ENHANCED QUENCH PROPAGATION VELOCITY

R.G. Mints[†‡], A.A. Akhmetov[†] and A. Devred[†]
[†]SSC Laboratory,[*] Dallas, TX, USA
[‡]School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv, Israel

Abstract—Quench propagation velocity in conductors having a large amount of stabilizer outside the multifilamentary area is considered. It is shown that the current redistribution process between the multifilamentary area and the stabilizer can strongly effect the quench propagation. A criterion is derived determining the conditions under which the current redistribution process becomes significant, and a model of effective stabilizer area is suggested to describe its influence on the quench propagation velocity. As an illustration, the model is applied to calculate the adiabatic quench propagation velocity for a conductor having a multiply connected stabilizer, consisting of an inner core and an outer sheath.

I. INTRODUCTION

The development of conductors with aluminium superstabilizer for applications, such as detector magnets for high energy physics [1], energy storage devices [2-4], and others, has led to new problems. One of them is the effect of current redistribution process between the superconductor and stabilizer on the quench propagation [5, 6].

The quench propagation velocity is determined by the Joule heating in the vicinity of the transition front. During the transition from the superconducting to the resistive state, the current is redistributed from the superconductor to the stabilizer. This redistribution occurs in two phases. First, the current is expelled from the superconducting filaments to the copper in the multifilamentary area. Second, the current diffuses into the stabilizer outside the multifilamentary area. If the interfilament spacing is small, the first phase is very fast. On the other hand, if most of the stabilizer is located outside of the multifilamentary area, the second phase can be relatively long. In the vicinity of the transition front, where the quench-driving heat release occurs, the current may thus remain confined in a small fraction of stabilizer around the multifilamentary area. This results in a relatively high local value of Joule heating, leading to high quench propagation velocity [6].

In this paper, we shall consider the case where the quench propagation is effected by the current redistribution process. We shall introduce the characteristic velocity at which this process becomes significant. We shall introduce a model of effective stabilizer area for fast quench propagation. We apply this model to calculate the adiabatic quench velocity for a conductor having a multiply connected stabilizer, consisting of an inner core and an outer sheath, as depicted in Fig. 1 [7].

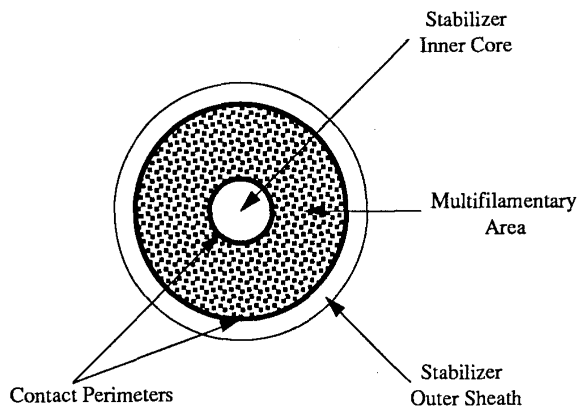

Figure 1: The conductor cross–section.

II. HIGHLY STABILIZED CONDUCTORS

Most of the papers on quench propagation velocity consider the current redistribution process as instantaneous (see the review in reference [8]). To discuss the applicability of this assumption, let us estimate the characteristic times of the phenomena involved. The current redistribution time, t_d, may be estimated as

$$t_d = \frac{\mu_0 d^2}{\rho_n}, \tag{1}$$

where ρ_n is the resistivity, and d is the effective thickness of the stabilizer. In case of the conductor shown in Fig. 1 there are two effective thicknesses: d_i for the inner core and d_o for the outer sheath, given by

$$d_i = \frac{A_n^i}{P_n^i}, \qquad and \qquad d_o = \frac{A_n^o}{P_n^o}, \tag{2}$$

where A_n^i and A_n^o, and P_n^i and P_n^o are the cross-sectional areas, and contact perimeters of the stabilizer (see Fig. 1).

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The characteristic time associated with the quench propagation, t_p, is given by

$$t_p = \frac{L}{v}, \tag{3}$$

where v is the quench propagation velocity, and L is the thickness of the zone where the quench–driving heat release occurs. In other words, L is the thickness of the region, in the vicinity of the transition front from the resistive to the superconducting state, where the Joule heating determining the propagation velocity takes place.

