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Surface-barrier and magnetization relaxation in layered superconductors

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The energy of a pointlike vortex is calculated for a layered superconductor with very weak interlayer Josephson coupling. An energy barrier existing near the sample surface is found. Magnetization relaxation due to thermally activated penetration of pointlike vortices is considered. An initial avalanche-type decay of magnetization is predicted.

I. INTRODUCTION

Magnetization relaxation measurements are an effective method to investigate the flux dynamics phenomena, current-voltage characteristics, and critical current in superconductors.[1-3] In type-II superconductors magnetization relaxation is determined by the vortices penetration, flow, and pinning. The flux penetration process begins when the external magnetic field H becomes higher than a certain edge field H^*. In the case of a cylinder subjected to a parallel field the value of H^* is bigger than the lower critical field H_{c1} and smaller than the thermodynamical critical field H_c.[4] The difference between H^* and H_{c1} depends on the interaction of the vortices with the pinning centers and the sample surface. In continuous superconductors the attraction of the Abrikosov vortices to the sample surface results in the Bean-Livingston barrier.[5] This barrier prevents penetration of the Abrikosov vortices inside the sample. The value of H^* determined by the Bean-Livingston barrier depends on the roughness of the surface. It is maximal in the case of an ideal flat surface. In the absence of the pinning centers in the bulk the edge field H^* is equal to the thermodynamical critical field H_c.[4]

Logarithmic relaxation of the magnetization M is one of the main results of the well-known Anderson flux-creep model.[6] A nonlogarithmic magnetization relaxation was considered in the vortex-glass[7] and collective creep (pinning) (Ref. 8) models. A considerable deviation from the logarithmic relaxation law is observed in a lot of experimental studies for the high-temperature superconductors.[9-16] In particular, a pronounced maximum on the curve of the magnetization relaxation rate is found in Ref. 11. Experimental study[15] presented the evidence that the Bean-Livingston barrier is the origin of the irreversibility observed in high-quality $YBa_2Cu_3O_7$ single crystals near the critical temperature T_c. It is emphasized in Ref. 15 that the Bean-Livingston barrier is especially effective for the high-temperature superconductors. A crossover from the bulk pinning to the surface pinning is observed and discussed in detail in Ref. 16. The considerable deviation from the logarithmic decay of the magnetization is one of the consequences of the specific quasi-two-dimensional nature of superconductivity in the high-temperature superconductors.

The most prominent high-temperature superconductors and, in particular, the Bi- and Tl-based compounds, consist of a periodic stack of the two-dimensional CuO layers (*ab* planes) where the superconductivity presumably resides. These materials are extremely anisotropic and, in particular, the density of the superconducting current in the direction perpendicular to the layers (*c* direction) is much less than in the *ab* planes. The discovery of the layered anisotropic high-T_c superconductors stimulated many theoretical studies of layered superconductors with weak interlayer Josephson coupling. In particular, the specific pointlike (or pancake) vortices were introduced and investigated.[8,17,18] Each of these pointlike vortices is residing only in one of the superconducting layers. The self-energy of an isolated pointlike vortex is proportional to $\ln(L/\xi)$, where L is the characteristic size of the sample in the *ab* plane and $\xi=\xi_{ab}$ is the coherence length in the *ab* plane. Thus, the self-energy of an isolated pointlike vortex diverges when $L/\xi\rightarrow\infty$ and it cannot exist in the bulk of a macroscopic sample.

Interaction of a pointlike vortex with the sample surface as well as for the Abrikosov vortex consists of repulsion and attraction. The repulsion results from the interaction with the Meissner screening current. The attraction results from the increase of the superconducting current density of the pointlike vortex caused by the sample surface. The correlation between these two interactions is determined by the external magnetic field H. At a certain value of $H=H_1$ the competition of attraction and repulsion can lead to a stable state localized near the sample surface. The existence of this stable state affects the flux penetration process and, in particular, the magnetization relaxation.

In this paper we study the process of the pointlike vortices penetration into a layered superconductor. We show that this process results in a specific thermally activated mechanism of magnetization relaxation, if the external magnetic field H is from the interval $H_{c1}<H<H^*$. We use for calculations the Lawrence-Doniach model[19] in the limit of very weak interlayer Josephson coupling. We consider the case characteristic for the extremely anisotropic high-temperature superconductors, i.e., $\xi \ll \lambda$, $d \ll \lambda$, $\xi_c \ll d$, where $\lambda = \lambda_{ab}$ is the London penetration depth in the ab plane, d is the distance between the layers, and ξ_c is the coherence length in the c direction.

