Spiral superconducting Nb–Ti filaments: electric field and critical current

M. Polák, R.G. Mints*, M. Majoroš and I. Hlásnik

Electrotechnical Institute, Electro-Physical Research Center, Slovak Academy of Sciences, 842 39 Bratislava, Dúbravská 9, Czechoslovakia

Received 5 August 1986; revised 7 November 1986

As a first step to a better understanding of the voltage–current characteristics of fine filament multifilamentary composite superconductors, the electric field and critical current along a single spiral-shaped filament were studied. When the spiral superconducting filament was carrying constant current and placed in a magnetic field, the local critical current and electric field vary due to the change of the angle between the magnetic field and filament axis. A theory is presented which enables us to calculate the mean and local values of the electric field and critical current of spiral samples for arbitrary orientation between the magnetic field and spiral axis. The presented experimental results are in good agreement with this theory.

Keywords: superconductors; Nb–Ti; physical properties

A twisted multifilamentary Nb–Ti composite was formed with a certain number of spiral superconducting filaments embedded in the normal metal matrix. In previous work concerning a single spiral superconducting filament in a field perpendicular to the spiral axis[1] we have shown that the electric field, as well as critical current density, varies along the filament. As a consequence of the electric field variation along the spiral filaments a redistribution of the filament currents takes place inside the composite. Therefore the current–voltage characteristics of the composite can differ from that of the individual filament, as shown by Février and Renard[2].

The aim of the present work is to obtain information about the electric field distribution along the spiral filament sample and to compare the critical currents of the spiral sample with those of the straight sample at arbitrary orientation between the magnetic field and the spiral axis.

Theory

At present it is well established that the current–voltage characteristics of hard superconductors in a wide range of electric fields, E, (at least for 10^{-3} μV cm^{-1} $< E <$ $10^2\mu$V cm^{-1}) have the following form (see, for example, Reference 3 and the references therein)

$$j = j_c + j_1 \quad \ln(E/E_o) \tag{1}$$

where: j is the current; $j_c = j_c(B,T,\beta, E_o)$, is the critical current corresponding to the electric field E_o, the magnetic induction B, and temperature T; the parameter $j_1(B,T,\beta)$ characterizes the slope of the E–j characteristic and β is the angle between the magnetic field and the wire axis. Usually E_o = 0.1 up to 1 μV cm^{-1}.

Figure 1 represents the dependence of the critical current, j_c, and the parameter j_1 on the angle $\phi = (\pi/2)-\beta$ (this notation allows us to simplify the equations deduced below) for a straight Nb–Ti monofilamentary wire, CFTH-14 (manufactured by Thomson-Brandt, France)[4], which was used for preparing a spiral sample. The outer diameter of the copper stabilized wire was 52 μm and the filament diameter was 36 μm. Note that for this wire, in the interval $0° < \phi < 60°$, the dependence $j_c(\phi)$ can be quite well approximated by the following formula

$$j_c(\phi) = j_c(\phi{=}0°)\,[\,1 + 0.75\,(\phi^2 + \phi^4)] \tag{2}$$

It should be noted that $j_c(\phi)$ and $j_1(\phi)$ are even functions of ϕ.

The curves shown in *Figure 1* do not have universal validity for Nb–Ti wires. In Nb–Ti wires with elongated defects the functions depend on the aspect ratio of these defects, as shown in Reference 5.

In this work we analyse the electric field distribution and critical currents of a monofilamentary wire shaped into a spiral with diameter $2R$ and twist pitch l_p (see *Figure 2*) for arbitrary orientation of B. We assume that the magnetic field is in the x–z plane and forms an angle ($0° < \alpha < 90°$) with the x axis.

Then the angle ϕ can be determined from the equation

$$\sin\phi = \sin\alpha\cos\zeta + \cos\alpha\sin\zeta\cos\frac{2\pi z}{l_p} \tag{3}$$

where

$$tg\,\zeta = \frac{2\pi R}{l_p} \tag{4}$$

The dependence of ϕ on z for the spiral with l_p = 10 R (corresponding to the sample used in our experiments) is shown in *Figure 3*. The function $\phi(z)$ has a minimum at the point $z = l_p/2$ and the corresponding value $\phi_{min} = \alpha - \zeta$. Due to the change of the angle ϕ along the

*On leave from the Institute of High Temperatures, USSR Academy of Sciences, 127 412 Moscow, Korovinskoe shosse, No. 412, USSR

0011–2275/87/040183–06 $03.00

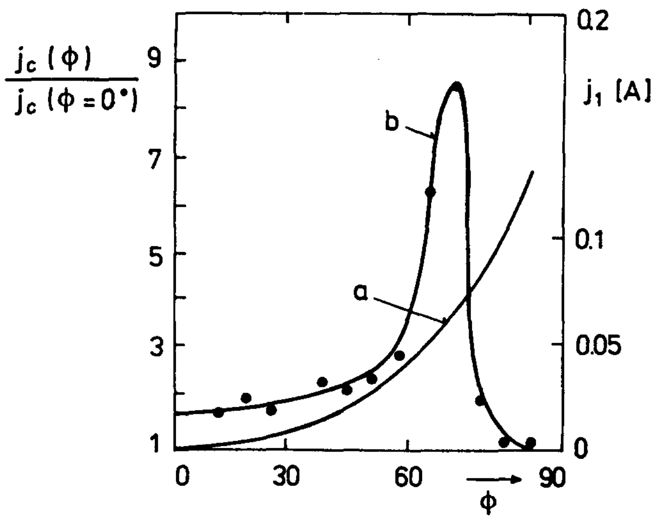

Figure 1 Measured dependence of: (a) $j_c(\phi)/j_c(\phi=0°)$; (b) j_l on angle ϕ, for B=4 T

spiral wire according to Equation (3), both j_c and j_l depend on z.

