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Propagation of the normal zone in composite superconductors with high contact resistance

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Abstract. The propagation of the normal zone in composite superconductors with high thermal resistance and high electrical contact resistance between the superconducting and the normal components of the conductor is analysed. It is shown that in certain conditions propagation can result from successive multiple splittings of the resistive domain generated. The minimum initiation current of this process has been found and has been analysed as a function of the composite parameters and external conditions.

1. Introduction

The conditions under which a normal zone can exist and can propagate in composite superconductors with contact thermal resistance and electrical resistance between the superconductor proper and the normal metal matrix have been repeatedly discussed in the literature (e.g. Al'tov *et al* 1975, Keilin and Ozhogina 1977, Kremlev 1980). Calculation of the minimum normal zone existence current I_m and the minimum normal zone propagation current I_p received the most attention.

Akhmetov and Mints (1982, 1983a) analysed the conditions for the existence of resistive domains (finite-sized regions of the normal zone) in composite superconductors with high thermal resistance and high electrical contact resistance. It was demonstrated both analytically and by computer simulation that single resistive domains are stable if the transport current I in a specimen is in the range $I_r < I < I_f \leqslant I_m$. The minimum existence current I_r of a resistive domain can be much lower than the current I_m, which is calculated for a wide range of composite parameters under the assumption that the normal (resistive) zone fills the whole superconductor (Akhmetov and Mints 1982, 1983a). If $I \geqslant I_f$ a single resistive domain initiated by a disturbance is unstable. Over a time of the order of the characteristic time of temperature relaxation in the superconductor, a superconducting region is formed at the centre of the resistive domain, that is, the domain splits in two. Our short note (Akhmetov and Mints 1983b) demonstrated that the two daughter domains recede to a certain distance and then each of them splits in two. This periodic process of splitting in two of the two outermost resistive domains continues until a string of resistive domains stretches across the whole specimen.

Resistive domains in composite superconductors with high thermal resistance and high electrical contact resistance were detected and studied by Akhmetov *et al* (1983), Akhmetov and Baev (1984) and Keilin and Kruglov (1984). The results reported are in good qualitative and quantitative agreement with the theory (Akhmetov and Mints 1982, 1983a, b).

In the present paper we analyse in detail the propagation of the normal zone in composite superconductors with high thermal resistance and high electrical contact resistance in the case of $I \geqslant I_f$.

2. Basic equations

The distribution of the normal and superconducting zones along a composite superconductor is obtained from the heat-transfer and continuity equations which describe the evolution of temperature and of electric current density. In this paper we consider the two specimens shown in figure 1. These specimens are composed of ribbons of equal width: superconductor (s), transition layer (i) and normal metal (n). The composite shown in figure 1(a) consists of three such ribbons. This composite is denoted below by TRC (for three-ribon composite); d_s, d_i and d_n denote the thicknesses of the superconductor, the transition layer and the normal metal ribbons respectively. Obviously, the superconducting and the normal components of a TRC are in direct contact with the coolant. The composite shown in figure 1(b) is made up of five ribbons and is denoted

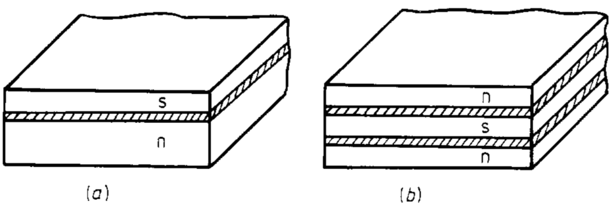

Figure 1. Specimen geometry. The shaded area indicates the transition layer. (a) TRC and (b) FRC.

by FRC (for five-ribbon composite). The ribbon thickness in a FRC is $\frac{1}{2}d_n$ for each of the two ribbons made of normal metal, d_i for the transition layer and d_s for the superconducting ribbon. The total thickness of the normal metal matrix in the FRC is therefore d_n, the same as in the TRC. However, in the FRC only the normal metal is in direct contact with the coolant.

We shall assume that the thermal and the electrical resistances are restricted to the transition layers, and that $d_i \ll d_n, d_s$. Consequently, when the parameters are typical of superconducting composites, both the temperature and the current density in the normal metal and in the superconductor change only along the x axis of the conductor. For the TRC the heat transfer and continuity equations (see e.g. Kremlev 1980, Akhmetov and Mints 1982, 1983a, b) of interest are

$$\nu_n \dot{T}_n = \kappa_n T_n'' - (W_0/d_n)(T_n - T_0) + \rho_n j_n^2 + \tfrac{1}{2}\rho_i d_i d_n (\mathrm{d}j_n/\mathrm{d}x)^2 - (\kappa_i/d_i d_n)(T_n - T_s) \tag{1}$$