In case of instantaneous current redistribution, the power Q of the Joule heating in the conductor is given by

$$Q = \rho \frac{I}{A} \begin{cases} 0, & T < T_{ci}; \\ I_c \dfrac{T - T_{ci}}{T_c - T_0}, & T_{ci} < T < T_c; \\ I, & T_c < T, \end{cases} \tag{4}$$

where

$$T_{ci} = T_c - i\,(T_c - T_0), \qquad i = \frac{I}{I_c}. \tag{5}$$

Here T_0 is the coolant temperature, T_c is the critical temperature at the given field and T_0, I is the transport current, I_c is the critical current at the given field and T_0, ρ is the longitudinal electrical resistivity of the conductor, and A is the conductor cross-sectional area. An expression of ρ is given by

$$\rho = \frac{A \rho_n \rho_s}{A_n \rho_s + A_s \rho_n}, \tag{6}$$

where A_n is the total cross-sectional area of stabilizer, and A_s and ρ_s are the cross-sectional area and the resistivity of the multifilamentary area. In this paper, we shall represent the power of the Joule heating in the conductor as a step function of temperature

$$Q = \rho \frac{I^2}{A} \begin{cases} 0, & T < T_t; \\ 1, & T_t < T. \end{cases} \tag{7}$$

The transition temperature, T_t, is determined so that the propagation velocity derived using Eq. (7) is equal to that derived using Eq. (4). It can be shown [9] that

$$T_t = T_{ci} + \alpha(i)\,(T_c - T_{ci}), \tag{8}$$

where α is a dimensionless parameter that only depends on i. This dependence is presented in Fig. 2.

In most cases of practical interest, the cooling conditions are weak. Then, L is determined by the thermal diffusion along the conductor, and can be estimated as [6]

$$L = \frac{\kappa}{Cv}, \tag{9}$$

where κ and C are the thermal conductivity and the heat capacity per unit volume averaged over the conductor cross-section, and taken at the given field and T_t.

Thus, the current redistribution process can be considered as instantaneous, only if the dimensionless parameter

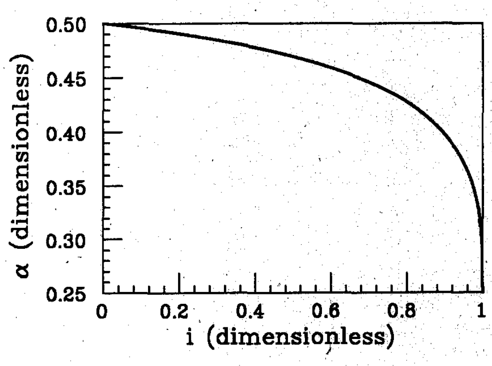

Figure 2: The dependence of the parameter α on the dimensionless current i.

$\tau = t_d/t_p$ is less than one. Using Eqs. (1) to (3), and Eq. (9), it is convenient to rewrite τ as

$$\tau = \frac{\mu_0 C d^2}{\kappa \rho_n} v^2 = \left(\frac{v}{v_c}\right)^2, \tag{10}$$

where we have introduced the characteristic velocity v_c

$$v_c = \frac{1}{d}\sqrt{\frac{\kappa \rho_n}{\mu_0 C}}. \tag{11}$$

For the conductor considered above there are two characteristic velocities: one for the inner core, v_c^i, and one for the outher sheath, v_c^o. Using Eq. (8) and the data from [7] (assuming an interfilament spacing of $2\,\mu m$, a critical current density $j_c(5\,T, 4.2\,K) = 2.75 \cdot 10^9\,A/m^2$ and a constant external magnetic field of $5\,T$) one gets: $v_c^i \approx 50 m/s$ and $v_c^o \approx 25 m/s$.

Let us now consider the case of instantaneous current redistribution. The quench propagation velocity is determined by the Joule heating in the vicinity of the transition front. The thickness of this region is L, and, the power of the quench-driving heat release, q, can be written as

$$q = \rho \frac{I^2}{A} L. \tag{12}$$

On the other hand, q is equal to the heat flux which heats up the superconducting zone to the transition temperature. It follows

$$q = vA\Delta H, \tag{13}$$

where

$$\Delta H = \int_{T_0}^{T_t} C\,dT, \tag{14}$$

is the difference in enthalpy per unit volume of conductor between T_0 and T_t. For adiabatic cooling conditions, the value of L is given by Eq. (9). Equating the two expressions of q, and substituting the expression of L, lead to

$$v = \frac{I}{A}\sqrt{\frac{\kappa \rho}{C \Delta H}}. \tag{15}$$

In this model, the maximum of the quench propagation velocity, v_m, is obtained for $I = I_c$. Let us estimate v_m for the conductor considered above. Using the data from [7] and Eq. (15), one gets: $v_m \approx 50 m/s$.