As one can see in *Figure 1*, the lower ϕ is, the lower the critical current, j_c, is. For $\alpha < \zeta$ there are two points on the curve $\phi(z)$ where ϕ=0 and the minimum value of $j_c[\phi(z)]$ equals $j_c(\phi=0)$. For $\alpha > \zeta$ the minimum value of $j_c[\phi(z)]$ equals $j_c(\phi_{min}) = j_c(\alpha - \zeta)$ (see *Figure 3*).

According to Equation (1) the electric field, $E(z)$, may be written as

$$E(z) = E_0 \exp\left\{\frac{j - j_c[\phi(z)]}{j_l[\phi(z)]}\right\} \tag{5}$$

As can be seen from Equation (5) the value of $E(z)$ is maximum at the points where $(j-j_c[\phi(z)])/(j_l[\phi(z)])$ is a maximum. It can be shown that $E(z)$ is maximum at points where $j_c[\phi(z)]$ is a minimum, supposing that the current, j, is higher than the minimum value of $j_c[\phi(z)]$.

Let us define the critical current of the spiral sample, $\tilde{j}_c$, as the current at the mean electric field along the spiral sample, $\tilde{E} = U/l = E_0$, where U is the voltage along the length l. For one complete spiral the mean value, $\tilde{E}$, can be written as

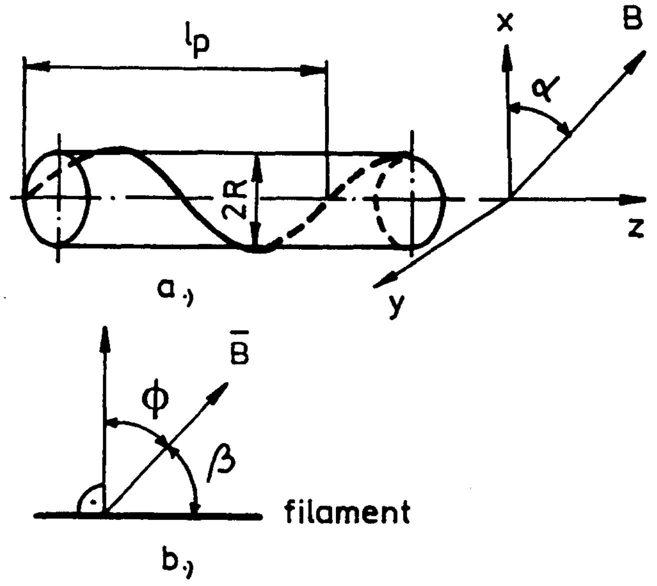

Figure 2 Orientation of the spiral sample and the filament in the co-ordinate system

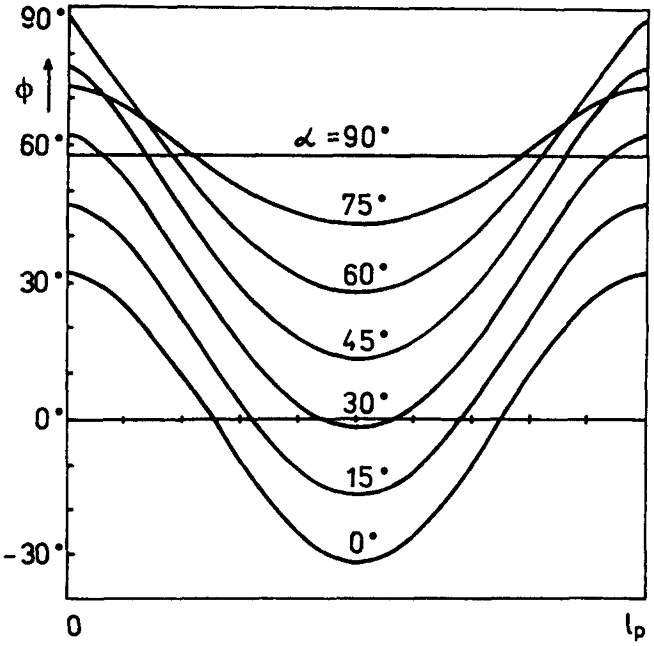

Figure 3 Dependence, $\phi(z)$, for the spiral with l_p/R = 10 and different angles of α

$$\tilde{E} = \frac{E_0}{l_p}\int_0^{l_p} \exp\left\{\frac{j - j_c[\phi(z)]}{j_l[\phi(z)]}\right\} dz \tag{6}$$