We consider here, as an illustration, the magnetization relaxation for the following problem. A semi-infinite layered superconductor is cooled down to a certain temperature T below the critical temperature in a zero magnetic field. Then, a magnetic field $\mathbf{H}$ parallel to the sample surface and perpendicular to the superconducting layers is instantaneously turned on and kept constant. In this case the outline of the scenario of thermally activated magnetization relaxation is as follows. The energy of a single pointlike vortex G_v has a minimum G_m, which is detached by an energy barrier G_g from the surface. When the external magnetic field H exceeds a certain value $H_1 \approx H_{c1}$, the minimum energy G_m becomes negative. It results in a thermally activated penetration of the pointlike vortices inside the sample, where they reside in the vicinity of the energy minimum. The rate of the pointlike vortices penetration inside the sample depends exponentially on the ratio of $G_g/k_B T$, where k_B is the Boltzmann constant. The contribution of the pointlike vortices to the magnetization δM is proportional to the number of them and leads to the decay of the magnetization. The interaction of the incoming pointlike vortex with the pointlike vortices residing in the sample affects the value of G_g. The shift of the energy barrier δG_g due to this interaction is negative and proportional to the number of the pointlike vortices in the sample. As a result, the magnetization relaxation rate increases with the decrease of the magnetization and it leads to the avalanche-type initial magnetization relaxation.

The paper is organized according to the outline of the scenario of thermally activated magnetization relaxation. In Sec. II, we calculate the energy of a single pointlike vortex G_v near the surface of a semi-infinite layered superconductor. In Sec. III, we consider the contribution δM to the magnetization resulting from the pointlike vortices residing in the vicinity of the energy minimum G_m. In Sec. IV, we study the diffusion of the pointlike vortices nucleating near the sample surface and we calculate the thermally activated flow of these vortices inside the bulk. In Sec. V, we consider the dependence of the shift of the energy barrier δG_g on the density of the pointlike vortices residing in the sample. In Sec. VI, we combine the results obtained in the previous sections, derive and solve the magnetization relaxation equation. In Sec. VII, we summarize the overall conclusions.

II. POINTLINE VORTEX ENERGY NEAR THE SAMPLE SURFACE

We calculate first the energy G_v of a single pointlike vortex residing in one of the superconducting layers near the sample surface. We use for calculations the Lawrence-Doniach model[19] in the limit of very weak interlayer Josephson coupling, i.e., we neglect the superconducting current in the c direction. In this case, the superconducting layers are coupled only electromagnetically via the magnetic field existing between the layers. This approach is valid, when the space scale of the phenomena under consideration is less than the Josephson length $\lambda_J = d\xi_{ab}/\xi_c$. We take into account only the electromagnetic coupling between the superconducting layers. It means that the space scale of the phenomena under consideration is determined by the penetration depth λ. Thus we assume that $\lambda < \lambda_J$. This assumption is valid, for example, for the Bi- and Tl-based high-temperature superconductors.

Consider a semi-infinite layered superconductor subjected to a magnetic field $\mathbf{H}$ parallel to the surface and perpendicular to the layers. Suppose that the z axis is parallel to $\mathbf{H}$ and the x axis is perpendicular to the surface. The free-energy functional F for an infinite set of electromagnetically coupled parallel superconducting layers then has the form[19,20]

$$F = \frac{d}{2\pi}\left[\frac{\Phi_0}{4\pi\lambda}\right]^2 \sum_n \int d^2 r \left[\nabla\phi_n - \frac{2e}{c}\mathbf{A}\right]^2 + \frac{1}{8\pi}\int d^3 r (\mathbf{H}-\mathbf{B})^2 , \tag{2.1}$$

where n is the number of the layer, ϕ_n is the phase of the order parameter in the nth layer, $\mathbf{A}=(A_x, A_y)$ is the vector potential, $\mathbf{B}=\nabla\times\mathbf{A}$ is the magnetic field, and $\nabla=(\nabla_x,\nabla_y)$ is the gradient in the layers plane.

