers To find the value of δG_g we suppose that the point-like vortices inside the sample are distributed randomly along the line $x = x_m$ in each of the superconducting layers The result of the calculation shows that δG_g is proportional to the average linear concentration N of the point-like vortices

$$\delta G_g = -\frac{Nd}{2\gamma_g M_0}\left(\frac{\Phi_0}{4\pi\lambda}\right)^3, \tag{15}$$

where $M_0 = H/4\pi$, and γ_g is a number of the order of one The main contribution to the decrease of the energy barrier results from the attractive interaction with the point-like vortices residing in the nearest $\lambda/d \gg 1$ layers The entire number of these point-like vortices is of the order of $N x_m \lambda/d \gg 1$ The formula given by Eq (15) is valid if the average distance $l = N^{-1}$ between the point-like vortices in each of the superconducting layers is less than the penetration depth, i e , $N\lambda < 1$

Similar calculations show that the increase of the magnetization δM due to the point-like vortices residing in the superconducting layers near the sample surface is also proportional to N

$$\delta M = N\frac{\Phi_0}{8\pi\lambda}\ln\left(\frac{4\pi\lambda^2}{\Phi_0}H\right) \tag{16}$$

Note, that Eq (16) is an exact one, i e , it is valid for any values of N

Finally, using the Eqs (15) and (16) we can present the dependence of δG_g on δM as

$$\delta G_g = -\frac{\delta M}{\gamma_g M_0}d\left(\frac{\Phi_0}{4\pi\lambda}\right)^2\ln^{-1}\left(\frac{4\pi\lambda^2}{\Phi_0}H\right) \tag{17}$$

Thus, the energy barrier decreases with the increase of the magnetization This dependence results in the avalanche-type thermally activated magnetization relaxation

To find the equation determining the magnetization relaxation we consider the diffusion of the point–like vortices in the layer $x < x_m$ To estimate the appropriate diffusion coefficient we treat the motion of the point–like vortices as a viscous flux flow We consider also the dependence of the energy of the point–like vortices G_v on x as an external potential Finally, this approach results in the following equation

$$\frac{dM}{dt} = \frac{\alpha M_0}{\tau} \exp\left(-\frac{\delta G_g}{k_B T}\right), \tag{18}$$

where

$$\alpha = \gamma_g \frac{k_B T}{d} \left(\frac{4\pi\lambda}{\Phi_0}\right)^2 \ln\left(\frac{4\pi\lambda^2}{\Phi_0} H\right), \tag{19}$$

$$\tau = \gamma_\tau \frac{\lambda^2}{\rho_n c^2} \exp\left(\frac{G_g}{k_B T}\right), \tag{20}$$

γ_τ is a number of the order of one, ρ_n is the resistivity in the normal state, k_B is the Boltzman constant, and

$$\tau_0 = \frac{\lambda^2}{\rho_n c^2}, \tag{21}$$

is the characteristic time constant

The solution of Eq (18) has the form

$$M = M_0\left[-1 + \alpha \ln\left(\frac{\tau}{\tau - t}\right)\right] \tag{22}$$

Thus, the thermally activated point–like vortices penetration into the sample lead to a specific avalanche–type dependence of the magnetization on time The characteristic time of the magnetization decay τ strongly (exponentially) depends on the energy barrier and the temperature The dimensionless amplitude α of the magnetization relaxation is proportional to the temperature and slowly (logarithmically) depends on the applied magnetic field

The dependence given by the Eq (22) is valid until

$$\delta M = \alpha M_0 \ln\left(\frac{\tau}{\tau - t}\right) < M_0, \tag{23}$$

and the density of vortices N is less than a certain critical value $N_c \sim \lambda^{-1}$ When N becomes of the order of N_c Abrikosov vortices self–assemble from the point–like vortices and then penetrate inside the bulk. It follows from the Eqs (16) and (23), that a noticeable increase of N starts, when $t \to \tau$ The penetration of Abrikosov vortices inside the bulk changes the time dependence of the magnetization decay to the regular logarithmic law The latter leads to the magnetization relaxation rate decreasing in time Thus, at $t \approx \tau$ the magnetization relaxation rate has a maximum

4. SUMMARY

We have shown that in an external magnetic field higher than H_0 the energy of a point–like vortex G_v has a minimum G_m detached from the sample surface by an energy barrier G_g The value of G_m becomes negative in magnetic field higher than H_1 We have calculated the time dependence of the magnetization relaxation due to the thermally activated penetration of point–like vortices inside the sample We have shown that this process results in an avalanche–type initial decay of the magnetization and a specific maximum in the magnetization relaxation rate at a certain moment of time $t \approx \tau$ We have estimated that the interlayer Josephson coupling doesn't change the obtained results considerably

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