According to the definition of $\tilde{j}_c$, we can write the integral equation for $\tilde{j}_c$

$$\frac{1}{l_p}\int_0^{l_p} \exp\left\{\frac{\tilde{j}_c - j_c[\phi(z)]}{j_l[\phi(z)]}\right\} dz = 1 \tag{7}$$

In general, the solution of Equation (7) can be found only by numerical calculations. As $j_c \gg j_l$ for all angles ϕ, the integral Equation (7) can be solved analytically by the saddle point method[6] (see Appendix). The results are as follows

$$\tilde{j}_c(\alpha) = j_c(0) + 0.5 j_l(0) \ln\left|\frac{\pi(\sin^2\zeta - \sin^2\alpha) j_c''(0)}{2 j_l(0)}\right| \quad \text{for } \alpha < \zeta \tag{8}$$

$$\tilde{j}_c(\alpha) = j_c(0) + 0.25 j_l(0) \ln\left|\frac{4.5 \sin^2(2\zeta) j_c''(0)}{j_l(0)}\right| \quad \text{for } \alpha = \zeta \tag{9}$$

$$\tilde{j}_c(\alpha) = j_c(\alpha-\zeta) + 0.5 j_l(\alpha-\zeta) \ln\left|\frac{2\pi \sin\zeta \cos\alpha\, j_c'(\alpha-\zeta)}{\cos(\alpha-\zeta) j_l(\alpha-\zeta)}\right|$$

$$\text{for } 90° > \alpha > \zeta \tag{10}$$

where: $j_c' = \mathrm{d}j_c/\mathrm{d}\phi$ and $j_c'' = d^2 j_c/\mathrm{d}\phi^2$.

It can be shown, that for spirals of commercial Nb–Ti wires the second term in Equations (8)–(10) contributes only a few per cent to j_c and with this accuracy we can write

$$\tilde{j}_c(\alpha) = j_c(0) \quad \text{for} \quad \alpha \le \zeta \tag{11}$$

$$\tilde{j}_c(\alpha) = j_c(\alpha - \zeta) \quad \text{for} \quad \alpha > \zeta \tag{12}$$

Note, that for $\alpha = \pi/2$, Equation (12) gives the exact value for j_c and E is constant along the spiral wire.

Experimental details

The aims of the experiments described in the following text were:

1. to compare the critical currents of a spiral sample in a field perpendicular to the spiral axis [$\tilde{j}_c(\alpha=0)$] with those of a straight sample in a field perpendicular to its axis [j_c ($\phi=0$)];
2. to obtain the dependence of $\tilde{j}_c$ of the spiral sample on the angle α and to compare this with the theory presented above; and
3. to verify experimentally that the local critical current and electric field vary along the spiral sample.

The CFTH-14 described above was used for preparing all samples. Three samples were prepared. Sample 1 (see *Figure 4a*) with a distance of 450 mm between the potential taps was wound on the cylindrical fibre-epoxy former. If placed in the field parallel to the former axis the wire is exposed to the magnetic field practically perpendicular to the wire axis.

Sample 2 (see *Figure 4b*) was prepared so that at first the measured superconducting wire was wound on insulated Cu wire with a diameter of 1 mm with a twist pitch of 1.75 mm. Then this superconducting spiral on the Cu core was wound on a sample holder with an identical shape to that used for sample 1. The total length of the superconducting wire between the potential taps was 890 mm. If placed in a field parallel to the former axis, the magnetic field was practically perpendicular to the spiral axis.

Sample 3 is shown in *Figure 5*. The superconducting wire was placed in the spiral groove machined on the surface of the cylindrical fibre-epoxy former with a diameter of 3 mm. The twist pitch of the spiral, l_p, was 15 mm so that angle ζ = 32.1°. The ends of the sample were soldered to the cylindrical copper contacts and three pairs of potential taps were attached to the sample. The distance between the taps was 2 mm. The sample holder was constructed so that the angle could be changed between 0 and 90°.

Results and discussion

Comparison of the critical currents j_c and $\tilde{j}_c$

The *E–j* characteristics of samples 1 and 2 at 4.2 K for different magnetic fields are shown in *Figure 6*. The critical current obtained from *Figure 6* for the straight

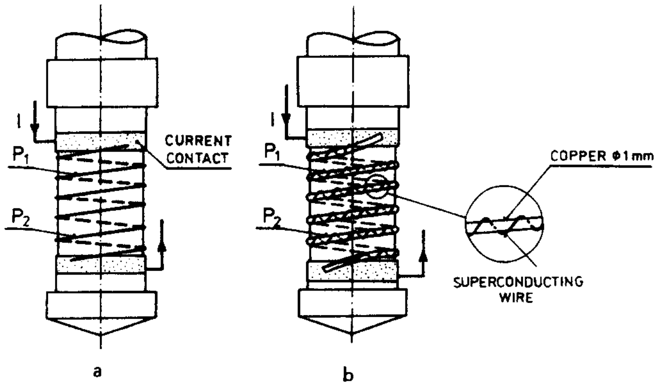

Figure 4 (a) Sample 1 in perpendicular field and (b) spiral sample 2 in a field perpendicular to the spiral axis. Angle ζ for the spiral sample is ≈ 61°