Variation of Eq. (2.1) with respect to the vector potential $\mathbf{A}$ results in the generalized London equation. Inside the superconductor it has the following form:[19,20]

$$\nabla^2 \mathbf{A} = \frac{d}{\lambda^2}\sum_n \delta(z-nd)\left[\mathbf{A} - \frac{\Phi_0}{2\pi}\nabla\phi_n\right] . \tag{2.2}$$

Solving Eq. (2.2) we can find the magnetic field $\mathbf{B}$ and energy G for any configuration of vortices residing in the sample.

Let us now consider a single pointlike vortex near the sample surface. In this case the magnetic field $\mathbf{B}(\mathbf{r})$ may be written as

$$\mathbf{B}(r) = \mathbf{H}\exp\left[-\frac{x}{\lambda}\right] + \mathbf{b}(\mathbf{r}) . \tag{2.3}$$

Here the first term represents the magnetic field penetration into the sample in the absence of vortices. The decay of this field is due to the Meissner screening current. The magnetic field $\mathbf{b}(\mathbf{r})$ results from the pointlike vortex, and can be calculated by means of Eq. (2.2) and the method of images. It means that, to the pointlike vortex located at (x,y), we add an image pointlike antivortex located at $(-x,y)$ and take for $\mathbf{b}(\mathbf{r})$ the sum of the field induced by the vortex and antivortex. The field $\mathbf{b}(\mathbf{r})$ automatically vanishes on the sample surface and the boundary condition $\mathbf{B}(0,y,z)=\mathbf{H}$ is satisfied. Using Eqs. (2.1)–(2.3) we calculate the energy of a single pointlike vortex. Finally, the dependence of G_v on x has the form

$$G_v=\left[\frac{\Phi_0}{4\pi\lambda}\right]^2 d\ln\left[\frac{2x}{\xi}\right]+\frac{\Phi_0}{4\pi}Hd\left[\exp\left[-\frac{x}{\lambda}\right]-1\right],\quad x\geq\xi\ . \tag{2.4}$$

The first term in Eq. (2.4) represents the attraction between the pointlike vortex and its image. It is minimal near the surface and increases monotonically with increase of x. The second term in Eq. (2.4) represents the repulsion of the pointlike vortex from the surface due to the external magnetic field and the associated screening current. It is maximal at $x=0$ and decreases monotonically with the increase of x. The dependence $G_v(x)$ is shown in Fig. 1 for different values of the magnetic field H. The function $G_v(x)$ increases monotonically if $H<H_0$, where

$$H_0=e\frac{\Phi_0}{4\pi\lambda^2}\ . \tag{2.5}$$

For $H>H_0$ the curve $G_v(x)$ has a maximum G_g at $x=x_g$ and a minimum G_m at $x=x_m$. The explicit formulas for G_g, x_g, G_m, and x_m can be derived by means of Eq. (2.4) in the case when $H\gg H_0$ and $\ln(\lambda/\xi)\gg 1$:

$$G_g\approx\left[\frac{\Phi_0}{4\pi\lambda}\right]^2 d\ln\left[\frac{\Phi_0}{2\pi}\frac{1}{\lambda\xi H}\right], \tag{2.6}$$

$$x_g\approx\frac{\Phi_0}{4\pi\lambda}\frac{1}{H}<\lambda\ , \tag{2.7}$$

$$G_m\approx\left[\frac{\Phi_0}{4\pi\lambda}\right]^2 d\ln\left[2e\frac{\lambda}{\xi}\ln\left[\frac{4\pi\lambda^2}{\Phi_0}H\right]\right]-\frac{\Phi_0}{4\pi}dH\ , \tag{2.8}$$

$$x_m\approx\lambda\ln\left[\frac{4\pi\lambda^2}{\Phi_0}H\right]>\lambda\ . \tag{2.9}$$

It follows from Eq. (2.8) that the value of the minimum energy G_m becomes negative when $H>H_1$, where

$$H_1\approx\frac{\Phi_0}{4\pi\lambda^2}\ln\left[2e\frac{\lambda}{\xi}\ln\ln\left[\frac{\lambda}{\xi}\right]\right]\ . \tag{2.10}$$

Note that the magnetic field H_1 is of the same order, but higher than the lower critical field H_{c1}.

