NORMAL ZONE SOLITON IN LARGE COMPOSITE SUPERCONDUCTORS

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INTRODUCTION

The study of normal zone of finite size (normal domains) in superconductors, has been continuously a subject of interest in the field of applied superconductivity (see for example review[1], and references therein). It was shown that in homogeneous superconductors normal domains are always unstable, so that if a normal domain nucleates, it will either expand or shrink. While testing the stability of large cryostable composite superconductors, a new phenomena was found, the existence of stable propagating normal solitons[2]. The formation of these propagating domains was shown to be a result of the high Joule power generated in the superconductor during the relatively long process of current redistribution between the superconductor and the stabilizer[2-9].

Theoretical studies were performed to investigate the propagation of normal domains in large composite superconductors, in the cryostable regime. Huang and Eyssa[3] performed numerical calculations simulating the diffusion of heat and current redistribution in the conductor, and showed the existence of stable propagating normal domains. They compared the velocity of normal domain propagation with the experimental data[2], obtaining a reasonable agreement. Dresner[4,5] presented an analytical method to solve this problem, if the time dependence of the Joule power is given. He performed explicit calculations of normal domain velocity assuming that the Joule power decays exponentially during the process of current redistribution.

In this paper, we propose a system of two one-dimensional diffusion equations describing the dynamics of the temperature and the current density distributions along the conductor. Numerical simulations of the equations reconfirm the existence of propagating domains in the cryostable regime, while an analytical investigation supplies an explicit formula for the velocity of the normal domain.

THE BASIC EQUATIONS

Let us consider for simplicity a rectangular conductor, carrying a transport current I, and consisting of a plane layer of a superconductor (which can be also a composite), electrically and thermally bonded to a stabilizing normal metal. The thickness of the superconductor and the stabilizer are denoted by d_s and d_n respectively.

A complete treatment of the problem of normal zone in composite superconductors requires the solution of the heat diffusion equation which defines the dynamics of the temperature, and the Maxwell equations, which define the dynamics of the current distribution. These equations form a set of three dimensional and time dependent non-linear equations,

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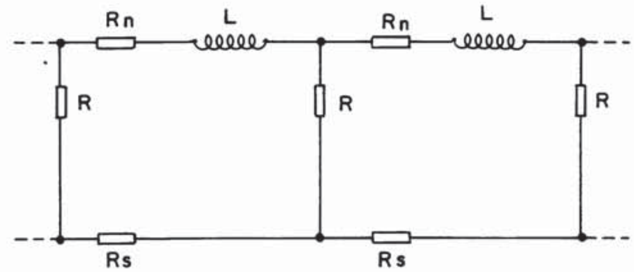


Fig. 1. Effective electrical circuit describing the current distribution in the conductor.

which are difficult for either analytical or numerical investigation. Here we show that for large composite superconductors, it is possible to reduce the complexity of the problem while preserving the main physical features: (i) The total longitudinal flow consists of parallel flows through the superconductor and the stabilizer; (ii) A perpendicular (redistribution) current is allowed to flow from one component to the other at any point along the conductor; (iii) Variations in the longitudinal current have a finite duration, which is of the order of the relaxation time of current redistribution in the stabilizer, τ_m. It is proportional to the ratio of the inductance and the resistance of a unit length of the conductor, $\tau_m \propto \mu_0 d_n^2/\rho_n$, where ρ_n is the resistivity of the stabilizer. For example, in [2], τ_m is estimated as approximately 0.3 sec. For larger conductors, it can be in the range $1 - 10$ sec.

We model the process of current redistribution by the effective electrical circuit sketched in Fig. 1. Here, each component is described by a discrete chain of resistors. $R_n = \rho_n \Delta x/d_n$, represents the stabilizer (Δx is an arbitrary discretization length, x is the axis along the conductor). $R_s = \rho_s \Delta x/d_s$, represents the superconductor, ($\rho_s(j_s, T)$ is the resistivity of the superconductor, j_s is the current density in the superconductor, T is the temperature). The two chains are linked through a third kind or resistors, $R = \gamma_R \rho_n d_n/\Delta x$ (γ_R is a numerical factor of the order of one, which depends on the geometry of the conductor). Finally, we attribute to the normal resistors an inductance $\mathcal{L} = \gamma_{\mathcal{L}} \mu_0 d_n \Delta x$, where $\gamma_{\mathcal{L}} \sim 1$ is another numerical factor. Applying the Kirchhoff's laws on this circuit, we obtain the equation for the current density in the superconductor,

$$\left(\frac{\gamma_{\mathcal{L}} \mu_0 d_n^2}{\rho_n}\right) \frac{\partial j_s}{\partial t} = \gamma_R d_n^2 \frac{\partial^2 j_s}{\partial x^2} - j_s \left(1 + \frac{\rho_s d_n}{\rho_n d_s}\right) + j, \tag{1}$$

where $j = I/d_s$.