$$\nu_s \dot{T}_s = \kappa_s T_s'' - (W_0/d_s)(T_s - T_0) + j_s E_s + \tfrac{1}{2}\rho_i (d_i d_n^2/d_s)(\mathrm{d}j_n/\mathrm{d}x)^2 - (\kappa_i/d_i d_s)(T_s - T_n) \tag{2}$$

$$\rho_i d_i d_n j_n'' - \rho_n j_n + E_s = 0. \tag{3}$$

Here ν is the specific heat, κ is the thermal conductivity, W_0 is the coefficient of heat transfer to the coolant at a temperature T_0, ρ is the resistivity, j is the current density and E is the electric field strength.

The system of equations for the FRC similar to equations (1)–(3) is

$$\nu_n \dot{T}_n = \kappa_n T_n'' - (2W_0/d_n)(T_n - T_0) + \rho_n j_n^2 + \tfrac{1}{4}\rho_i d_i d_n (\mathrm{d}j_n/\mathrm{d}x)^2 - (2\kappa_i/d_i d_n)(T_n - T_s) \tag{4}$$

$$\nu_s \dot{T}_s = \kappa_s T_s'' + j_s E_s + \tfrac{1}{4}\rho_i (d_i d_n^2/d_s)(\mathrm{d}j_n/\mathrm{d}x)^2 - (2\kappa_i/d_i d_s)(T_s - T_n) \tag{5}$$

$$\tfrac{1}{2}\rho_i d_i d_n j_n'' - \rho_n j_n + E_s = 0. \tag{6}$$

The analysis is simplified by using the dimensionless variables

$$\theta = (T - T_0)/(T_c - T_0) \qquad i_n = j_n d_n / j_c d_s$$

$$i_s = j_s/j_c \qquad \varepsilon_s = E_s d_n / \rho_n j_c d_s$$

and the parameters

$$i = i_s + i_n \qquad W = W_0 + (\kappa_i/d_i)$$

$$h = \kappa_i/d_i W \leqslant 1 \qquad \alpha = \rho_n j_c^2 d_s^2 / 2W_0(T_c - T_0)d_n.$$

Here T_c is the critical temperature and j_c is the critical current density of the superconductor. Note that if the thermal contact resistance is high, that is, if the heat transfer to the coolant is much greater than the heat exchange between the normal and the superconducting components of the composite, we have $W_0 > \kappa_i/d_i$, hence $h < 1$.

The system of equations describing the temperature and current distributions in the TRC can be rewritten in a more convenient form as

$$\tau_n \dot{\theta}_n = l_n^2 \theta_n'' - \theta_n + 2\alpha(1-h)i_n^2 + \alpha(1-h)l_i^2(\mathrm{d}i_n/\mathrm{d}x)^2 + h\theta_s \tag{7}$$

$$\tau_s \dot{\theta}_s = l_s^2 \theta_s'' - \theta_s + 2\alpha(1-h)i_s\varepsilon_s + \alpha(1-h)l_i^2(\mathrm{d}i_n/\mathrm{d}x)^2 + h\theta_n \tag{8}$$

$$l_i^2 i_n'' - i_n + \varepsilon_s = 0 \tag{9}$$

where the characteristic relaxation times τ_n, τ_s, the temperature variation lengths l_n, l_s and the length l_i over which current is leaked from the superconducting to the normal metal are $\tau_n = d_n \nu_n / W$, $\tau_s = d_s \nu_s / W$, $l_n^2 = \kappa_n d_n / W$, $l_s^2 = \kappa_s d_s / W$ and $l_i^2 = (\rho_i/\rho_n) d_i d_n$ respectively. Note that if the electrical contact resistance is high then $l_s \ll l_n \ll l_i$.

Using the same notation equations (4)–(6) can be rewritten to describe the distribution of temperature and current in the FRC as

$$\tfrac{1}{2}\tau_n \dot{\theta}_n = \tfrac{1}{2} l_n^2 \theta_n'' - \theta_n + \alpha(1-h)i_n^2 + \tfrac{1}{4}\alpha(1-h)l_i^2(\mathrm{d}i_n/\mathrm{d}x) + h\theta_s \tag{10}$$

$$\tfrac{1}{2}\tau_s \dot{\theta}_s = \tfrac{1}{2} l_s^2 \theta_s'' - h\theta_s + \alpha(1-h)i_s\varepsilon_s + \tfrac{1}{4}\alpha(1-h)l_i^2(\mathrm{d}i_n/\mathrm{d}x) + h\theta_n \tag{11}$$