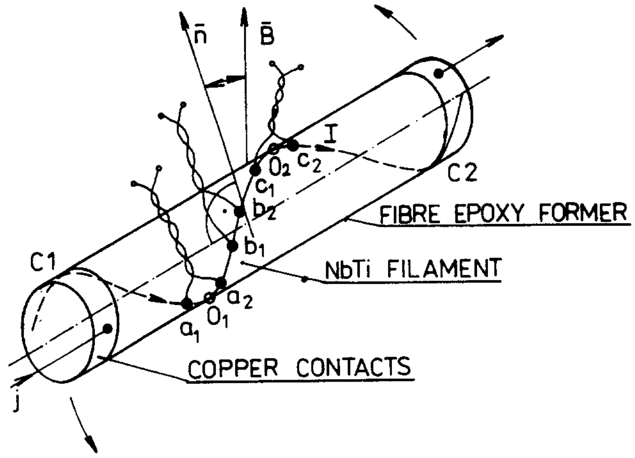

Figure 5 Position of the potential taps on spiral sample 3

sample, j_c, at E_o=0.1 μV cm^{-1} and for the spiral sample, $\tilde{j}_c$, at E_o = 0.1 μV cm^{-1} are compared in *Table 1*.

One may see that $\tilde{j}_c$ measured on the spiral sample is only a few per cent (1.6–2.8%) greater than j_c as predicted by theory. As an example, we can compare calculated and measured values of $\tilde{j}_c$ at B = 4 T. For calculating $\tilde{j}_c$ for sample 2 at B = 4 T, the parameters to be inserted in Equation (8) are as follows: $j_c(0)$ = 1.242 A, $j_1(0)$ = 0.013 A, ζ = 61°, $j_c''(\phi)=j_c(0)\,(1.5+9\,\phi^2)$, $j_c''(0)$ = 1.242 A × 1.5 = 1.863 A. Then, from Equation (8) we obtain $\tilde{j}_c$ = 1.275 A, which agrees well with the experimental value, 1.272 A. Similar agreement between theory and experiment was found for other values of B.

Measurements of $\tilde{j}_c$ (α) on spiral sample 3

Sample 3 was turned by steps of α = 6°. In each position current–voltage characteristics were measured and $\tilde{j}_c$ determined. The measured ratio $\tilde{j}_c(\alpha)\,/\,\tilde{j}_c(\alpha = 0°) = f(\alpha)$ at 4.07 T and 3.01 T is shown in *Figure 7*.

The theoretical curve shown in *Figure 7* has been calculated for B = 4 T using Equations (8)–(10). The parameters $j_c(0)$ and $j_1(\phi)$ measured on the straight sample are: j_c (ϕ=0) = 1.242 A at B=4.0 T, and the measured parameter, $j_1(\phi)$, is shown in *Figure 1*. From *Figure 7* we see that for $\alpha \leq \zeta$ (ζ=32.1°) the critical current, $\tilde{j}_c$ (α), is practically equal to j_c (α=0°). This can be explained by the fact that in this range of angle α, the minimum value of $|\phi|$ is ϕ_{min} = 0° (see *Figure 3*) and the critical current of the spiral sample is controlled by j_c (ϕ=0). For $\alpha > \zeta$ the

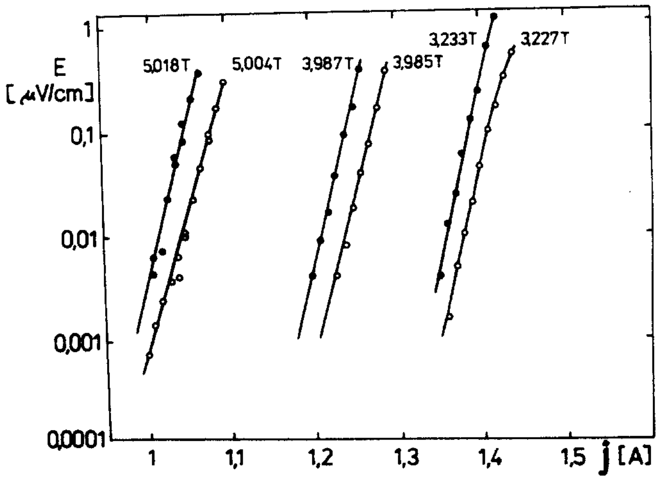

Figure 6 Measured *E–j* characteristics of: ●, the straight sample; and ○, sample 2 in fields of ≈ 5,4 and 3 T

Table 1 Critical current obtained from *Figure 6* for the straight sample, j_c, at E_o = 0.1 μV cm^{-1} and for the spiral sample, $\tilde{j}_c$, at E_o = 0.1 μV cm^{-1}

Sample	Critical current (A)	Magnetic field, *B*(T)		
		5	4	3.2
1	j_c (A)	1.050	1.242	1.388
2	$\tilde{j}_c$ (A)	1.081	1.272	1.411

minimum angle ϕ is $\phi_{min} = \alpha - \zeta$, and $\tilde{j}_c(\alpha)$ should be approximately equal to $j_c(\alpha - \zeta)$. From *Figure 7* it can be verified that this is true with an accuracy better than 5° in α, which is within the limits of measurement accuracy.