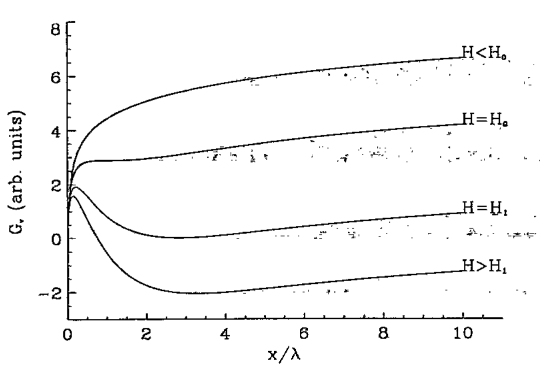

FIG. 1. Dependence of the pointlike vortex energy G_v on x for different values of the external magnetic field H.

Let us estimate the characteristic value of G_g. To do it we use the data obtained for a monocrystal $Bi_2Sr_2CaCu_2O_8$:[21] $\lambda\approx 3\times 10^{-5}$ cm, $\xi=\xi_{ab}\approx 1.5\times 10^{-7}$ cm, $d\approx 1.5\times 10^{-7}$ cm. Using Eq. (2.6) we find that, for $T\ll T_c$, the value of H_1 is equal to $H_1=0.013$ T, and

$$\frac{G_g}{k_BT}=\frac{300}{T}\ln\left[\frac{0.7}{H}\right], \tag{2.11}$$

where the temperature is given in K, and the magnetic field H is given in T. It follows from Eq. (2.11) that for $T\ll T_c$ the energy barrier G_g is much higher than k_BT.

Thus, if the magnetic field $H>H_1$, the minimum energy of the pointlike vortex G_m is negative, and the energy barrier G_g is finite. At nonzero temperature it results in the thermally activated penetration of the pointlike vortices into the sample. This mechanism of magnetization relaxation is a new mechanism which is specific for the layered superconductors.

The pointlike vortices penetrated into the sample reside in the vicinity of the energy minimum. The presence of these vortices affects the magnetization leading to its decay.

III. MAGNETIZATION RESULTING FROM POINTLIKE VORTICES

We now calculate the magnetization δM resulting from the pointlike vortices residing in the superconducting layers in the vicinity of the plane x_m, i.e., in the vicinity of the energy minimum G_m. These pointlike vortices induce the surface current I, which is the same in each of the layers. The magnetization δM is then given by the formula

$$\delta M=\frac{I}{cd}\ . \tag{3.1}$$

We calculate the surface current I in order to find δM. To do it, let us suppose that in the nth superconducting layer the pointlike vortices reside on the line $x=x_m$, $z=nd$ with the average linear density N. Note that the value of N is the same for each of the layers. We denote the y coordinate of the kth pointlike vortex residing in the nth layer as

$$y_{kn}=kl+u_{kn}\ . \tag{3.2}$$

Here $l=N^{-1}$ is the average distance between the pointlike vortices in the layers plane. We suppose that the values u_{kn} are distributed randomly with the distribution function $f(u)$ and

$$\int f(u)du=1\ ,\quad \int uf(u)du=0\ ,\quad \int u^2f(u)du\approx l^2\ . \tag{3.3}$$

We use the formula for the Lorentz force F_L acting on a pointlike vortex to derive the expression for the surface current I. As the value of I is the same in each of the layers, we calculate I for the layer with the number $n=0$. The Lorentz force is given by the formula[4]

$$F_L=\frac{\Phi_0}{c}J\ , \tag{3.4}$$

where J is the surface current density in the layer with $n=0$ at the position of the pointlike vortex, and

$$I=\int_0^\infty J\,dx \ . \tag{3.5}$$

Note that as well as I the value of J is the same in each of the layers.

We use the method of images to find the surface current density J. It follows from this method that inside the sample J is a sum of the current densities induced by the pointlike vortices $J_{nk}^{(v)}$ and their images (antivortices) $J_{nk}^{(a)}$, i.e.,

$$J=\sum_n\sum_k [J_{nk}^{(v)}+J_{nk}^{(a)}] \ . \tag{3.6}$$

Combination of Eqs. (3.6) and (3.4) results in the formula determining the value of J as

$$J=\frac{c}{\Phi_0}\sum_n\sum_k [F_{Lnk}^{(v)}+F_{Lnk}^{(a)}] \ , \tag{3.7}$$

where $F_{Lnk}^{(v)}$ is the Lorentz force resulting from the kth vortex in the nth layer, and $F_{Lnk}^{(a)}$ is the Lorentz force resulting from the kth antivortex in the nth layer. Integration of both sides of Eq. (3.7) over x from $x=0$ to ∞ leads to an expression determining the value of I:

$$I=\frac{c}{\Phi_0}\sum_n\sum_k [G_{kn}^{(v)}(\infty)+G_{kn}^{(a)}(\infty)] \ . \tag{3.8}$$