Under usual conditions, the thermal relaxation time over the cross section is much shorter than the thermal relaxation time between the conductor and the coolant. In this case the temperature of the superconductor and the stabilizer practically coincide, and depend only of the coordinate along the conductor. The temperature, $T(x)$, satisfies the following equation

$$c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x}\right) - W(T) + Q(T), \tag{2}$$

where c is the heat capacity, and κ is the heat conductivity, both average values, obtained by weighting the values in each component with its relative cross sectional area. The term $W(T)$ is the rate of heat transfer to the coolant per unit volume, which is expressed by means of the heat transfer coefficient $W(T) = h(T)(T - T_0)/d$, where $d = d_s + d_n$, and T_0 is the temperature of the coolant. The rate of Joule heating per unit volume, $Q(T, j_s)$, has three contributions: from the current in the superconductor (in the normal state), from the current in the stabilizer, and from the perpendicular current. As a result $Q(T)$ is given by

$$Q(T) = \frac{1}{d}\left[d_s \rho_s j_s^2 + \frac{d_s^2 \rho_n}{d_n}(j - j_s)^2 + \gamma_R d_n d_s^2 \rho_n \left(\frac{\partial j_s}{\partial x}\right)^2\right]. \tag{3}$$

Let us as usually introduce the following dimensionless quantities: θ, the temperature, and i_s, the current density in the superconductor

$$\theta \equiv \frac{T - T_0}{T_c - T_0} \quad , \quad i_s \equiv \frac{j_s}{j_c}, \tag{4}$$

where j_c is the critical current density in the superconductor at the temperature T_0. We define also the following characteristic scales,

$$L_{th}^2 \equiv \frac{(d_n + d_s)\kappa}{h} \qquad \tau_{th} \equiv \frac{(d_n + d_s)c}{h}, \qquad L_m^2 \equiv \gamma_R d_n^2, \qquad \tau_m \equiv \frac{\gamma_{\mathcal{L}} \mu_0 d_n^2}{\rho_n}, \tag{5}$$

and the dimensionless parameters

$$\alpha \equiv \frac{d_s^2 \rho_n j_c^2}{d_n h (T_c - T_0)} \quad , \quad \xi(\theta, i_s) \equiv \frac{\rho_s d_n}{\rho_n d_s}, \tag{6}$$

$$i \equiv j/j_c, \qquad \tau \equiv \tau_m/\tau_{th}, \qquad \lambda \equiv L_m/L_{th}. \tag{7}$$

where ξ is the ratio of the resistances of the superconductor and of the stabilizer per unit length, and α is the Stekly parameter[12]. Using data from[1,2] we obtain the typical values of $\xi \sim 10 \div 100$, $\tau \sim 10 \div 100$ and $\lambda \sim 0.1 \div 1$. Expressing time in units of τ_{th}, and length in units of L_{th}, equations (1),(2) take the form

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} - \theta + \alpha (i - i_s)^2 + \xi(\theta, i_s) \alpha i_s^2 + \alpha \lambda^2 \left(\frac{\partial i_s}{\partial x} \right)^2 \tag{8}$$

$$\tau \frac{\partial i_s}{\partial t} = \lambda^2 \frac{\partial^2 i_s}{\partial x^2} - (1 + \xi(\theta, i_s))\, i_s + i, \tag{9}$$

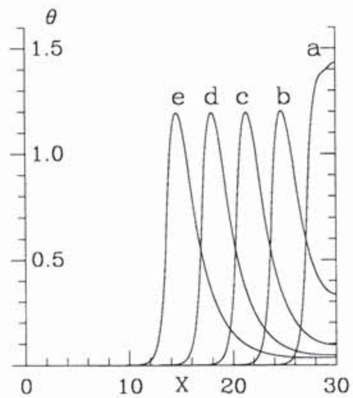
This set of equations describes in general a multistable system, where the existence, and properties of the stationary states depend on the explicit form of $\xi(\theta, i_s)$. In this paper, we take for simplicity the case of constant h, and the resistivity of the superconductor as a step function at the normal transition $\rho_s(j_s, T) = \rho_s\, \eta(j_s - j_c(T))$, with $j_c(T) = j_c\,(1 - (T - T_0)/(T_c - T_0))$, where $\eta(x)$ is the Heavyside step function. It is easy to show that under these assumptions, the condition for cryostablity is $\alpha < 1$.