Experimental verification of critical current and electric field variation along the spiral sample

According to the theory presented above we have calculated the angle ϕ, the ratio $j_c[\phi(z)]/j_c(\phi=0)$ and the ratio $E(z)/E(\phi=0)$ as a function of the co-ordinate z for spiral sample 3 (see *Figure 8*). Then we measured the voltage–current characteristics between the potential taps, a_1–a_2, b_1–b_2 and c_1–c_2, in magnetic fields with B=1 up to 5 T. The critical currents and electric fields measured between taps a_1–a_2, b_1–b_2 and c_1–c_2 characterize the wire in the vicinity of points $z = l_p/4$, $z = l_p/2$ and $z = 3\,l_p/4$, respectively.

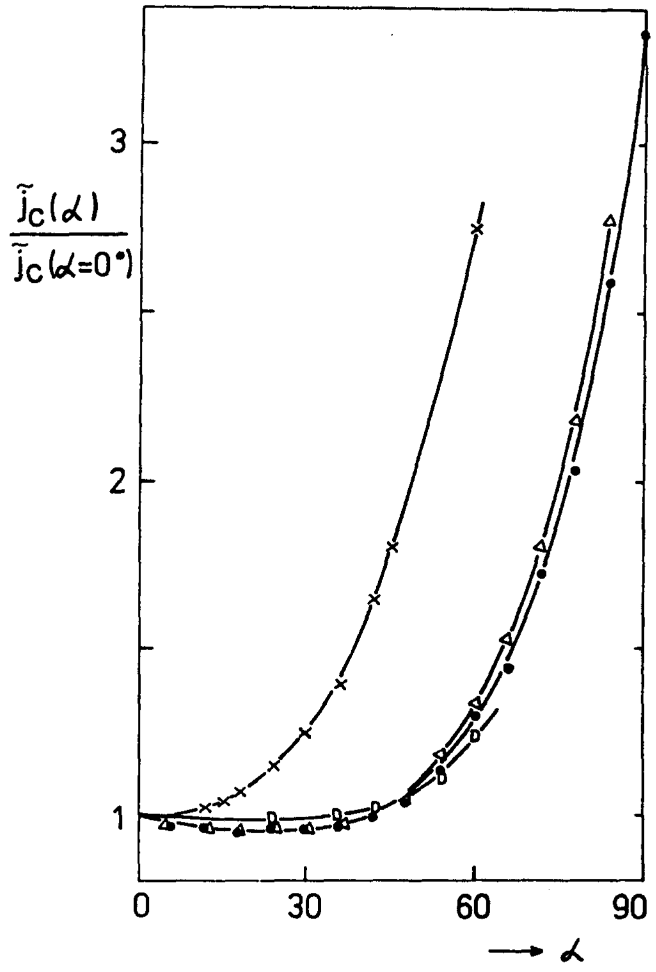

Figure 7 Angular dependence of ratio $\tilde{j}_c(\alpha)/\tilde{j}_c(\alpha=0°)$ for spiral sample 3 at: ●, 4.07 and △, 3.01 T (measured); and D, 4 T (theory). ×, This curve represents the ratio $j_c(\phi)/j_c(0)$ for the straight sample as a function of angle ϕ