Here $G_{kn}^{(v)}(\infty)$ is the energy of interaction between the pointlike vortex residing in the layer $n=0$ at $x\to\infty$ and the kth pointlike vortex residing in the nth layer, and $G_{kn}^{(a)}(\infty)$ the energy of interaction between the pointlike vortex residing in the layer $n=0$ at $x\to\infty$ and the kth pointlike antivortex residing in the nth layer. It is convenient to rewrite Eq. (3.8) in the following way:

$$I=\frac{c}{\Phi_0}[G_1+G_2] \ , \tag{3.9}$$

where

$$G_1=\sum_k [G_{k0}^{(v)}(\infty)+G_{k0}^{(a)}(\infty)] \tag{3.10}$$

is the energy of interaction with the pointlike vortices and antivortices residing in the layer with $n=0$, and

$$G_2=\sum_{n\neq 0}\sum_k [G_{kn}^{(v)}(\infty)+G_{kn}^{(a)}(\infty)] \tag{3.11}$$

is the energy of interaction with the pointlike vortices and antivortices residing in the layers with $n\neq 0$.

We calculate the sums given by Eqs. (3.10) and (3.11) using the following formulas:[8]

$$G_p=-2qd\left[\frac{\Phi_0}{4\pi\lambda}\right]^2\ln\left[\frac{\rho}{\xi}\right] \ , \quad z=0 \ , \quad \xi\ll\rho \ , \tag{3.12}$$

$$G_p=q\frac{d^2}{2\lambda}\left[\frac{\Phi_0}{4\pi\lambda}\right]^2\exp\left[-\frac{|z|}{\lambda}\right]\ln\left[\frac{\rho}{\xi}\right] \ , \quad z\neq 0 \ , \ \lambda\ll\rho \ . \tag{3.13}$$

Here G_p is the energy of the interaction of two pointlike vortices when $q=1$, and G_p is the energy of the interaction of a pointlike vortex and pointlike antivortex when $q=-1$:

$$\rho=[(x_1-x_2)^2+(y_1-y_2)^2]^{1/2} \ , \quad z=z_1-z_2 \ , \tag{3.14}$$

(x_1,y_1,z_1) and (x_2,y_2,z_2) are the coordinates of the interacting pointlike vortices (antivortices).

We calculate first the value of $G_1(x)$ for x_m, $l\ll x$. According to the method of images for each pointlike vortex with coordinates (x_m,y_{kn}) there is a pointlike antivortex with coordinates $(-x_m,y_{kn})$. Finally, it leads to the following expression for $G_1(x)$:

$$G_1(x)=\left[\frac{\Phi_0}{4\pi\lambda}\right]^2 d\sum_k \ln\left[\frac{(x+x_m)^2+(y-y_{k0})^2}{(x-x_m)^2+(y-y_{k0})^2}\right] \ . \tag{3.15}$$

For x_m, $l\ll x$, Eq. (3.15) reads

$$G_1(x)=\left[\frac{\Phi_0}{2\pi\lambda}\right]^2 dx_m\sum_k\frac{x}{x^2+y_{k0}^2} \ . \tag{3.16}$$

In the limit $x\to\infty$ the main contribution in the sum in Eq. (3.16) comes from the interval, where $k\gg 1$. It allows one to substitute the sum in Eq. (3.16) by the integral. Finally, we obtain the value of $G_1(x)$ in the form

$$G_1=\left[\frac{\Phi_0}{2\pi\lambda}\right]^2 dNx_m\int_{-\infty}^{\infty}\frac{xN}{x^2N^2+(k+Nu_{k0})^2}dk \ . \tag{3.17}$$