$$\tfrac{1}{2} l_i^2 i_n'' - i_n + \varepsilon_s = 0. \tag{12}$$

Similarly, for equations (10)–(12) for the FRC the characteristic relaxation times $\tilde{\tau}_n$, $\tilde{\tau}_s$, the temperature variation lengths $\tilde{l}_n$, $\tilde{l}_s$ in the normal and superconducting components and the length $\bar{l}_i$ over which the current is leaked from the superconductor to the normal metal are $\bar{\tau}_n = \frac{1}{2}\tau_n$, $\bar{\tau}_s = (1/2h)\tau_s$, $\bar{l}_n = (1/\sqrt{2})\, l_n$, $\bar{l}_s = (1/\sqrt{2}h)\, l_s$ and $\bar{l}_i = (1/\sqrt{2})\, l_i$ respectively. Consequently, $\bar{\tau}_s$ and $\bar{l}_s$ in the FRC differ considerably from their counterparts τ_s and l_s in the TRC if the thermal contact resistance is high ($h < 1$). However, for typical parameters of composite superconductors the lengths l_s, l_n and l_i rank in the same order: $\bar{l}_s < \bar{l}_n < \bar{l}_i$.

In further numerical calculations we also need to know the explicit form of the functions $j_c = j_c(\theta_s)$ and $E_s = E_s(\theta_s, j_s)$. In this paper we assume for simplification that the critical current density is a linear function of temperature and that the current–voltage characteristic has the form

$$E_s = \begin{cases} 0 & \theta_s < 1 - i_s \\ \rho_s j_s & \theta_s \geqslant 1 - i_s. \end{cases} \tag{13}$$

In this case

$$\varepsilon_s = \frac{1}{r^2}\begin{cases} 0 & \theta_s < 1 - i + i_n \\ (i - i_n) & \theta_s \geqslant 1 - i + i_n \end{cases}$$

where

$$r^2 = \rho_n d_s / \rho_s d_n .$$

Typically, $r^2 \ll 1$. In the numerical calculations which follow we assume $r = 0.03$ and $\alpha = 2$.

3. Steady-state temperature distribution in the FRC

In this section we consider the steady-state temperature distributions corresponding to single resistive domains in the FRC. A similar problem was solved for the TRC by Akhmetov and Mints (1982, 1983a).

Our first assumption is that the term $h\theta_n$ in equation (11) is negligibly small. Obviously, this term accounts for the thermal coupling of the normal and superconducting components of the FRC. Given the current–voltage characteristic of the superconductor (equation (13)) we can then readily find the explicit form of the distributions of temperature $\theta_s = \theta_s(x)$ and current $i_n = i_n(x)$. Let ε_s be non-zero in the range $|x| < l$; $|x| < l$ implies that $\theta_s(x) > 1 - i + i_n(x)$. In this case l is found in a straightforward manner from the condition $\theta_s(l) = 1 - i + i_n(l)$. Here we shall not give explicitly the resultant and rather unwieldy transcendental equation for finding the dependence $l = l(\mathrm{i})$. For an arbitrary ratio of the parameters of the superconducting component this equation can be solved only numerically. In figures 2(*a*), (*b*) and (*c*) the results of such computations have been plotted for the following set of parameters: $l_n/l_s = \bar{l}_i/\bar{l}_s = 33$ and $h = 0.05, 0.1$ and 0.2 respectively. As in the case of the TRC the dependence $l = l(i)$ for a single resistive domain consists of two branches: one growing (upper branch) and one falling (lower branch). Note that in the constant-current mode the lower branch of the $l = l(i)$ curve corresponds to an unstable domain that can be stabilised, for instance, in the constant-voltage mode. The minimum existence current i_r of a resistive domain in the limiting case ($l \ll l_s$) is then found in the analytical form, namely

$$i_r^2 = \frac{4h}{\alpha(1-h)}\left(\frac{l_s}{\sqrt{h}\, l_i} + r^2\right). \tag{14}$$