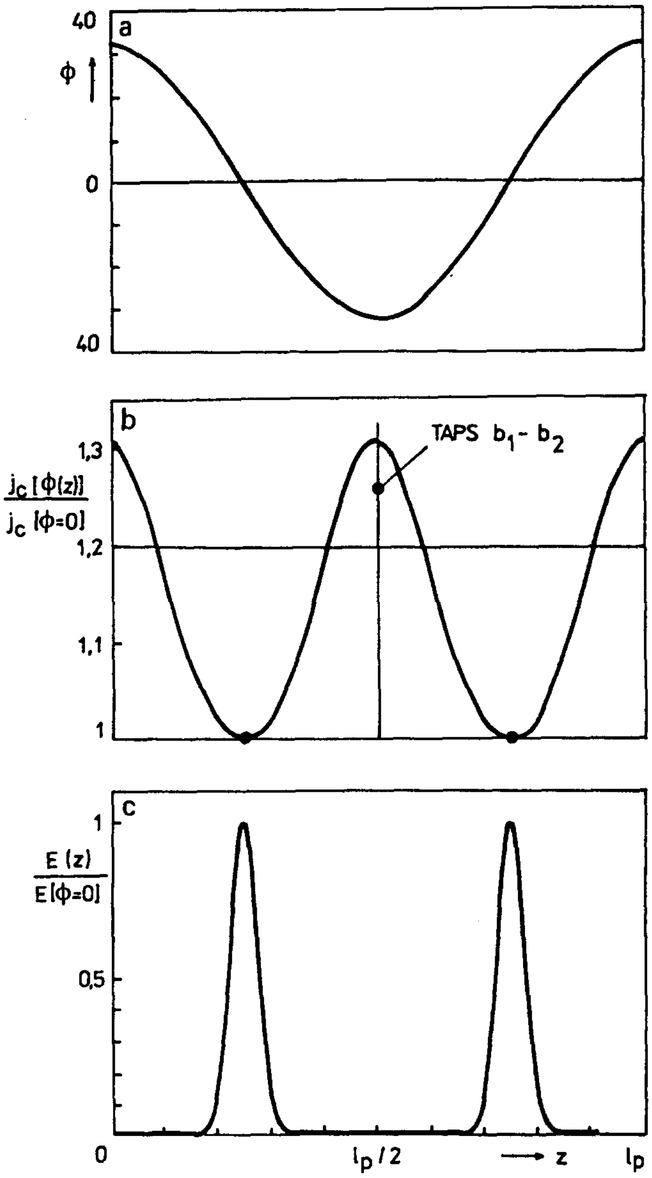

Figure 8 Variation of: (a) ϕ; (b) j_c $[\phi(z)]$ / j_c $[\phi=0]$; (c) $E(z)/E[\phi=0]$ along spiral sample 3 for α = 0 (magnetic field perpendicular to the spiral axis)

The measured critical currents, j_c, at E_o = 1 μV cm^{-1} are shown as a function of B in *Figure 9*. The measured ratio $j_c[\phi(z)]/j_c(\phi=0)$ at B=4 T for $z = l_p/4$, $l_p/2$ and $3\,l_p/4$ is shown in *Figure 8b*. The 2 mm gap between the potential taps is relatively large compared with the twist pitch length, and from the voltage, U, measured between them only the mean value of the electric field between the taps can be calculated. Thus, the real electric field in $z = l_p/4$ and $z = 3\,l_p/4$ is greater than that calculated from $U_{a_1-a_2}$ and $U_{c_1-c_2}$, and in $z = l_p/2$ it is smaller than calculated from $U_{b_1-b_2}$. Consequently, the measured ratio, j_c $[\phi(z = (l_p/2))]/j_c$ $[\phi(z = (l_p/4))]$, is smaller, as could be expected from the theory.

In *Figure 8c* it can clearly be seen that the electric field decreases very rapidly for $z \lessgtr l_p/4$ and $z \lessgtr (3/4)l_p$. The experiment confirmed that the voltage between b_1–b_2 was unmeasurably small for currents which give rise to measurable voltages between a_1–a_2 and c_1–c_2.

Thus, the described experiments confirmed that the critical current, as well as electric field, varies along the spiral sample and the theory presented describes this variation.

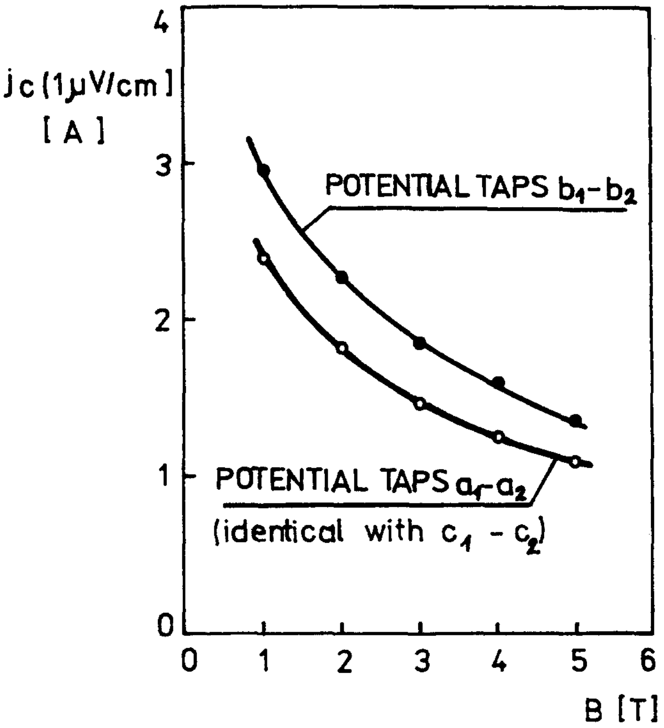

Figure 9 Critical current, j_c, for E_o=1 μV cm^{-1} measured between the potential taps a_1–a_2 and b_1–b_2 on the spiral sample as a function of B. Values of j_c between a_1–a_2 and c_1–c_2 are identical

Conclusions

It was shown that the electric field distribution in the spiral superconductor in a homogeneous magnetic field is very inhomogeneous. Because, in a multifilamentary wire the mean longitudinal electric field along the composite is constant, we suppose that the high inhomogeneity of E in filaments must be compensated for by high inhomogeneity of E in the normal matrix. This means, that in those parts of the composite where $j_c(z)$ is maximum, i.e. E is minimum, the current density in the matrix is maximum.