It follows from Eq. (3.17) that $G_1=G_1(\infty)$ is given by the formula

$$G_1=\left[\frac{\Phi_0}{2\pi\lambda}\right]^2 dNx_m \ . \tag{3.18}$$

An analogous calculation for G_2 results in

$$G_2=-\tfrac{1}{2}G_1 \ . \tag{3.19}$$

We now combine Eqs. (2.9), (3.1), (3.9), (3.18), and (3.19) to calculate the magnetization δM resulting from the pointlike vortices residing in the sample. Finally, we find

$$\delta M=\frac{\Phi_0}{8\pi\lambda}N\ln\left[H\frac{4\pi\lambda^2}{\Phi_0}\right] \ . \tag{3.20}$$

Note that Eq. (3.20) does not depend on the distribution function $f(u)$. The accuracy of Eq. (3.20) is of the order of $l/L\ll 1$, where L is the characteristic size of the sample in the superconducting layers plane. It means that Eq. (3.20) is an exact formula for any macroscopic sample.

Thus it follows from Eq. (3.20) that in order to find the magnetization relaxation equation we have to know the rate of N. To calculate it we treat the diffusion of the pointlike vortices in the stripe $0<x<x_g$.

IV. DIFFUSION EQUATION FOR POINTLIKE VORTICES

We consider here, as an illustration, the magnetization relaxation for the following problem. A semi-infinite layered superconductor is cooled down to a certain temperature T below the critical temperature in a zero magnetic field. Then, a magnetic field $\mathbf{H}$ parallel to the sample surface is instantaneously turned on. This process results in the initial magnetization $\mathbf{M}_0$, which is equal to

$$\mathbf{M}_0=-\frac{1}{4\pi}\mathbf{H} . \tag{4.1}$$

We suppose also that the superconducting layers are perpendicular to $\mathbf{H}$ and the field is from the interval $H_1<H<H^*$. In this case there is only one mechanism of the magnetization $\mathbf{M}$ relaxation, i.e., the thermally activated penetration of the pointlike vortices into the sample. It follows from Eq. (3.20) that the rate of this relaxation process is determined by the rate of N, which is equal to the pointlike vortices flow j inside each of the superconducting layers:

$$\frac{dN}{dt}=j . \tag{4.2}$$

We consider now the diffusion of the pointlike vortices in the stripe $0<x<x_g$ to find the value of the flow j. The magnetic flux penetrates into the bulk through the sample surface. It means that the pointlike vortices nucleate in the superconducting layers at $x=0$. We consider here the case when the energy of a single pointlike vortex $G_v(0)\approx G_v(\xi)\gg k_BT$. At the same time the minimum of G_v is localized on the line $x=x_m$ in each of the superconducting layers. If the external magnetic field H is from the interval $H>H_1$, the value of G_m is negative. Thus, for $H>H_1$ a flow of the pointlike vortices towards $x=x_m$ exists in each of the superconducting layers. This flow is determined by the probability p of the thermally activated penetration of the pointlike vortices in the region $x\approx x_m$. The value of p depends exponentially on the ratio of the energy barrier G_g and k_BT and in the case when $G_g\gg k_BT$ we have

$$p\propto\exp\left[-\frac{G_g}{k_BT}\right]\ll 1 . \tag{4.3}$$

This strong inequality allows one to determine j by means of a quasistationary diffusion equation. It means that the pointlike vortices flow j has the form

$$j=-D\frac{\partial n}{\partial x}-\mu n\frac{\partial G_v}{\partial x} . \tag{4.4}$$

Here $n(x)$ is the density of the pointlike vortices in the superconducting layers, D is the diffusion coefficient, and μ is the mobility of the pointlike vortices. The connection between D and μ is given by the Einstein relation, and thus $D=k_BT_\mu$. We use here the value of μ calculated in the viscous flux-flow model.[22] It follows from this model that for the pointlike vortices flow the mobility is given by the formula

$$\mu=\gamma_\mu\frac{c^2\rho_n\xi^2}{\Phi_0^2 d} , \tag{4.5}$$

where γ_μ is a numerical factor of the order of 1, and ρ_n is the resistivity in the normal state.

