Initiation of traveling normal domains in large composite superconductors

V. S. Kovner, Raz Kupferman, and R. G. Mints
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, 69978 Ramat-Aviv, Israel

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We consider the initiating energy for traveling normal domains in large composite superconductors. We perform numerical simulations of normal zone initiation using the effective circuit model. The initiating energy is obtained as a function of transport current and four dimensionless parameters characterizing the composite and cooling conditions. We suggest an analytical expression determining the initiating energy for the traveling normal domains in the region of parameters of practical interest.

I. INTRODUCTION

Large composite superconductors have been recently tested for use in superconducting magnetic energy storage (SMES) systems.[1] These conductors are composed of superconducting multifilament strands embedded in a large stabilizer, made from a normal metal, with high thermal and electrical conductivity. Due to the large size and relatively low electrical resistivity of the stabilizer, if a normal zone nucleates in the superconductor, the current in this region redistributes into the stabilizer. This process is followed by a significant decrease of the joule power and the recovery of superconductivity. Despite the above stabilizing mechanism, it was found experimentally that normal domains of finite size can propagate along the conductor for transport currents larger than a certain threshold current I_d.[1]

The origin of such traveling normal domains can be explained qualitatively by the following arguments. If a part of the superconductor undergoes a normal transition, most of the current stays confined in the superconductor during the relatively long process of current redistribution into the stabilizer. The joule power is then much higher than after the current is redistributed across the conductor. This heat release results in a "hot" region at the front of the normal zone, and causes the expansion of the normal domain. After the current is redistributed in the stabilizer, the superconductor cools down towards the stable state and superconductivity recovers (in the cryostable regime).

The dynamics of a traveling normal domain was investigated in a number of theoretical studies. Huang and Eyssa[2,3] performed numerical simulations for the diffusion of heat and the redistribution of current in the conductor in the presence of a normal zone. Their simulations showed the formation of a stable traveling normal domain. They compared the calculated velocity of this domain propagation with the experimental data,[1] obtaining reasonable agreement. Dresner[4] proposed an analytical method to calculate the propagation velocity of a traveling normal domain, assuming the time dependence of the joule power. He performed explicit calculations approximating by an exponential term the decay of the joule power during the process of current redistribution. In Refs. 5 and 6, we investigated both numerically and analytically the nucleation and propagation of a traveling normal domain in large composite superconductors using an effective circuit model. We proposed explicit equations for the velocity of the domain and for the threshold current I_d.

We consider now the influence of a transient external perturbation of a total energy, Q_p, on a large composite superconductor in the cryostable regime. We suppose that this perturbation creates a normal nucleus. In case $I<I_d$, the superconducting state is stable with respect to such perturbations. For $I>I_d$, the superconducting state is metastable. This means that it is stable against perturbations with sufficiently small Q_p, so that the normal nucleus disapears after the perturbation is over. If the value of Q_p exceeds a certain critical value Q_{in} (the initiating energy), the final state is a state with traveling normal domains. In general, Q_{in} depends not only on the parameters of the superconductor and the coolant, but also on the time dependence of the perturbation and on its spatial distribution. An important particular case is when the length of the pulse is much shorter than the characteristic thermal length of the system and the duration of the pulse is much shorter than the thermal relaxation time of the system. In this case, the initiating energy depends only on the parameters of the composite and cooling conditions. A large number of experimental and theoretical (usually numerical) studies concerning normal zone initiation by localized pulses were carried out (see, e.g., Refs. 7–9). However, these studies considered the noncryostable regime, and calculated the minimum energy initiating the thermal quench of superconductivity Q_c. In particular, simple analytical formulae for the quench energy were obtained by Pasztor and Schmidt,[10] Dresner,[11] and Gurevich *et al.*[12]

In this article we consider the initiating energy for large composite superconductors. We treat the cryostable regime in case, when it is unstable against the perturbations resulting in traveling normal domains. The article is organized as follows: In Sec. II we review the effective circuit model,[5,6] which is used for the numerical simulations. We modify the equations such that they include an additional term representing external heating. In Sec. III we present the numerical results and show the dependence of Q_{in} on the transport current, the parameters of the conductor, and the cooling conditions. We suggest an explicit formula for Q_{in}, which we compare to numerical results. A brief summary is given in Sec. IV.