RESULTS

Numerical Simulations

In order to study the propagating normal domains, we performed first numerical simulations of eq. (8), (9). We chose as initial conditions a nucleus of length $2L_{th}$, in which the temperature is raised to the critical value $\theta = 1$. As we are interested in cryostable conductors, the simulation were performed for $\alpha < 1$.

For a given set of parameters, we found the existence of a threshold current i_d, above which propagating domains are formed. For $i_d < i < 1$, the normal zone starts to expand while the current in this region diffuses out into the stabilizer. After it reached a certain length, the center of the normal zone starts to cool down but the outer sides continue to expand. As a result, superconductivity recovers at the center of the normal zone, and we find two separated normal domains propagating away in opposite directions. The system approaches a steady state where two normal domains are propagating with constant velocity, while superconductivity recovers behind. A sequence of temperature distributions for currents $i > i_d$ is shown in Fig. 2. The temperature at the front of the propagating domain reaches a steady shape after a time interval of the order of the thermal relaxation time, τ_{th} (one in dimensionless units). The "tail" of the profile reaches its steady shape after a relatively long time interval, which is of the order of the current distribution relaxation time, τ_m. The velocity of propagation attains its final value much faster than the time required to obtain the steady profile. It is consistent with the well known fact that the velocity of the normal zone is determined only by the temperature at the front of the domain[1]. In Fig. 3 we show a sequence of temperature profiles that were obtained for current below i_d, but very close to it. The initial



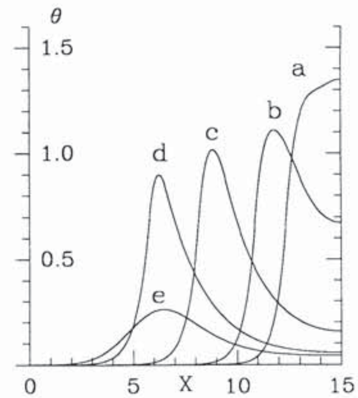


Fig. 2. Temperature distribution for $i > i_d$. The parameters are : $\tau = 100$, $\xi = 100$, $\lambda = 0.3$, $i = 0.34$, a) $t = 1$, b) $t = 2$, c) $t = 3$, d) $t = 4$, e) $t = 5$.

Fig. 3. Temperature distribution for $i < i_d$. The parameters are : $\tau = 100$, $\xi = 100$, $\lambda = 0.3$, $i = 0.33$, a) $t = 1$, b) $t = 2$, c) $t = 4$, d) $t = 6$, e) $t = 7$.

heat release in the normal zone forms a propagating domain, whose temperature gradually decreases, until it reaches the critical temperature, where superconductivity recovers. As i is decreased further below i_d, the complete recovery of superconductivity occurs faster.

To study the dependence of the velocity of propagation on the various parameters, we show in Fig. 4 plots of $v(i)$ for different values of α and τ (in units of $v_{th} \equiv L_{th}/\tau_{th}$). The velocity is a monotonically increasing function of the current, i, and it is finite at the threshold in agreement with[2–4]. The values of the threshold current, i_d, and the threshold velocity, v_d, depend both on τ and α. Above i_d, the dependence of the velocity on τ becomes less significant, and it is mainly determined by α.

Analytical Solution of the Steady State

For $i > i_d$, the temperature and the current density distributions of the steady state are the stationary solutions of equations (8), (9) with $i_s = i_s(x + vt)$ and $\theta = \theta(x + vt)$, which correspond to a frame of reference moving along the conductor with velocity v

$$\frac{\partial^2\theta}{\partial x^2} - v\frac{\partial\theta}{\partial x} - \theta + \alpha(i - i_s)^2 + \xi\alpha i_s^2\,\eta(i_s + \theta - 1) + \alpha\lambda^2\left(\frac{\partial i_s}{\partial x}\right)^2 = 0 \tag{10}$$

$$\lambda^2\frac{\partial^2 i_s}{\partial x^2} - v\tau\frac{\partial i_s}{\partial x} - (1 + \xi\,\eta(i_s + \theta - 1))\,i_s + i = 0, \tag{11}$$