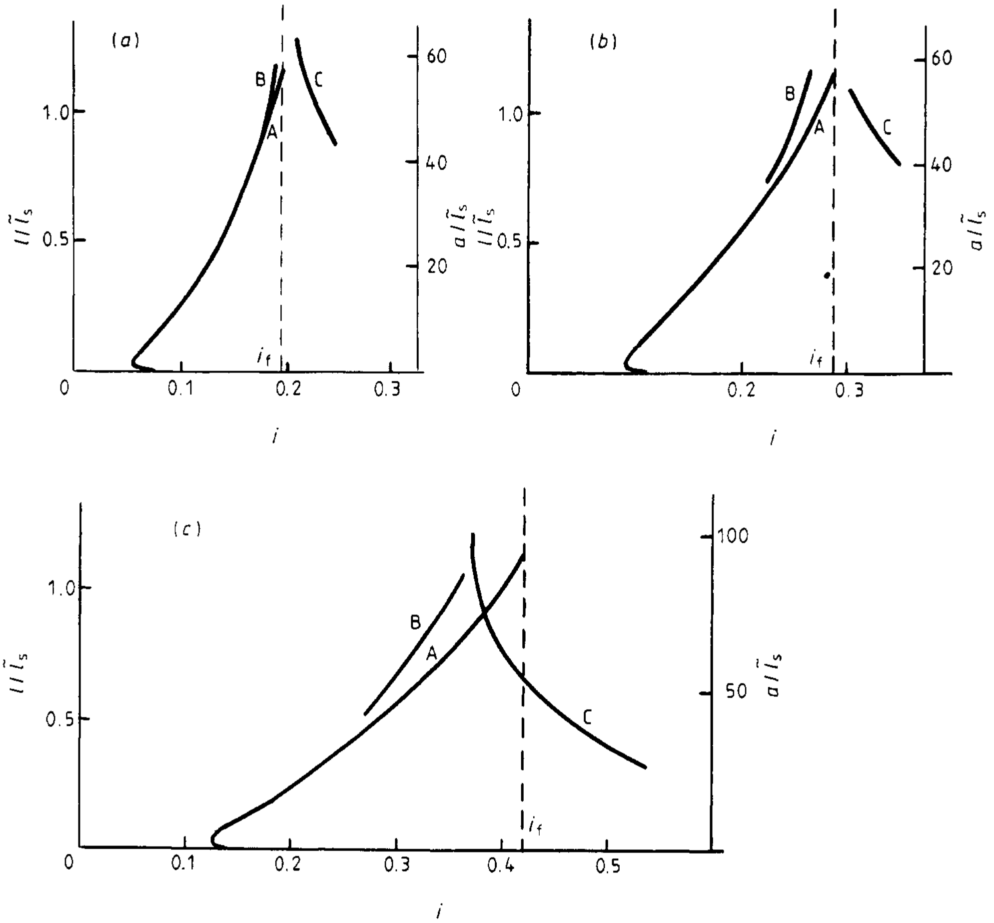

Figure 2. Curves $l = l(i)$ and $a = a(i)$ for the FRC with parameters $l_n/l_s = \tilde{l}_i/\tilde{l}_s = 33$. $l = l(i)$; A, $\theta_n \equiv 0$; B, θ_n is allowed to rise; C, $a = a(i)$. (*a*) $h = 0.05$, (*b*) $h = 0.1$, (*c*) $h = 0.2$.

Having found the function $l = l(i)$, the current–voltage characteristic of the FRC can be found from the relation

$$\varphi = \frac{i}{(1+r^2)^2} \frac{\tanh(l/\tilde{l}_b)}{\tilde{l}_b/\tilde{l}_i + \tanh(l/\tilde{l}_b)} + \frac{l}{\tilde{l}_i} \frac{i}{1+r^2}$$

where

$$\varphi = (1/2\tilde{l}_i) \int_{-\infty}^{+\infty} \varepsilon_s \, dx \qquad \text{and} \qquad \tilde{l}_b^2 = \tfrac{1}{2} l_i^2 [r^2/(1+r^2)].$$

Note that the quantity $\tilde{l}_b$ defines the length over which the current leaks from the superconducting component of the FRC, in its normal state, to the normal component of the FRC.

Figure 3 plots the current–voltage characteristics of the FRC and of an equivalent TRC (i.e. having identical values of all physical characteristics) calculated for the following set of parameters: $l_n/l_s = 33$, $l_i/l_s = 75$, $\tilde{l}_i/\tilde{l}_s = 33$ and $h = 0.2$. The range in which resistive