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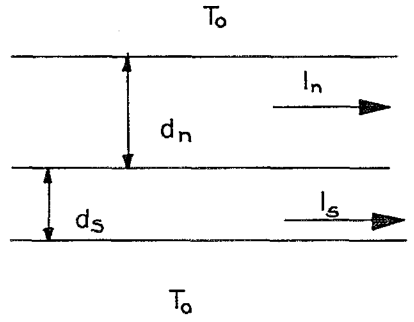

FIG. 1. The composite considered.

II. THE MAIN EQUATIONS

In this section, we review the effective circuit model.[5,6] This model describes the dynamics of the temperature and current density distributions in a composite superconductor in the presence of a normal zone.

Let us consider a rectangular conductor consisting of two ribbons of equal width, a superconductor of thickness, d_s, and a stabilizer (normal metal) of thickness, d_n (Fig. 1). The conductor carries transport current I, and is kept in thermal contact with a heat reservoir of temperature T_0.

In order to obtain the initiating energy $Q_{\rm in}$, the dynamics of the temperature and the current density distributions in the composite has to be considered. A complete treatment of this problem requires the solution of the heat diffusion equation for the temperature field coupled to the set of Maxwell equations for the current density distribution. These equations form a set of three-dimensional time dependent nonlinear equations, which is difficult for either analytical or numerical investigation. A one-dimensional model describing this process was proposed in.[5,6] This model takes into account the main physical features of the problem, and can be described by the electrical circuit sketched in Fig. 2. The upper chain of resistors represents the stabilizer, each resistor of resistance, $R_n = \rho_n \Delta x/d_n$, where ρ_n is the resistivity of the stabilizer and Δx is an arbitrary descretization length. Similarly, the lower chain of resistors represents the superconductor and each resistor of resistance, $R_s = \rho_s \Delta x/d_s$. Here, ρ_s is the resistivity of the superconductor, which vanishes in the superconducting phase, and is finite in the normal phase. Both chains are linked through a chain of resistors $R = \gamma_R \rho_n d_n/\Delta x$, where γ_R is a numerical factor of the order of one, depending on the geometry of the conductor. Finally, the inclusion of a characteristic time scale in the electric current diffusion process is accomplished by taking into account the inductance of the stabilizer (the inductance of the superconductor is neglected) $\mathscr{L} = \gamma_l \mu_0 d_n \Delta x$. Here, γ_l is another numerical factor. This model yields a set of two one-dimensional diffusion equations for the current density distribution in the superconductor $j_s(x,t)$ and for the temperature field $T(x,t)$

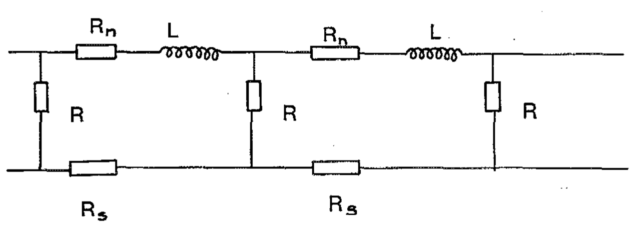

FIG. 2. Effective electrical circuit describing current distribution in the conductor.

$$\left(\frac{\mathscr{L} d_n}{\rho_n}\right)\frac{\partial j_s}{\partial t} = \gamma_R d_n^2 \frac{\partial^2 j_s}{\partial x^2} - j_s\left(1 + \frac{\rho_s d_n}{\rho_n d_s}\right) + j, \tag{2.1}$$

and

$$c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) - W(T) + Q(T) + Q_p(x,t), \tag{2.2}$$

where c is the heat capacity and k is the heat conductivity both taken to be constant. The parameter $j \equiv I/d_s$ is the current density in the superconductor far from a normal domain, where all the current flows through the superconductor. The function $W(T)$ is the rate of heat transfer to the coolant per unit volume, which can be written in the form $W(T) = h(T)(T-T_0)/d$, where $d \equiv d_s + d_n$. The function $Q(T)$ is the rate of joule heating per unit volume having three contributions: From the joule heating in the superconductor when it is in the normal state, from the current in the stabilizer, and from the perpendicular current. As a result (see, also, Fig. 2), $Q(T)$ is given by

$$Q(T) = \frac{1}{d}\left[d_s \rho_s j_s^2 + \frac{d_s^2 \rho_n}{d_n}(j - j_s)^2 + \gamma_R d_n d_s^2 \rho_n \left(\frac{\partial j_s}{\partial x}\right)^2\right]. \tag{2.3}$$

