ON DISSIPATIVE STRUCTURES IN NONUNIFORM MEDIA

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It is found that in many physical, chemical and biological systems the nonuniform active media may be considered as a "matrix" controlling the selection of the dissipative structures of definite type.

Dissipative structures are well known in many physical, chemical and biological systems [1,2]. Usually, one-dimensional dissipative structures are described by means of the system of equations

$$\frac{\partial\hat{\psi}}{\partial t} = \frac{\partial}{\partial x}\hat{D}\frac{\partial\hat{\psi}}{\partial x} - \hat{f}(\hat{\psi}), \qquad (1)$$

where $\hat{\psi}$ and $\hat{f}$ are the n-space vectors, $\hat{D}$ is a square matrix of order n. Electric and magnetic fields strength, concentrations of chemical reagents, temperature and other characteristics may be the components of the vector $\hat{\psi}$ depending on the system under consideration. The functions f_i ($i = 1, 2, ..., n$) describe the interaction of the components of the vector $\hat{\psi}$ (including the self-action). Usually the matrix $\hat{D}$ is diagonal and the corresponding diagonal elements are the proper "diffusion" coefficients.

Stable stationary one-dimensional dissipative structures (periodic, stochastic and so on) may exist in uniform infinite media only if the system is multicomponent ($n \geqslant 2$) [1,2]. In these cases the medium is only a "passive" background for the development of the corresponding phenomena. For nonuniform systems the medium may take an active part in the process of dissipative structure formation. The description of dissipative structures in nonuniform media is an important problem as any system is more or less nonuniform.

In the present paper we found that stable stationary dissipative structures may exist in a nonuniform medium even if they are absent in the uniform one. The one-component ($n = 1$) systems are an interesting example of such a situation. In this case even weak nonuniformities may lead to the arising of various dissipative structures.

Let us first consider the one-component system ($n = 1$) in the case when $l \ll L$, where l is the characteristic length of the nonuniformities and L is the characteristic length of variation of the function $\psi = \psi(x)$. For $l \ll L$, one may write eq. (1) in the form

$$\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial x}D\frac{\partial\psi}{\partial x} - f(\psi,\beta) + \sum_k F_k(\psi)\delta(x - x_k), \qquad (2)$$

where β is the parameter describing the external influence on the system, x_k is the coordinate of the kth nonuniformity, and $F_k = F_k(\psi)$ is the self-consistent "strength" of the kth nonuniformity. The dissipative structures under consideration exist if the equation $f(\psi,\beta) = 0$ has more than one root. This is so, for example, for active media, where the function $f = f(\psi)$ is N-like [3] (see fig. 1) [‡1]. In this case the system

[‡1] The dissipative structures existing in nonuniform quasilinear systems were studied in ref. [4].

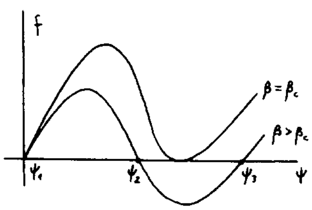

Fig. 1. The dependence $f = f(\psi,\beta)$ on ψ.

may be in two stable uniform states $\psi = \psi_1$, and $\psi = \psi_3$. The "phases" $\psi = \psi_1$ and $\psi = \psi_3$, in principle, may interchange along the sample.

There is only one stable nonuniform solution of eq. (2) in the case of an infinite uniform medium ($F = 0$). This solution $\psi = \psi(x - vt)$ describes the "interface" boundary, the domain wall ($\psi(-\infty) = \psi_1$, $\psi(\infty) = \psi_3$) moving with velocity v [3]. The velocity of the domain wall v depends on the parameter β and changes sign at $\beta = \beta_p$ [$v(\beta_p) = 0$]. To find the parameter β_p one has the equation $S(\psi_3, \beta_p) = 0$ [3], where

$$S(\psi) = \int_{\psi_1}^{\psi} D(\psi')f(\psi')\,d\psi' . \qquad (3)$$

Other nonuniform distributions in the case of infinite uniform media are unstable, including solitary waves, domains ($\psi(\pm\infty) = \psi_1$, $\psi(x) > \psi_1$) and various periodic structures.