where v still has to be determined. We define $x = 0$ to be the point where the normal transition occurs, and $x = \ell$ to be the point where superconductivity recovers. Eq.(11) is non-linear, but it can be solved in the three regions $x < 0$ ($\eta = 0$), $0 \le x \le \ell$ ($\eta = 1$) and $\ell \le x$ ($\eta = 0$). In each region, it becomes a linear equation with constant coefficients. The explicit expressions for $i_s(x)$ can be then substituted into eq.(10) yielding a linear equation for $\theta(x)$. Finally, the boundary conditions, and the matching conditions, form a closed set of equations, for the integration constants, v and ℓ. A considerable simplification of this procedure can be obtained by ignoring the recovery of superconductivity, thus, performing the above procedure only in the two regions $x < 0$ and $x > 0$. As was shown in Section 3, the recovery of superconductivity occurs far behind the propagating front, hence, does not affect the propagation.

We start with eq.(11). The boundary conditions at infinity are

$$i_s(-\infty) = i, \qquad i_s(\infty) = i/(1 + \xi). \tag{12}$$

The solution is given by

$$i_s(x) = \begin{cases} i - A\,e^{k_+ x} & x < 0 \\ i/(1+\xi) + B\,e^{-k_- x} & x > 0 \end{cases}, \tag{13}$$

where

$$k_+ \equiv \frac{v\tau + \sqrt{(v\tau)^2 + 4\lambda^2}}{2\lambda^2}, \qquad k_- \equiv \frac{-v\tau + \sqrt{(v\tau)^2 + 4\lambda^2(1+\xi)}}{2\lambda^2}. \tag{14}$$

Substituting the current distribution (13) into eq. (10), with the following boundary conditions at infinity,

$$\theta(-\infty) = 0, \qquad \theta(\infty) = \alpha\xi i^2/(1+\xi), \tag{15}$$

we obtain

$$\theta(x) = \begin{cases} C\,e^{\omega_+ x} - \dfrac{A^2(\alpha + \alpha\lambda^2 k_+^2)}{4k_+^2 - 2vk_+ - 1}\,e^{2k_+ x} & x < 0 \\ D\,e^{-\omega_- x} + \dfrac{\alpha\xi i^2}{1+\xi} - \dfrac{B^2(\alpha + \alpha\xi + \alpha\lambda^2 k_-^2)}{4k_-^2 + 2vk_- - 1}\,e^{-2k_- x} & x > 0 \end{cases}, \tag{16}$$

where

$$\omega_\pm \equiv \frac{\pm v + \sqrt{v^2 + 4}}{2}. \tag{17}$$

The four unknown constants, A, B, C, D and the velocity v, are determined by four matching conditions, and by the requirement of self-consistency at the transition point, $i_s(0) = 1 - \theta(0)$. A closed equation is obtained for the velocity,

$$(1-i)(\omega_+ + \omega_-) = -\frac{(\omega_+ - 2k_+)k_-^2}{(k_- + k_+)^2}\frac{\xi^2 i^2}{(1+\xi)^2}\frac{\alpha + \alpha\lambda^2 k_+^2}{4k_+^2 - 2vk_+ - 1} - \tag{18}$$
$$-\frac{k_-(\omega_+ + \omega_-)}{k_- + k_+}\frac{\xi i}{1+\xi} - \frac{(\omega_- - 2k_-)k_+^2}{(k_- + k_+)^2}\frac{\xi^2 i^2}{(1+\xi)^2}\frac{\alpha(1+\xi) + \alpha\lambda^2 k_-^2}{4k_-^2 + 2vk_- - 1} + \omega_- \frac{\alpha\xi i^2}{1+\xi}.$$

Calculations of $v(i)$ show, in agreement with the dynamical simulations, that for any given set of parameters, eq. (18) has solutions of v only if i is larger than a threshold value i_d. For

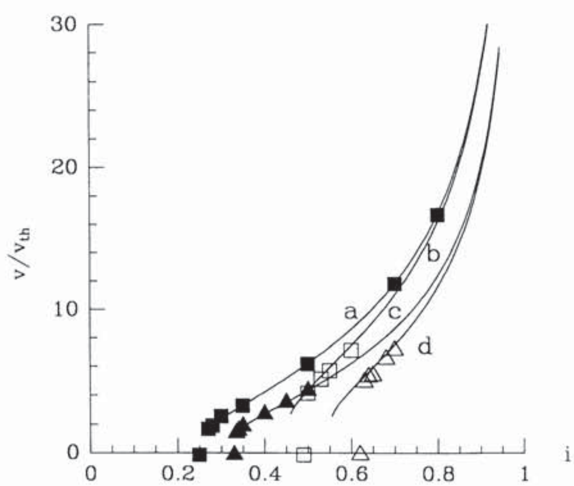


Fig. 4. Velocity of the normal domain in units of v_{th} versus current. The dots represent the results obtained in the numerical simulations, the solid lines are the solutions of eq. (3.10). The parameters are: $\xi = 100$, a) $\tau = 100, \alpha = 0.9$, b) $\tau = 10, \alpha = 0.9$, c) $\tau = 100, \alpha = 0.5$, d) $\tau = 10, \alpha = 0.5$.