For adiabatic cooling conditions, a criterion defining highly stabilized conductors may be derived by comparing v_m and v_c. Let us define dimensionless parameter β

$$\beta = \left(\frac{v_m}{v_c}\right)^2. \tag{16}$$

Then, a highly stabilized conductor is a conductor with β larger than one. Combining Eqs. (11) and (15) leads to the following criterion

$$\beta = \frac{\rho}{\rho_n}\left(\frac{d}{A}\right)^2 \frac{\mu_0 I_c^2}{\Delta H_c} \geq 1, \tag{17}$$

where ΔH_c is calculated by means of Eq. (14) at $I = I_c$. For the conductor considered above: $\beta_i \approx .91$ for the inner core, and $\beta_o \approx 3.7$ for the outer sheath. It thus appears that actual conductors can exibit quench propagation velocities larger than v_c.

III. EFFECTIVE STABILIZER AREA MODEL

Let us now consider the case where the current redistribution has to be taken into account while calculating the quench propagation velocity, *i.e.*, $v \gg v_c$. Then, in the vicinity of the transition front, the current remains confine to a certain fraction of stabilizer around the multifilamentary area, leading to non-uniform quench-driving heat release. The cross-sectional area occupied by the current is determined by the parameter, τ. The larger τ, *i.e.*, the larger the ratio of v to v_c, the smaller the fraction of stabilizer where the current has diffused.

In most cases of practical interest, the cooling is weak and, at the same time, the ratio of the transverse thermal diffusivity to the magnetic flux diffusivity is high. It results that the temperature distribution over the conductor cross-sectional area is uniform, even if the heat release is non-uniform.

The main difference between highly stabilized and conventional conductors is thus the non-uniformity of the quench-driving heat release. To find the exact expression of the Joule heating, we should solve the system of Maxwell's and heat diffusion equations. For most cases of practical interest, it cannot be done analytically, and is a complicated problem for numerical analysis.

In this paper, we shall calculate the Joule heating considering that the current is uniformly redistributed between the multifilamentary area and a certain area of the stabilizer, which we shall introduce as an effective area, A_{eff}. As the fraction of the stabilizer where the current has diffused depends on the quench propagation velocity, the effective area of the stabilizer is determined by the ratio v/v_c. In case of the conductor shown in Fig. 1 we have

$$A_{\text{eff}} = A^i_{\text{eff}} + A^o_{\text{eff}}, \tag{18}$$

$$A^i_{\text{eff}} = A^i_n\, f_i\left(\frac{v}{v^i_c}\right), \qquad A^o_{\text{eff}} = A^o_n\, f_o\left(\frac{v}{v^o_c}\right), \tag{19}$$

where A^i_{eff} and A^o_{eff} are the effective stabilizer areas, and v^i_c and v^o_c are the critical velocities for the inner core and outer sheath.

To find an expressions for f_i and f_o, let us first discuss the asymptotic behavior of f_i and f_o. When the ratios v/v^i_c and v/v^o_c are small, the current redistribution process is almost instantaneous, and the current occupies the whole stabilizer cross-sectional area, *i.e.*, A^i_{eff} tends towards A^i_n, and A^o_{eff} tends towards A^o_n. We thus have

$$f_i\left(\frac{v}{v^i_c}\right) = 1, \qquad \text{for} \quad \frac{v}{v^i_c} \to 0, \tag{20 a}$$

$$f_o\left(\frac{v}{v^o_c}\right) = 1, \qquad \text{for} \quad \frac{v}{v^o_c} \to 0. \tag{20 b}$$

On the other hand, when the ratios v/v^i_c and v/v^o_c are large, the current only diffuses into thin layers of stabilizer, l_i and l_o, and the current redistribution process can be treated as in the case of a semi-infinite slab of stabilizer. Then, l_i and l_o are determined by the magnetic flux diffusion length for a characteristic time of the order of t_p

$$l_i = \sqrt{D_m t_p} = \frac{A^i_n}{P^i_n}\frac{v^i_c}{v}, \tag{21 a}$$

$$l_o = \sqrt{D_m t_p} = \frac{A^o_n}{P^o_n}\frac{v^o_c}{v}. \tag{21 b}$$

Thus, the effective areas, *i.e.*, the cross-sectional areas of stabilizer occupied by the current are

$$A^i_{\text{eff}} = l_i P^i_n = A^i_n \frac{v^i_c}{v}, \tag{22 a}$$

$$A^o_{\text{eff}} = l_o P^o_n = A^o_n \frac{v^o_c}{v}, \tag{22 b}$$

and, it comes

$$f_i\left(\frac{v}{v^i_c}\right) = \frac{v^i_c}{v}, \qquad \text{for} \quad \frac{v}{v^i_c} \to \infty. \tag{23 a}$$

$$f_o\left(\frac{v}{v^o_c}\right) = \frac{v^o_c}{v}, \qquad \text{for} \quad \frac{v}{v^o_c} \to \infty. \tag{23 b}$$