The function $Q_p(x,t)$ is the power of the external heating per unit volume. The total energy of the pulse is given by

$$Q_p = A\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dt\, Q_p(x,t), \tag{2.4}$$

where A is the cross-sectional area of the composite. For convenience, we use the following dimensionless variables; the temperature

$$\theta \equiv \frac{T - T_0}{T_c - T_0}, \tag{2.5}$$

and the current density in the superconductor

$$i_s \equiv \frac{j_s}{j_c}, \tag{2.6}$$

where T_c is the critical temperature of the superconductor. We define $L_{\rm th}$ the characteristic thermal length and $\tau_{\rm th}$ the characteristic thermal relaxation time,

$$L_{\rm th}^2 \equiv \frac{(d_n + d_s)k}{h}, \quad \tau_{\rm th} \equiv \frac{(d_n + d_s)c}{h}, \tag{2.7}$$

the characteristic length of the current redistribution L_m and the corresponding relaxation time τ_m

$$L_m^2 \equiv \gamma_R d_n^2, \quad \tau_m \equiv \frac{\mathscr{L} d_n}{\rho_n}. \tag{2.8}$$

We assume here the "step model" for the resistivity of the superconductor,[9]

$$\rho_s(j_s,T)=\rho_s\eta[j_s-j_c(T)], \tag{2.9}$$

where η is the Heaviside step function ($\eta=0$ if $x<0$ and $\eta=1$ if $x>0$), and $j_c(T)$ is the critical current density in the superconductor given by

$$j_c(T)=j_c\left(1-\frac{(T-T_0)}{(T_c-T_0)}\right)=j_c(1-\theta). \tag{2.10}$$

We treat perturbations with length $L_q \ll L_{\text{th}}$ and duration $\tau_q \ll \tau_{\text{th}}$. In this case, the function $Q_p(x,t)$ is proportional to a product of two delta functions:

$$Q_p(x,t)=\frac{Q_p}{A}\,\delta(x)\delta(t), \tag{2.11}$$

where Q_p is the total energy of the pulse.

Finally, we introduce three dimensionless parameters

$$\xi\equiv\frac{\rho_s d_n}{\rho_n d_s},\quad \alpha\equiv\frac{d_s^2\rho_n j_c^2}{d_n h(T_c-T_0)},\quad q_p\equiv\frac{Q_p}{Q_h}, \tag{2.12}$$

where ξ is the ratio of the resistances of the superconductor and the stabilizer per unit length, α is the ratio of characteristic rates of joule heating and heat flux to the coolant (Stekly parameter), and q_p is the dimensionless total energy of the pulse, where

$$Q_h\equiv cAL_{\text{th}}(T_c-T_0).$$

Equations (2.1) and (2.2) in the dimensionless form are given by

$$\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial x^2}-\theta+\alpha(i-i_s)^2+\xi\alpha i_s^2\eta(i_s+\theta-1)+\alpha\lambda^2\left(\frac{\partial i_s}{\partial x}\right)^2$$
$$+q_p\delta(z)\delta(t), \tag{2.13}$$

$$\tau\frac{\partial i_s}{\partial t}=\lambda^2\frac{\partial^2 i_s}{\partial x^2}-[1+\xi\eta(i_s+\theta-1)]i_s+i, \tag{2.14}$$

where time is measured in units of τ_{th} and length in units of L_{th}, the dimensionless parameters i, τ, and λ are defined by

$$i\equiv\frac{j}{j_c},\quad \tau\equiv\frac{\tau_m}{\tau_{\text{th}}},\quad \lambda\equiv\frac{L_m}{L_{\text{th}}}. \tag{2.15}$$

III. RESULTS AND DISCUSSION

The value of the dimensionless initiating energy $q_{\text{in}}=Q_{\text{in}}/Q_h$ was obtained for a given set of parameters i, α, ξ, τ, and λ by means of numerical simulations of Eqs. (2.13) and (2.14) for different values of the energy q_p. The initial conditions were taken as follows:

$$\theta(x,0)=0,\quad i_s(x,0)=i.$$

The large time behavior of the system determines whether $q_p<q_{\text{in}}$ or $q_p>q_{\text{in}}$. Namely, for $q_p<q_{\text{in}}$, the system tends back to the initial superconducting state, whereas for $q_p>q_{\text{in}}$ a pair of traveling normal domains propagate in opposite directions along the system. The values of the parameters were taken from refs. 1 and 4. Typical values of ξ,