Let us consider nonuniform media containing only "isolated" nonuniformities. It means that the distance between the nonuniformities is large compared with the characteristic length L. In this case dissipative structures may exist consisting of separate variously interchanging fragments, localized domains and domain walls. Each of the fragments is stable relative to small perturbations $\delta\psi = \delta\psi(x, t)$ if the value of the parameter β is from a certain region $\beta_1 < \beta < \beta_2$.

To find the values of the parameters β_1 and β_2 let us consider the localization conditions of the domain and domain wall on an "isolated" point nonuniformity. In this case the sum in the right part of eq. (2) contains only the self-consistent term $F(\psi)\delta(x)$. It is easy to analyse qualitatively the solution of the stationary ($\partial\psi/\partial t = 0$) equation (2) corresponding to the localized domains and domain walls by means of "mechanical" analogy. In fact, if $\partial\psi/\partial t = 0$, then eq. (2) is analogous to the equation describing the motion of a particle with mass D under the action of the "force" $f(\psi,\beta)$. The term $F(\psi)\delta(x)$ corresponds to an impact changing the momentum of the particle at the "time" moment $x = 0$ at the "point" $\psi = \psi_m \equiv \psi(0)$. The change of the "momentum" is equal to $-F(\psi_m)$.

Using the "mechanical" analogy it is easy to obtain that the existence of nontrivial ($\psi_m \neq 0$) solutions of the equation

$$S(\psi_m,\beta) = \tfrac{1}{8}F^2(\psi_m,\beta) \qquad (4)$$

is the necessary condition of a domain localization on the nonuniformity. Note, that $\psi_m \equiv \psi(0)$ is the value of the function $\psi = \psi(x)$ at the point $x = 0$ where the nonuniformity is located. Analogously, one may obtain the following equation for the case of a domain wall localized on the nonuniformity:

$$S(\psi_m,\beta) = \tfrac{1}{2}\,[S_3(\beta)/F(\psi_m,\beta) + \tfrac{1}{2}F(\psi_m,\beta)]^2 , \qquad (5)$$

where $S_3 \equiv S(\psi_3,\beta)$. In general, eqs. (4), (5) have several solutions corresponding to different types of the localized distributions $\psi = \psi(x,\psi_m)$ and allow one to find the values β_1 and β_2 for each of them. It is easy to obtain with the aid of eq. (2) that the inequality

$$\partial\psi_m/\partial\beta > 0 \qquad (6)$$

is the stability condition for localized domains and domain walls relative to small perturbations $\delta\psi$ in the case when $f(\psi,\beta)$ is similar to the function shown in fig. 1. Variation of the parameter β leads to jump-like appearance and disappearance of the stable distributions $\psi = \psi(x,\beta)$, at the corresponding values of β (β_1 and β_2).

Let us characterize the self-consistent "strength" of the nonuniformities with the dimensionless parameter Γ:

$$\Gamma \sim (l/L)\,\Delta f/f_m , \qquad (7)$$

where Δf is the characteristic variation of the function f due to the presence of the nonuniformity, $f_m = f(\psi_m,\beta)$. Usually the value of the parameter Γ is small ($\Gamma < 1$) due to the small ratio $l/L \ll 1$. It follows from eqs. (4), (5) that for weak nonuniformities ($\Gamma \ll 1$) the localization of domains and domain walls is possible if $|\beta - \beta_p| \ll \beta_p$. The difference $\beta_2 - \beta_1$ is of the order of $\Gamma^2\beta_p$ for the case of a localized domain and of the order of $\Gamma\beta_p$ for the case of a localized domain wall.