The inequality $k_BT\ll G_g$ results in zero pointlike vortices density at $x=x_g$. The solution of Eq. (4.4) matching this boundary condition has the form:

$$n(x)=\frac{j}{D}\exp\left[-\frac{G_v(x)}{k_BT}\right]\int_x^{x_g}\exp\left[\frac{G_v(x')}{k_BT}\right]dx' . \tag{4.6}$$

The inequality $G_v(0)\approx G_v(\xi)\gg k_BT$ allows one to estimate the value of $n(0)$ as

$$n(0)=\frac{\gamma_n}{\xi^2}\exp\left[-\frac{G_v(0)}{k_BT}\right] , \tag{4.7}$$

where γ_n is a numerical factor of the order of 1. Using Eq. (4.6) and the boundary condition (4.7) we find the pointlike vortices flow j and the rate of N in the form

$$j=\frac{dN}{dt}=\gamma_j\left[\frac{k_BT}{d}\right]^{1/2}\left[\frac{4\pi\lambda}{\Phi_0}\right]^2\frac{M_0}{\tau_0}\times\exp\left[-\frac{G_g}{k_BT}\right] . \tag{4.8}$$

Here γ_j is a numerical factor of the order of 1, M_0 is the absolute value of the initial magnetization

$$M_0=\frac{1}{4\pi}H , \tag{4.9}$$

and

$$\tau_0=\frac{\lambda^2}{\rho_n c^2} \tag{4.10}$$

is the characteristic time constant appearing in the theory of nonequilibrium superconductivity.[22]

The combination of Eqs. (3.20) and (4.8) determines the magnetization relaxation due to the thermally activated penetration of the pointlike vortices inside the sample. To find the dependence $N(t)$ and thus δM, we have to take into account that the interaction of the incoming pointlike vortex with the pointlike vortices residing in the sample results in a shift of the energy barrier δG_g. Thus to find the final form of the equation determining the magnetization relaxation we have to calculate δG_g as a function of δM.

V. ENERGY BARRIER SHIFT FOR POINTLIKE VORTICES

The interaction of the incoming pointlike vortex with the pointlike vortices residing in the vicinity of the energy minimum at $x\approx x_m$ determines the value of δG_g. The calculation of δG_g is similar to the calculation of δM (see Sec. III). The interaction of the incoming pointlike vortex with the pointlike vortices and antivortices residing in the same superconducting layer increases the energy barrier by a certain amount δG_1. The interaction of the incoming pointlike vortex with the pointlike vortices and antivortices residing in the superconducting layers above and below decreases the energy barrier by a certain amount δG_2. The total energy barrier shift is equal to

$$\delta G_g=\delta G_1+\delta G_2 . \tag{5.1}$$

Using Eqs. (3.12) and (3.13) the expression for δG_1 can be written in the form

$$\delta G_1 = \left[\frac{\Phi_0}{2\pi\lambda} \right]^2 d \sum_k \frac{x_m x_g}{x_m^2 + (y - y_{k0})^2} . \tag{5.2}$$

It follows from Eq. (5.2) that δG_1 is an oscillating function of y. The energy barrier shift δG_1 has a minimum if

$$|y - y_{k0}| \sim l > x_m \sim \lambda . \tag{5.3}$$

The characteristic value of this minimum is of the order of

$$\delta G_1 \sim \frac{\Phi_0^3 d N^2}{\lambda^2 H} . \tag{5.4}$$

Using Eqs. (3.12) and (3.13) the expression for δG_2 can be written in the form

$$\delta G_2 = -\left[\frac{\Phi_0}{4\pi\lambda} \right]^2 \frac{d^2}{\lambda} \sum_n \exp\left[-|n| \frac{d}{\lambda} \right] \times \sum_k \frac{x_m x_g}{x_m^2 + (y - y_{kn})^2} . \tag{5.5}$$

The main contribution to the value of δG_2 results from the interaction with the pointlike vortices and antivortices located in different superconducting layers at the coordinates (x_m, y_{kn}), where $|y - y_{kn}| < x_m$. It follows from Eq. (5.5) that the total number of sufficient terms in the sums over k and n is of the order of $N x_m \lambda / d$ and the value of δG_2 is equal to

$$\delta G_2 = -\gamma_g \left[\frac{\Phi_0}{4\pi\lambda} \right]^3 \frac{Nd}{H} , \tag{5.6}$$

where γ_g is a numerical factor of the order of 1. Comparing Eqs. (5.4) and (5.6) we see that the main term in the energy barrier shift δG_g is determined by δG_2. Using Eq. (3.20) the value of δG_g can be written as the following function of δM:

$$\delta G_g = -\gamma_g \frac{\delta M}{2\pi M_0} \left[\frac{\Phi_0}{4\pi\lambda} \right]^2 d \ln^{-1} \left[\frac{4\pi\lambda^2 H}{\Phi_0} \right] . \tag{5.7}$$