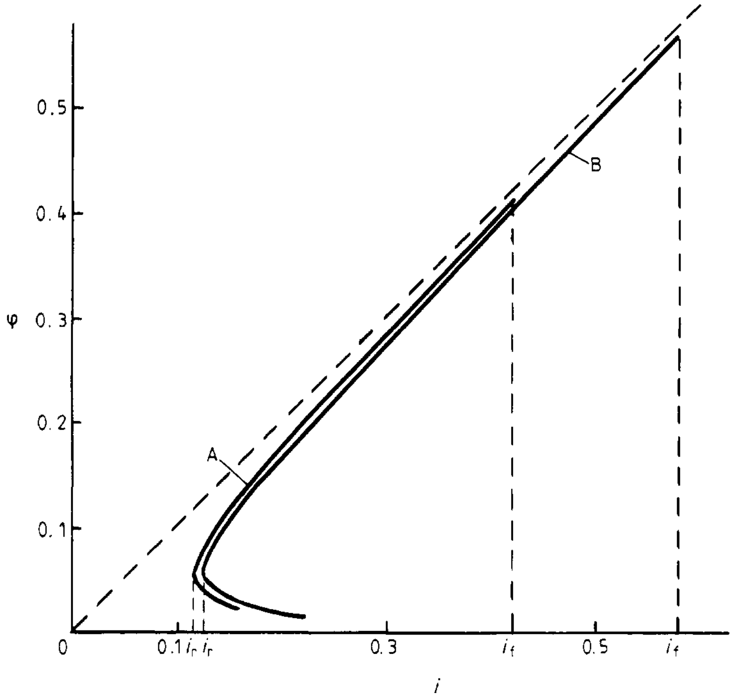

Figure 3. Current–voltage characteristics: A, for a FRC with parameters $l_n/l_s = \bar{l}_i/\bar{l}_s = 33$ and $h = 0.2$; B, for a TRC with parameters $l_n/l_s = 33$, $l_i/l_s = 75$ and $h = 0.2$.

domains exist in the FRC is clearly narrower than that in the TRC and is shifted toward lower currents.

As the transport current density tends to the value $i = i_f$, a typical dip appears on the curve $\theta_s = \theta_s(x)$ in the neighbourhood of the point $x = 0$, prior to the splitting of the domain in two, and the temperature $\theta_s(0)$ tends to the value $1 - i + i_n(0)$. No stationary resistive domains can exist in the FRC if $i \geqslant i_f$.

The effect of the thermal coupling of the normal and superconducting components of the FRC, the term $h\theta_n$ of equation (11) which was neglected earlier, was taken into account by a numerical solution of the non-stationary system of equations (10)–(12) which describes the temperature distributions $\theta_n = \theta_n(x, t)$ and $\theta_s = \theta_s(x, t)$. The resistive domain was generated by an initial thermal pulse localised in the superconducting component of the FRC ($\theta_n(x, 0) \equiv 0$ and $\theta_s(x, 0) = 1$ if $|x| \leqslant \frac{1}{2}\bar{l}_s$ or $\theta_s(x, 0) = 0$ if $|x| > \frac{1}{2}\bar{l}_s$).

The computations made it possible to analyse both the dynamics of formation of a resistive domain in the FRC and the thermal coupling of the normal and superconducting components of the FRC. The main results of these computations are as follows. The initial thermal pulse evolves to a stationary single resistive domain over a time of the order of $\bar{\tau}_s$. The length of the domain generated, $l = l(i)$, is somewhat greater than the value found when the thermal coupling of the normal and superconducting components of the FRC is neglected (see figure 2). However, the current–voltage characteristic of a specimen with the resistive domain remains practically unaffected.

Figure 4 shows the steady-state temperature distributions in the normal and super-

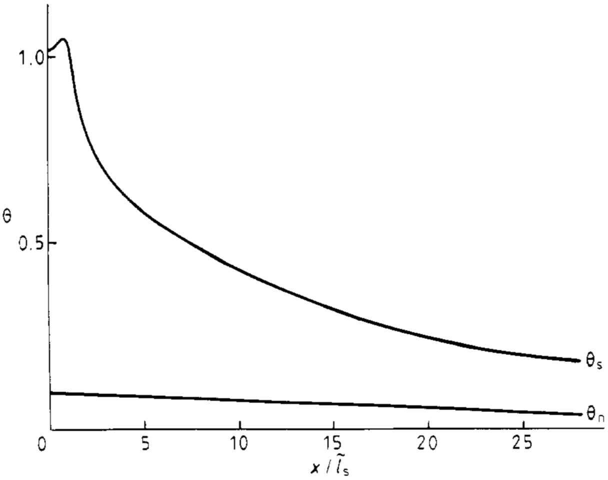

Figure 4. Steady-state temperature distributions in the normal and superconducting components of a FRC with parameters $l_n/l_s = \bar{l}_i/\bar{l}_s = 33$ and $h = 0.05$ for $i = 0.19$, $i_f = 0.2$.

conducting components of the FRC, calculated for the following set of parameters: $l_n/l_s = \bar{l}_i/\bar{l}_s = 33$, $h = 0.05$ and $i = 0.19$. The transport current density i is quite close here to $i_f = 0.2$. Consequently, the $\theta_s = \theta_s(x)$ curve has a characteristic dip in the neighbourhood of $x = 0$. Note that in contrast to the situation analysed earlier (Akhmetov and Mints 1983a) for the TRC, the temperature of the superconducting component in the FRC exceeds that of the normal component in any cross-section of the specimen.