Let us now consider some properties of the dissipative structures existing due to the localization of the domain walls. They, evidently, consist of separate interchanging fragments, "phases" $\psi = \psi_1$ and $\psi = \psi_3$ (fig. 2). The lengths of the fragments D_k are practically independent of β in the case when the localization conditions $\beta_1^{(k)} < \beta < \beta_2^{(k)}$ are fulfilled for each of the domain walls forming the dissipative structure. If the value of the parameter β_1 becomes equal to β_1 (or β_2) then the corresponding domain wall becomes

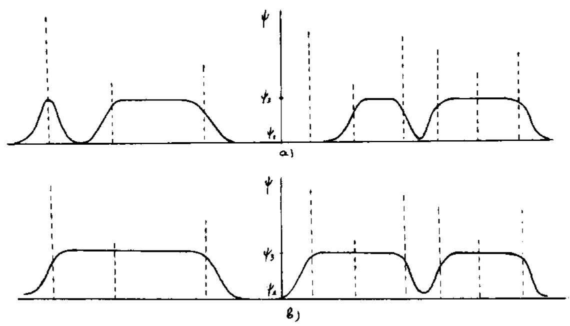

Fig. 2. Example of the jump-like reorganization of the dissipative structure due to the variation of the parameter β: (a) $\beta = \beta_a$, (b) $\beta = \beta_b$ ($\beta_a < \beta_b$). The dashed lines correspond to different nonuniformities.

delocalized and it leads to the jump-like reorganization of the dissipative structure. This reorganization process is irreversible and arises from both the variation of the parameter β and the action of a strong external perturbation.

Consider now the dissipative structures existing in media with slowly varying properties ($l \gg L$). In this case it is convenient to define the local value of the parameter $\beta_p = \beta_p(x)$ $[v(\beta_p(x)) = 0]$. Then the condition

$$\beta = \beta_p(D_k) \tag{8}$$

allows one to find the equilibrium positions $x = D_k$ for the domain wall. If there are two or more solutions of eq. (8), then stable fragments of the "phase" $\psi = \psi_3$ may exist in the regions where $\beta > \beta_p(x)$ (see fig. 3). The lengths of these fragments D_k increase with increase of the parameter β. The dependence $D_k = D_k(\beta)$ is jump-like if the function $\beta_p = \beta_p(x)$ is nonmonotonic and two solutions of eq. (8) merge at $\beta = \beta_+^{(k)}$ and vanish if $\beta > \beta_+^{(k)}$. For illustration, the process of jump-like reorganization of the dissipative structure is shown in fig. 3 for the case of two nonuniformities. The corresponding dependence $D_k = D_k(\beta)$ is given in the inset to fig. 3 for the straight and reverse run of β. If there are N minima on the curve $\beta_p = \beta_p(x)$ in the region $\beta > \beta_p(x)$ then the number of possible dissipative structures existing due to the localization of domain walls is equal to 2^N. The special kind of such dissipative structures is defined by the prehistory of the sample, the character and intensity of the external influence on the system and so on.

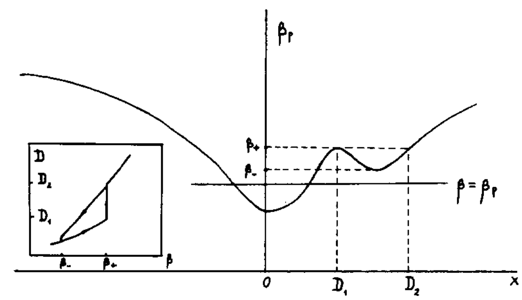

Fig. 3. The dependence $\beta_p = \beta_p(x)$ on x.

Thus, various stable dissipative structures may exist in nonuniform systems even if they are one-component. The special kind of these dissipative structures is defined mostly by the location of the nonuniformities The "strengths" of the nonuniformities determine here only the region of the values of where the dissipative structures of given type may exist. Such properties of the dissipative structures under consideration are the consequence of the initial uniform system being of the trigger-type (see fig. 1). In this case even the existence of weak nonuniformities may lead to a local transition of some parts of the sample from one stable state $\psi = \psi_1$ to another $\psi = \psi_3$. Such transitions arise due to the variation of β causing the jump-like reorganization of the dissipative structure or if the external influence is strong enough.

Thus, the dissipative structures in nonuniform media are characterized by the following main characteristics: (1) hard excitation, (2) jump-like reorganization and hysteresis, (3) essential dependence on the location and "strengths" of the nonuniformities.

The above approach allows one to consider the nonuniform active medium as a "matrix" controlling the selection of the dissipative structures of definite types in many physical, chemical and biological systems.

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