VI. MAGNETIZATION RELAXATION

Combining Eqs. (3.20), (4.8), and (5.7) we find the equation determining the magnetization relaxation in the form

$$\frac{dM}{dt} = \frac{\alpha}{\tau} \exp\left[-\frac{\delta M}{\alpha M_0} \right] , \tag{6.1}$$

where

$$\alpha = \gamma_\alpha \frac{k_B T}{d} \left[\frac{4\pi\lambda}{\Phi_0} \right]^2 \ln \left[\frac{4\pi\lambda^2 H}{\Phi_0} \right] , \tag{6.2}$$

$$\tau = \gamma_\tau \tau_0 \left[\frac{k_B T}{d} \left[\frac{4\pi\lambda}{\Phi_0} \right]^2 \right]^{1/2} \exp\left[\frac{G_g}{k_B T} \right] , \tag{6.3}$$

γ_α and γ_τ are numerical factors of the order of 1.

The solution of Eq. (6.1) has the form

$$M = M_0 \left[-1 + \alpha \ln \left[\frac{\tau}{\tau - t} \right] \right] . \tag{6.4}$$

Thus the thermally activated pointlike vortices penetration into the sample led to a specific initial avalanche-type dependence of the magnetization on time. The characteristic time of the magnetization decay τ strongly (exponentially) depends on the energy barrier and the temperature. The dimensionless amplitude α of the magnetization relaxation is proportional to the temperature and logarithmically depends on the applied magnetic field.

The dependence given by Eq. (6.4) is valid until

$$\delta M = \alpha M_0 \ln \left[\frac{\tau}{\tau - t} \right] < M_0 , \tag{6.5}$$

and the density of vortices N is less than a certain critical value $N_c \sim \lambda^{-1}$. When N becomes of the order of N_c the Abrikosov vortices self-assemble from the pointlike vortices and then penetrate inside the bulk. It follows from Eqs. (3.20) and (6.4) that a noticeable increase of N starts when $t \to \tau$.

The penetration of the Abrikosov vortices inside the bulk changes the time dependence of the magnetization decay to the regular logarithmic law. The latter leads to the magnetization relaxation rate decreasing in time. Thus at $t \approx \tau$ the magnetization relaxation rate has a maximum.

To estimate the values of the characteristic time constant τ and the dimensionless amplitude of the magnetization relaxation α we use $\gamma_\alpha = \gamma_\tau = 1$ and the data obtained for a monocrystal $Bi_2Sr_2CaCu_2O_8$:[21] $\lambda \approx 3 \times 10^{-5}$ cm, $\xi = \xi_{ab} \approx 1.5 \times 10^{-7}$ cm, $d \approx 1.5 \times 10^{-7}$ cm, $\rho_n \approx 10^{-5}$ Ω cm. Using Eqs. (2.6), (6.2), and (6.3) we find that if $T \ll T_c$ and $H \geq H_1 = 0.013$ T, then

$$\tau \approx 10^{-13} \left[\frac{T}{300} \right]^{1/2} \exp\left[\frac{300}{T} \ln \left[\frac{0.7}{H} \right] \right] s , \tag{6.6}$$

$$\alpha \approx \frac{T}{300} \ln(500H) , \tag{6.7}$$

where the temperature T is given in K and the magnetic field H is given in T. Substituting in Eqs. (6.6) and (6.7), $T = 25$ K and $H = 0.04$ T, we find $\tau \approx 30$ s and $\alpha \approx 0.25$. These numbers seem to be reasonable for the experimental observation of the main peculiarities of the magnetization relaxation due to the thermally activated penetration of the pointlike vortices.

VII. SUMMARY

To summarize, we have shown that in an external magnetic field higher than H_0 the energy of a pointlike vortex G_v has a minimum G_m detached from the sample surface by an energy barrier G_g. The value of G_m becomes negative in magnetic field higher than $H_1 \sim H_{c1}$. We have calculated the time dependence of the magnetization relaxation due to the thermally activated penetration of pointlike vortices inside the sample. We have shown that this process results in an avalanche-type initial decay of the magnetization and a specific maximum in the magnetization relaxation rate at a certain moment of time $t \approx \tau$. The value of τ exponentially depends on the ratio of the energy barrier G_g and $k_B T$.

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