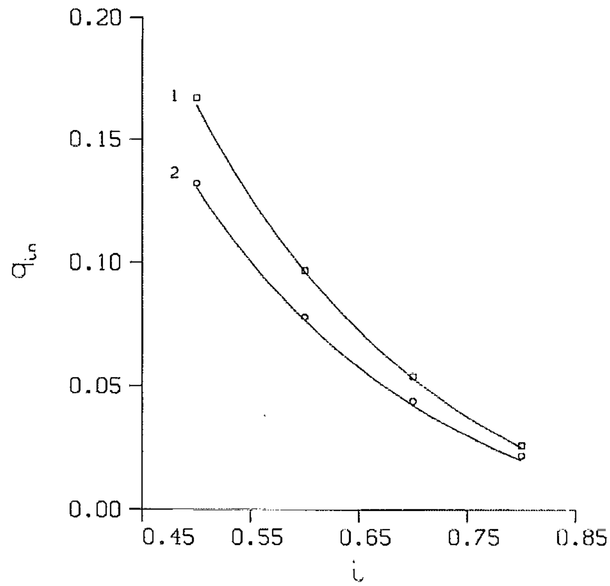

FIG. 3. Dimensionless initiating energy q_{in} as a function of current. Solid lines represent the values calculated by the Eq. (3.8). Points represent the results of numerical simulations. 1. $\alpha=0.9$, $\xi=120$, $\tau=90$. 2. $\alpha=0.9$, $\xi=190$, $\tau=90$.

τ, and λ can be then estimated as $\xi=100-200$, $\tau=10-100$, and $\lambda=0.1-1.0$. Specifically, as we are interested in cryostable conductors, the case $\alpha<1$ is considered.

We plot the initiating energy, q_{in}, as a function of i, τ, ξ, and α in Figs. 3–6. Note, that the dependence of q_{in} on λ was found to be practically negligible. In Fig. 3, the initiating energy q_{in} is plotted as a function of the dimensionless transport current i. It can be shown that for i approaching 1, the initiating energy q_{in} tends to zero proportionally to $(1-i)^{3/2}$. Note that the value of q_{in} does not depend on any of the parameters ξ, τ, and α when $i\to 1$. The dependence of the initiating energy q_{in} on τ is shown in Figs. 4(a) and 4(b). In the range $\tau<30$, q_{in} is a sharply decreasing function of τ. Above this range it varies relatively slowly. We present the dependence of q_{in} on ξ in Fig. 5. This dependence can be shown to be approximately proportional to $\xi^{-1/2}$. Finally, the dependence of q_{in} on α is shown in Fig. 6.

Let us now estimate the value of the initiating energy q_{in} from the following qualitative considerations. When a part of the superconductor undergoes a normal transition, the current is confined in the superconductor during a time interval of the order of τ_m/ξ. During this interval, the superconductor in the vicinity of the transition front is unstabilized as the value of i is higher than the minimum propagation current for the superconductor itself. The normal zone boundary propagates with a certain velocity v. Thus, a region with the length of the order of $v\tau_m/\xi$, in front of the normal domain, becomes temporary unstabilized. The effective Stekly parameter α_{eff} associated with this unstabilized superconductor is determined by the ratio of the characteristic rate of joule heating and characteristic heat flux to the coolant in this area, and is equal to $\alpha_{\text{eff}}=\alpha\xi\gg 1$.[5,6]