4. Propagation of the normal zone ($i \geqslant i_f$)

The propagation of the normal zone generated by an initial thermal pulse for transport current densities $i \geqslant i_f$ was investigated by a numerical solution of equations (7)–(9) for the TRC, and of equations (10)–(12) for the FRC. Some of the results of these computations for the TRC were published earlier in a short note (Akhmetov and Mints 1983a). The initial thermal pulse was located in the superconducting component of the two composites, namely $\theta_n(x, 0) \equiv 0$ and $\theta_s(x, 0) = 1$ if $|x| \leqslant \frac{1}{2}l_s$ (for the TRC) or $|x| \leqslant \frac{1}{2}\bar{l}_s$ (for the FRC); $\theta_s(x, 0) = 0$ if $|x| > \frac{1}{2}l_s$ (for the TRC) or $|x| > \frac{1}{2}\bar{l}_s$ (for the FRC). In the case of the TRC the computations were performed for the following sets of parameters: (i) $l_n/l_s = 50$, $l_i/l_s = 100$ and $h = 0.05$; (ii) $l_n/l_s = 33$, $l_i/l_s = 75$ and $h = 0.2$. In the case of the FRC the computations were performed for $l_n/l_s = \bar{l}_i/\bar{l}_s = 33$ and (i) $h = 0.05$, (ii) $h = 0.1$, (iii) $h = 0.2$.

The computations revealed the following pattern of normal zone propagation in the TRC and the FRC for $i \geqslant i_f$. Over a time of the order of τ_s (in the TRC) or $\bar{\tau}_s$ (in the FRC) a non-stationary resistive domain created at the start grows in size to a length about equal to l_b (or $\bar{l}_b$). The temperature of the superconducting component at the domain centre (at $x = 0$) decreases to $\theta_s(0) = 1 - i + i_n(0)$, the electric field $\varepsilon_s(0)$ vanishes by virtue of the current–voltage characteristic (equation (13)) and a superconducting interlayer is formed in the neighbourhood of the point $x = 0$. As a result the non-stationary resistive domain, triggered by the initial thermal pulse at $i \geqslant i_f$, splits in two. The two daughter

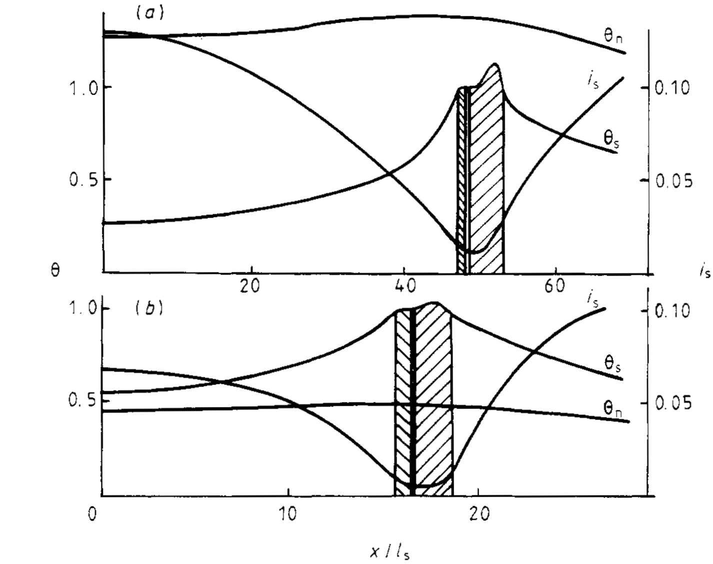

Figure 5. Distributions of temperature and current density i_s at the moment of splitting of a resistive domain: (a) in a TRC with parameters $l_n/l_s = 33$, $\bar{l}_i/\bar{l}_s = 75$ and $h = 0.2$ for $i = 0.70$; (b) in a FRC with parameters $l_n/l_s = l_i/l_s = 33$ and $h = 0.2$ for $i = 0.50$.

domains separate, recede and grow in size. At a distance of the order of l_i (or $\bar{l}_i$) each of them again splits in two. Figure 5 shows the distributions of temperature $\theta_n = \theta_n(x)$ and $\theta_s = \theta_s(x)$ and of current density $i_s = i_s(x)$ in the TRC and in the FRC at the moment when the electric field in the domains vanishes (the moment of splitting). Computations were carried out for the following parameters: $l_n/l_s = 33$, $l_i/l_s = 75$, $h = 0.2$ and $i = 0.7$ (for