It has been shown that the critical current of a spiral superconductor, $\tilde{j}_c$, is mainly controlled by the critical current at points with maximum value of $E[\phi(z)]$. For $\alpha \leq \zeta$, it leads to $\tilde{j}_c(\alpha) \cong j_c(0) \cong$ constant, and for $\alpha > \zeta$, the critical current $\tilde{j}_c(\alpha) \cong j_c(\alpha - \zeta)$. The critical current of the spiral sample in the field perpendicular to the spiral axis does not differ significantly from that for a straight sample in a perpendicular field.

References

1 **Polák, M., Mints, R.G. and Hlásnik, I.** *Proc Int Symp Basic Problems of Superconductivity Applications in Energetics* Smolenice, Czechoslovakia (October 1985)
2 **Février, A. and Renard, R.C.** *Adv Cryog Eng* (1978) **24** 363
3 **Mints, R.G. and Rakhmanov, A.L.** *Instabilities in Superconductors* Nauka, Moscow, USSR (1984)
4 **Polák, M. and Gentile, D.** *J Physique* (1984) **45** (supplement no. 1), C1-541
5 **Takács, S. and Pevala, A.** *Phys Status Solidi (a)* (1977) **41** K 175
6 **Copson, E.T.** *Asymptotic Expansions* Cambridge Press, UK (1965)

Appendix

The dependence $\tilde{j}_c(\alpha)$ can be obtained analytically when $\alpha<\zeta$, $90°>\alpha>\zeta$ and $\alpha=\zeta$. For solving the integral Equation (7) the saddle point method[6] may be applied. The applicability of this method is shown by the fact that along the spiral the quantity

$$q(z) = \frac{\tilde{j}_c - j_c[\phi(z)]}{j_l(z)} \gg 1$$

except for areas in the vicinity of points where $j_c[\phi(z)]$ is minimum. Then (see, for example, Reference 6)

$$\frac{1}{l_p}\int_0^{l_p} \exp[q(z)]\,dz \cong \frac{1}{l_p}\sum_{z_o} \exp[q_{max}] \left(\frac{2\pi}{\left|\left[\frac{d^2q}{dz^2}\right]_{z_o}\right|}\right)^{1/2} \quad (A1)$$

where z_o are points at which the quantity $q(z)$ is maximum and $q_{max}=q(z_o)$. Thus, for calculating $\tilde{j}_c(\alpha)$, the maximum of $q(z)$ and the values d^2q/dz^2 at the points z_o need to be found. Let us do this for the cases where: $\alpha < \zeta$, $90° > \alpha > \zeta$, $\alpha = 0$.

Case where $\alpha < \zeta$

Two points exist on the curve $\phi(z)$ where $\phi(z_o)=0$. In the vicinity of these points the ratio $q(z)= (\tilde{j}_c - j_c(z))/j_l(z)$ is maximum, and a small range of z in the vicinity of these points determines the integral in Equation (7). The derivative dq/dz is given by

$$\frac{dq}{dz} = -\frac{d\phi}{dz}\frac{j_c}{j_l}\left[\frac{j'_c}{j_c} + \frac{\tilde{j}_c - j_c}{j_c}\frac{j'_l}{j_l}\right] \quad (A2)$$

where

$$j'_c = \frac{dj_c}{d\phi}, j'_l = \frac{dj_l}{d\phi}$$

Note that $\tilde{j}_c - j_c \approx j_l$ (as will be shown later) and, consequently, the second term of Equation (A2), in square brackets, is small due to the low value of $j_l/j_c \ll 1$. Further, in order to simplify the final equations, we shall neglect it. Then the derivative dq^2/dz^2 needed for calculating the integral involved in Equation (7) is

$$\frac{d^2q}{dz^2} = -\left(\frac{d\phi}{dz}\right)^2_{z_o} \frac{j''_c(0)}{j_l(0)} \quad (A3)$$

where $j''_c = d^2j_c/d\phi^2$.

The values of z_o in which $\phi(z_o)=0$ are determined from the condition

$$\cos\left(\frac{2\pi z_o}{l_p}\right) = -\frac{tg\alpha}{tg\zeta} \quad (A4)$$

and

$$\left|\left[\frac{d\phi}{dz}\right]_{z_o}\right| = \frac{2\pi}{l_p}\sin\zeta\cos\alpha\left(1-\frac{tg^2\alpha}{tg^2\zeta}\right)^{1/2}$$

$$= \frac{2\pi}{l_p}(\sin^2\zeta - \sin^2\alpha)^{1/2} \quad (A5)$$

Substituting Equation (A5) into (A3), we find, finally, that

$$\left[\frac{d^2q}{dz^2}\right]_{z_0} = -\frac{4\pi^2}{l_p^2}(\sin^2\zeta - \sin^2\alpha)\frac{j_c''(0)}{j_1(0)} \quad (A6)$$

Using Equation (A6), the integral Equation (7) can be solved and for $\tilde{j}_c(\alpha)$ and $\alpha < \zeta$ we find

$$\tilde{j}_c(\alpha) = j_c(0) + 0.5 j_1(0) \ln\left|\frac{\pi(\sin^2\zeta - \sin^2\alpha) j_c(0)}{2 j_1(0)}\right|, \alpha < \zeta \quad (A7)$$