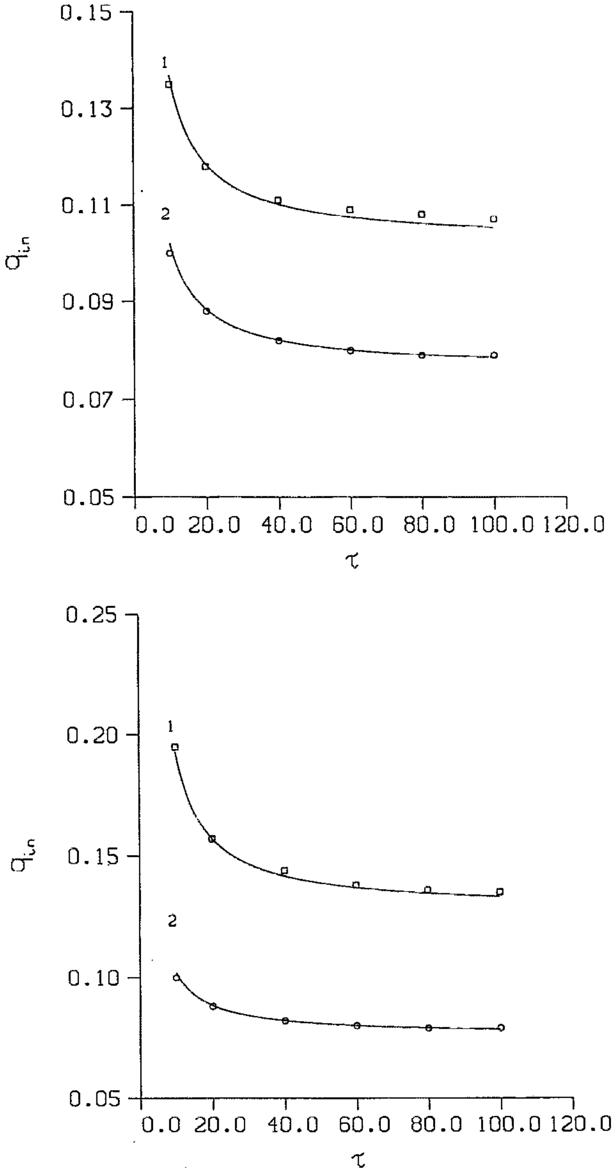

FIG. 4. Dimensionless initiating energy as a function of τ. (a) 1. $i=0.6$, $\alpha=0.9$, $\xi=100$. 2. $i=0.6$, $\alpha=0.9$, $\xi=180$. (b) 1. $i=0.5$, $\alpha=0.9$, $\xi=180$. 2. $i=0.6$, $\alpha=0.9$, $\xi=180$.

To initiate a propagating normal zone by a heat pulse, it is necessary to heat up to a temperature of the order of $T_c(i)$ a region with a certain length l_{in}. In case the current in the superconductor is constant, the value of $l_{\text{in}}=l_c$ can be estimated from the heat balance equation and for large α_{eff} it is equal to[9]

$$l_c \approx L_{\text{th}} \frac{\sqrt{1-i}}{i} \frac{1}{\sqrt{\alpha\xi}}. \tag{3.1}$$

The value of initiating energy in that case $q_{\text{in}}=q_c$ is equal to[9]

$$q_c \approx 2.3 \frac{(1-i)^{3/2}}{i\sqrt{\alpha\xi}}. \tag{3.2}$$

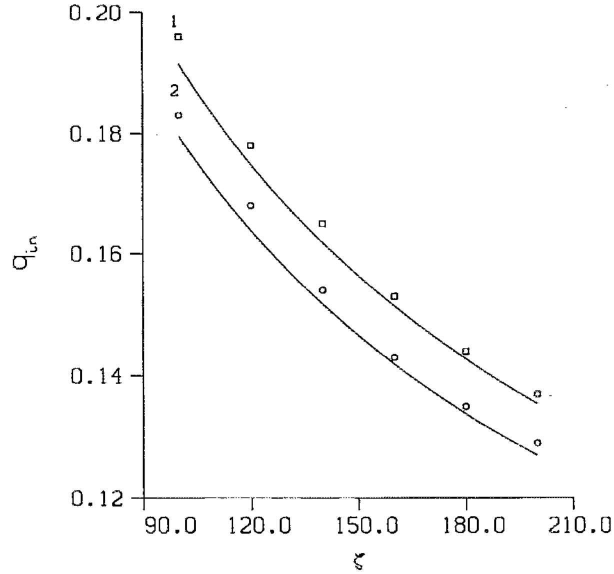

FIG. 5. Dimensionless initiating energy as a function of ξ. 1. $i=0.5$, $\alpha=0.8$, $\tau=90$. 2. $i=0.5$, $\alpha=0.9$, $\tau=90$.