$i_d < i < 1$, there are two roots, where only the largest corresponds to a stable solution (the second root is a decreasing function of i). At the threshold, there is only one root, v_d, which is has a finite value.

As stated before, the relevant range of parameters in the case of large cryostable composite superconductors is $\xi \gg 1, < \tau < \xi$ and $\lambda < 1$. The computations show that in this regime of parameters, the right-hand side in eq. (18) is dominated by the third term. Expanding the leading terms in powers of $1/v$, we find

$$v(i) \simeq \sqrt{\frac{\alpha \xi i^2}{1-i} - \frac{2\xi}{\tau}}, \qquad (v > 1). \tag{19}$$

Fig. 4 shows a comparison of the velocity obtained by the numerical simulations, with the velocity obtained by the analytical procedure. For large values of τ, ($\tau = 100$), the roots of the implicit eq. (18) give the velocity with a high degree of accuracy in the entire range of currents, and in particular, the threshold current, i_d, and the corresponding velocity, v_d. For smaller values of τ, ($\tau = 10$), the velocity calculated by (18) is still very close to the exact value, deviating from it in less than 5% for $i > i_d$, while the threshold current is about 10% less than the value obtained in the numerical simulations. This discrepancy is explained by the fact that when τ is smaller, the length of the domain is reduced, and the recovery of superconductivity becomes more significant in the determination of the velocity. Eq. (19) fits the roots of eq. (18), for large values of i. As this formula was obtained by expanding the implicit equation in powers of $1/v$, it is accurate only if the velocity, is sufficiently large. For $v > 4$, we see that formula (19) is accurate within few percents.

DISCUSSION

Let us now discuss the physical mechanism of normal zone propagation in large composite superconductors. Imagine that a part of the superconductor undergoes to the normal state. The current starts to redistribute between the superconductor and the stabilizer by diffusion, a process which has a characteristic duration of τ_m/ξ. After the redistribution of current is complete, the conductor cools down during a time period of the order of the thermal relaxation time, τ_{th} . When the temperature crosses the critical temperature, the superconducting state is recovered, and the current rediffuses back to the superconductor during a time period of the order of τ_m. As the redistribution of current requires a finite duration, the stabilizing mechanism suffers an effective delay time τ_m/ξ. The Joule power in the normal zone during this time interval is consequently high like in the case of a unstabilized superconductor. The effective Stekly parameter, $\tilde{\alpha}$, associated with this temporarily unstabilized superconductor is determined by the resistivity of the superconductor in the normal state, ρ_s, given by $\tilde{\alpha} = \rho_s j_c^2 d_s / h(T - T_0) = \alpha\xi$. In most cases of practical interest ($\alpha \leq 1$), $\alpha\xi \gg 1$. In the case of an unstabilized conductor, the normal zone expands with constant velocity v, if the current exceeds the minimum normal zone propagating current, $\tilde{i}_p$. In this range of parameters, $\tilde{i}_p$ is given by $\tilde{i}_p \sim \sqrt{2/\tilde{\alpha}}$, and the velocity, v, is given by the approximate expression[1,11],

$$v \simeq v_{th}\sqrt{\alpha\xi i^2/(1-i)}, \qquad\qquad v(i) \gg v_{th}.$$

This expression coincides with formula (19) when $\tau/\xi \gg 1$, as in this limit the delay time of the stabilizing mechanism becomes very long.

CONCLUSIONS

We proposed a system of two diffusion equations describing the normal zone nucleation and propagation in composite superconductors, with relatively long current redistribution time. We investigated the propagation of normal domains and showed that they exist if the current exceeds a threshold value. We found an analytical solution for the temperature and the current density distributions for the stationary normal domains, as well as an explicit expression for the velocity of propagation. The proposed system of equations can be generalized to investigate the role of the temperature dependence of the relevant parameters (e.g. heat transfer coefficient, heat capacity and conductivity), and the effect of contact resistance between the superconductor and the stabilizer.

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