Note that this procedure also enables us to calculate the current–voltage characteristic of the spiral superconductor, which takes the form

$$j = \tilde{j}_c + j_1(0) \ln (E/E_o) \quad (A8)$$

because the ratio of the maximum electric field to the value $E(\tilde{j}_c)$ can easily be found. As

$$E_{max} = E_o \exp\left|\frac{j - j_c(0)}{j_1(0)}\right| \quad (A9)$$

thus, one can determine

$$\frac{E_{max}(\tilde{j}_c)}{\tilde{E}(\tilde{j}_c)} = \left(\frac{\pi(\sin^2\zeta - \sin^2\alpha) j_c''(0)}{2 j_1(0)}\right)^{1/2} \gg 1, \alpha < \zeta \quad (A10)$$

The condition $E_{max} \gg \tilde{E}$ is, at the same time, a criterion of applicability of the expression (A7), and the electric field changes from zero along the length $2\, l_1$ in the vicinity of the points z_o only, i.e. in the interval $(z - z_o) < l_1$. The value of l_1 is of the order

$$l_1^2 \sim \frac{l_p^2}{2\pi^2} \frac{j_1(0)}{(\sin^2\zeta - \sin^2\alpha) j_c''(0)} \ll l_p^2, \alpha < \zeta \quad (A11)$$

Case where 90° > α > ζ

In this case there are no points on the curve $\phi(z)$ where $\phi = 0$. According to Equation (A2) the value of $q(z)$ is maximum at the point where ϕ is minimum ($\phi_{min} = \alpha - \zeta$), i.e. at $z = l_p/2$.

Thus, for $[d^2q/dz^2]_{l_p/2}$ we obtain

$$\left[\frac{d^2q}{dz^2}\right]_{l_{p/2}} = -\left[\frac{d^2\phi}{dz^2}\right]_{l_{p/2}} \frac{j_c'(\alpha-\zeta)}{j_1(\alpha-\zeta)} \quad (A12)$$

The values of $[d^2\phi/dz^2]_{l_p/2}$ can be found by differentiating Equation (3) according to z. Finally we obtain

$$\left[\frac{d^2\phi}{dz^2}\right]_{l_{p/2}} = \frac{4\pi^2 \sin\zeta \cos\alpha}{l_p^2 \cos(\alpha-\zeta)} \quad (A13)$$

Using Equations (A12) and (A13) we can calculate the integral in Equation (7) and for $\tilde{j}_c(\alpha)$ we obtain

$$\tilde{j}_c(\alpha) = j_c(\alpha-\zeta) + 0.5 j_1(\alpha-\zeta) \ln\left|\frac{2\pi \sin\zeta \cos\alpha\, j_c(\alpha-\zeta)}{\cos(\alpha-\zeta) j_1(\alpha-\zeta)}\right|, \quad \alpha > \zeta \quad (A14)$$

Analogically the voltage–current characteristic of the spiral superconductor can be found, which in this case has the form

$$j = j_c + j_1(\alpha-\zeta) \ln (E/E_o) \quad (A15)$$

As one has

$$E_{max} = E_o \exp\left|\frac{j - j_c(\alpha-\zeta)}{j_1(\alpha-\zeta)}\right| \quad (A16)$$

after a simple transformation for the ratio $E_{max}(\tilde{j}_c)/E(j_c)$ in Equation (A15) we obtain

$$\frac{E_{max}(\tilde{j}_c)}{\tilde{E}(\tilde{j}_c)} = \left(\frac{2\pi \sin\zeta \cos\alpha\, j_c'(\alpha-\zeta)}{\cos(\alpha-\zeta) j_1(\alpha-\zeta)}\right)^{1/2} \gg 1, \alpha > \zeta \quad (A17)$$

It is to be noted that the condition $E_{max} \gg \tilde{E}$ is the criterion for applicability of Equation (A14), and the electric field deviates from zero along the length $2l_2$ in the vicinity of the point $z = l_p/2$, i.e. in the region $| z - l_p/2 | < l_2$. The value l_2, as follows from calculations made previously, is of the order of

$$l_2^2 = \frac{l_p^2 \cos(\alpha-\zeta)\, j_1(\alpha-\zeta)}{2\pi^2 \sin\zeta \cos\alpha\, j_c'(\alpha-\zeta)} \quad (A18)$$

From Equation (A14) we see that $\tilde{j}_c(\alpha)$ increases if the angle α is increased. Thus, at $\alpha \sim \zeta$, the function $\tilde{j}_c(\alpha)$ exhibits a slight minimum.

Case where α = ζ

With this geometry, the maximum $q(z)$ occurs at $z = l_p/2$, where $\phi = 0$ and $d\phi/dz = 0$. Calculating the integral in Equation (7) we obtain

$$j_c(\alpha=\zeta) = j_c(0) + 0.25 j_1(0) \ln\left|\frac{4.5 \sin^2(2\zeta) j_c''(0)}{j_1(0)}\right| \quad (A19)$$