At the same time, the length of the unstabilized segment in front of the traveling normal domain l_d can be estimated as

$$l_d \approx v\frac{\tau_m}{\xi} = v t_{\text{th}} \frac{\tau}{\xi}. \tag{3.3}$$

For $\alpha_{\text{eff}} \gg 1$, the velocity v of the normal zone boundary propagation is equal to[5,6]

$$v \approx \frac{L_{\text{th}}}{\tau_{\text{th}}} \sqrt{\alpha\xi} \frac{i}{\sqrt{1-i}}. \tag{3.4}$$

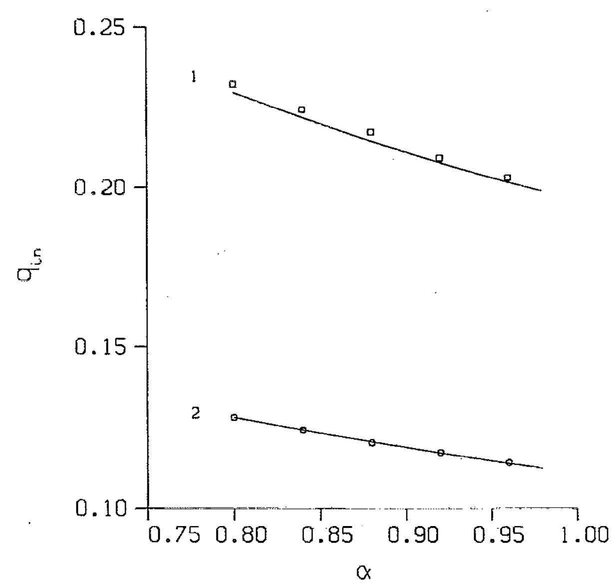

FIG. 6. Dimensionless initiating energy as a function of α. 1. $i=0.5$, $\xi=100$, $\tau=20$. 2. $i=0.6$, $\xi=100$, $\tau=20$.

Substituting Eq. (3.4) in Eq. (3.3), we find that the value of l_d can be estimated as

$$l_d \approx L_{\rm th}\tau\sqrt{\frac{\alpha}{\xi}}\frac{i}{\sqrt{1-i}}. \tag{3.5}$$

In case, when $l_c \ll l_d$, which corresponds to $\tau \gg 1$, formula (3.2) is a good approximation for the initiating energy $q_{\rm in}$. To estimate $q_{\rm in}$ for a wider range of τ, we have to take into account that the current is not constant in the normal domain due to the redistribution into stabilizer. To do it, we introduce the effective current $i_{\rm eff}$, which is a function of the ratio l_c/l_d and $i_{\rm eff} \to i$, when $l_c \ll l_d$. In the first (linear) approximation, we obtain

$$i_{\rm eff} = i\left(1-\gamma\frac{l_c}{l_d}\right), \tag{3.6}$$

where γ is a numerical factor of the order of one. Substituting Eqs. (3.1) and (3.5) in Eq. (3.6), we obtain

$$i_{\rm eff} = i\left(1-\gamma\frac{1-i}{\tau\alpha i^2}\right). \tag{3.7}$$

Substituting $i_{\rm eff}$ in Eq. (3.2) instead of i, we find the following expression for initiating energy

$$q_{\rm in} \approx \frac{2.3}{\sqrt{\tau\xi}}\frac{(1-i)^{3/2}(\alpha\tau i^2+0.75)^{3/2}}{\alpha i^2[\alpha\tau i^2-0.75(1-i)]}, \tag{3.8}$$

where we obtain the value $\gamma=0.75$ by best fitting to the numerical data. Equation (3.8) approximates the results of our numerical simulations with a maximum deviation less than 4% for the values of parameters $100<\xi<200$, $40<\tau<100$, $0.8<\alpha<1.0$, and the transport current $0.5<i<0.85$. The initiating energy $q_{\rm in}$, calculated by means of Eq. (3.7), is presented by solid lines in Figs. 3–6.

IV. SUMMARY

The initiating energy for traveling normal domains is obtained for a large composite superconductor as a function of transport current and for dimensionless parameters characterizing the composite and cooling conditions. An effective circuit model is used for numerical simulations. An analytical expression for initiating energy is suggested. The initiating energy obtained by means of this expression is in good agreement with the results of numerical calculations.

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