Having determined the asymptotic dependencies for small and large values of v/v^i_c and v/v^o_c, we shall now define f_i and f_o for the full range of velocities. To match smoothly Eqs. (20 a) and (23 a), and Eqs. (20 b) and (23 b) we suggest the following functions

$$f_i\left(\frac{v}{v^i_c}\right) = \frac{I_1\left(2\frac{v^i_c}{v}\right)}{I_0\left(2\frac{v^i_c}{v}\right)}, \tag{24 a}$$

$$f_o\left(\frac{v}{v_c^o}\right) = \tanh\left(\frac{v_c^o}{v}\right), \qquad (24\,b)$$

where $I_0(x)$ and $I_1(x)$ are modified Bessel functions of order $0, 1$. Note, that the Eq. (24a) is a generalization of Eq. (24b) for the case of cylindrical geometry.

IV. ADIABATIC QUENCH PROPAGATION

In this section, we shall apply the above model of effective stabilizer area to the computation of the adiabatic quench propagation velocity. To do it, we have to calculate the quench-driving heat release. In the case of adiabatic cooling conditions, it is given by Eq. (12). Then, substituting A_n by A_{eff} in Eq. (12), it comes

$$q = \frac{\rho_n \rho_s I^2}{A_{\text{eff}}\rho_s + A_s \rho_n}\, \frac{\kappa}{Cv}, \qquad (25)$$

where we have replaced L by Eq. (9). An equation determining v can be derived by equating Eqs. (25) and (13), and replacing A_{eff} by Eq. (18). It comes

$$v = \frac{I}{A}\sqrt{\frac{\kappa\rho}{C\Delta H}}\, \sqrt{\frac{1+r}{1 + r\left[x_n^i\, f_i\left(\frac{v}{v_c^i}\right) + x_n^o\, f_o\left(\frac{v}{v_c^o}\right)\right]}}, \qquad (26)$$

where

$$r = \frac{A_n \rho_s}{A_s \rho_n} \gg 1, \qquad x_n^i = \frac{A_n^i}{A_n}, \qquad x_n^o = \frac{A_n^o}{A_n}. \qquad (27)$$

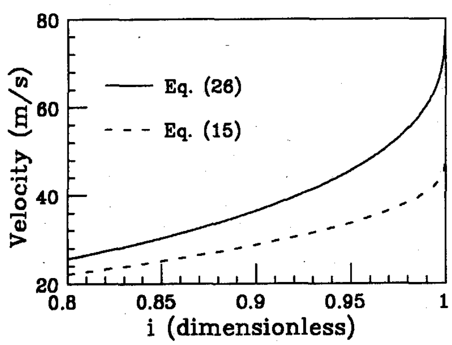

Figure 3: The dependence of the quench propagation velocity v on the dimensionless current i.

To illustrate these results, Fig. 3 shows plots of the quench propagation velocity as a function of the dimensionless current i for the conductor considered above. The solid line represents the velocity calculated by the combination of Eqs. (24a, b) and (26), which takes into account the current redistribution process. The dashed line represents the velocity calculated by means of Eq. (15), which

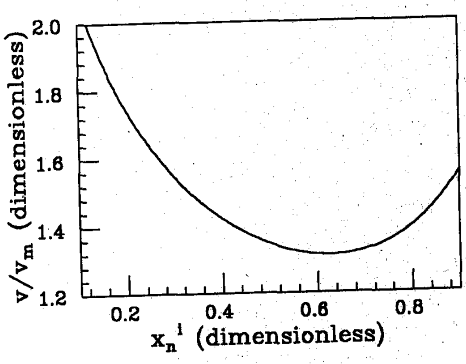

Figure 4: The dependence of the ratio $v(I_c)/v_m(I_c)$ on x_n^i.

assumes an instantaneous current redistribution. As can be seen in Fig. 3, the difference in the results can be up to 1.6 times. Note that Eq. (26) shows that the velocity depends on the distribution of stabilizer between the inner core and the outher sheath, *i.e.*, v is a function of x_n^i or $x_n^o = 1 - x_n^i$. This dependence is illustrated in Fig. 4 (for $I = I_c$). It can be seen that v goes through a minimum.

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