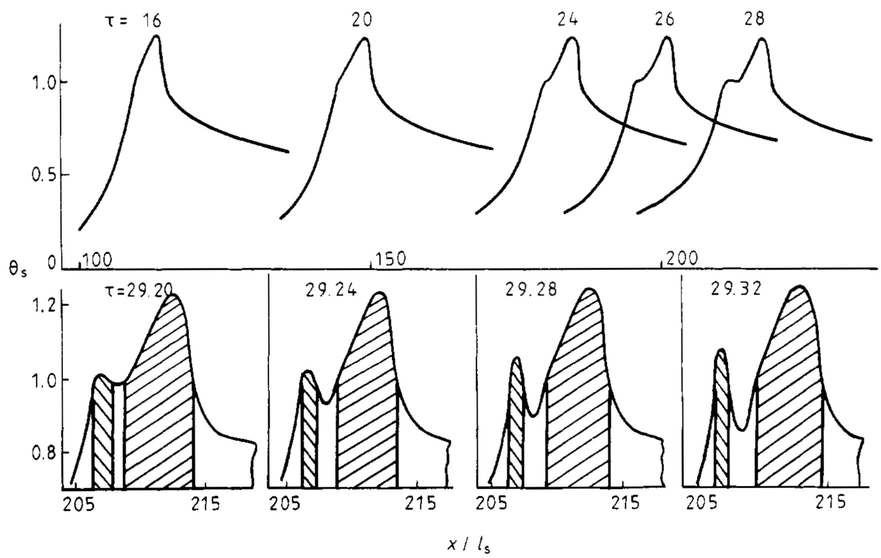

Figure 6. Evolution of temperature distribution in the superconducting component during the splitting of the resistive domain in a TRC with parameters $l_n/l_s = 50$, $l_i/l_s = 100$ and $h = 0.05$ for $i = 0.70$.

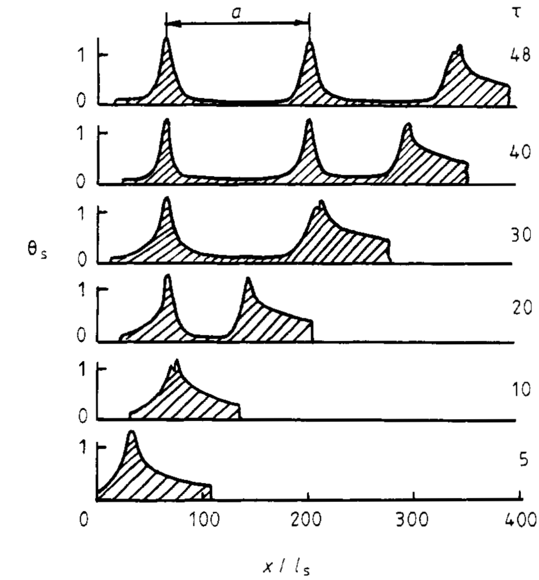

Figure 7. The process of formation of a string of resistive domains in a TRG with parameters $l_n/l_s = 50$, $l_i/l_s = 100$ and $h = 0.05$ for $i = 0.70$.

the TRC); $l_n/l_s = \bar{l}_i/\bar{l}_s = 33$, $h = 0.2$ and $i = 0.50$ (for the FRC). Figure 6 shows the temperature distribution $\theta_s = \theta_s(x)$ in the superconducting component of the TRC in the course of resistive domain splitting at different moments of dimensionless time $\tau = t/\tau_s$ ($l_n/l_s = 50$, $l_i/l_s = 100$, $h = 0.05$ and $i = 0.7$).

Further propagation of the normal zone proceeds as follows. The outermost resistive domains (the leading ones) continue moving away from the origin ($x = 0$). Their length increases and at a distance of the order of l_i (or $\bar{l}_i$) from the place of the previous splitting each of the two leading domains splits in two (into a leading and a drop-out domain). This process of formation and successive splittings of the leading domain is repeated periodically. At the moment of splitting the lengths of the leading and the drop-out domains are roughly $\frac{3}{4}$ and $\frac{1}{4}$ of the length of the parent domain (see figure 6). Note that the newly born leading domain invariably keeps moving away from the point $x = 0$. In contrast to this, the drop-out domain moves slowly (as compared with the leading one) in the opposite direction until the leading domain splits in two. At this moment the drop-out domain stops and assumes a fixed position. Figure 7 illustrates the above process of formation of a string of equidistant (to within the chosen computation accuracy) resistive domains in a TRC (the pattern is quite similar in an FRC). The parameters chosen for the computation were $l_n/l_s = 50$, $l_i/l_s = 100$, $h = 0.05$ and $i = 0.7$. Note that a similar 'self-organisation' of dissipative structures appears in a number of problems in physics, biophysics, chemistry and biology (see, e.g., Nicolis and Prigogine 1979).

Obviously, the formation of a periodic string of resistive domains results in an increased potential difference φ. As an example, figure 8 plots $\varphi = \varphi(t)$ (curve A) and $\varphi' = \varphi'(t)$ (curve B) calculated for a TRC with $l_n/l_s = 50$, $l_i/l_s = 100$ and $h = 0.05$, for two values of the transport current density: $i = 0.55$ and $i = 0.7$. Here $\varphi'(t)$ is the potential difference across the leading domain. Figure 8 clearly shows that as i increases

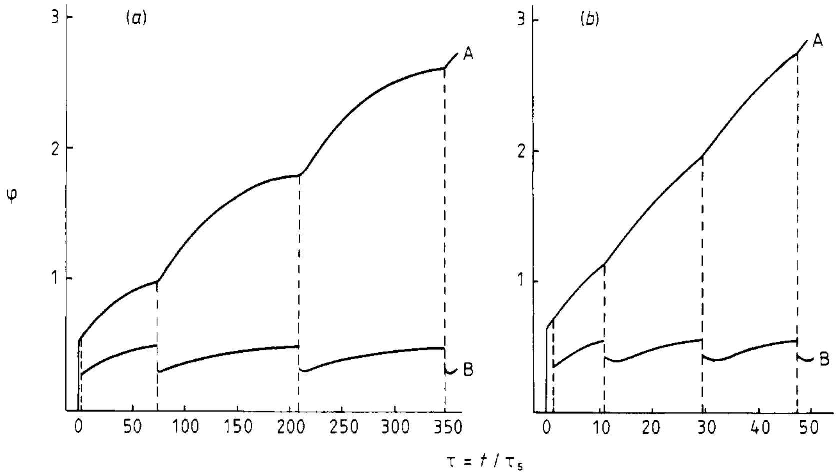

Figure 8. Functions $\varphi = \varphi(t)$ (A) and $\varphi' = \varphi'(t)$ (B) for a TRC with parameters $l_n/l_s = 50$, $l_i/l_s = 100$ and $h = 0.05$ for (*a*) $i = 0.55$ and (*b*) $i = 0.70$.

the function $\varphi(t)$ tends to linearity ($\varphi \sim t$). Note that the $\varphi = \varphi(t)$ and $\varphi' = \varphi'(t)$ curves of the FRC are similar to those of figure 8.

The mean velocity v of the leading domain in a superconducting composite with high contact resistance obviously determines (at $i \geqslant i_f$) the velocity of normal zone propagation. Figure 9 plots the ratio v/v_0 as a function of transport current density i, where $v_0 = l_s/\tau_s$ (for the TRC) or $v_0 = \tilde{l}_s/\tilde{\tau}_s$ (for the FRC). The following parameters were used in the calculations: $l_i/l_s = 75$, $l_n/l_s = \tilde{l}_i/\tilde{l}_s = 33$ and $h = 0.2$. For comparison, we show in the same figure the normal zone propagation velocity as a function of i in a composite

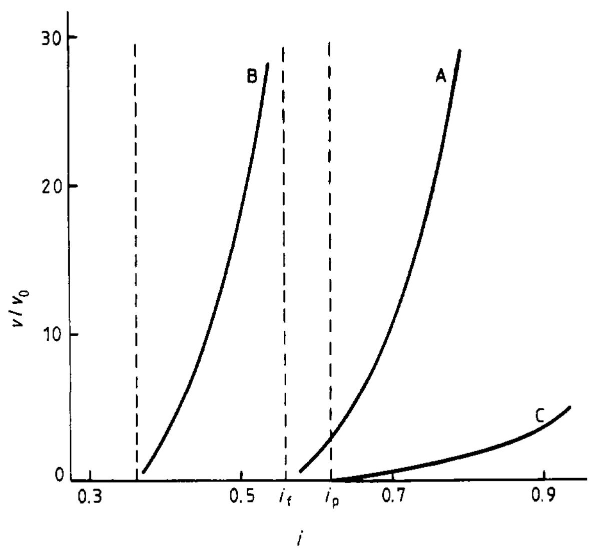

Figure 9. Functions $v = v(i)$: A, for a TRC with parameters $l_n/l_s = 33$, $l_i/l_s = 75$ and $h = 0.2$; B, for a FRC with parameters $l_n/l_s = \tilde{l}_i/\tilde{l}_s = 33$ and $h = 0.2$; C, for a composite equivalent to the TRC and the FRC with zero contact resistance.

superconductor equivalent to the TRC and the FRC but having zero thermal and electrical contact resistance between the normal and the superconducting components. Note also that in the range $0 < i - i_f \ll i_f$ of the transport current the advance of the leading domain in the interval between two successive splittings is essentially non-uniform. The velocity of the leading domain is a maximum at the moment of formation; it then decreases and reaches a minimum at the moment of splitting. At $i \gg i_f$ the velocity of the leading domain is practically constant.

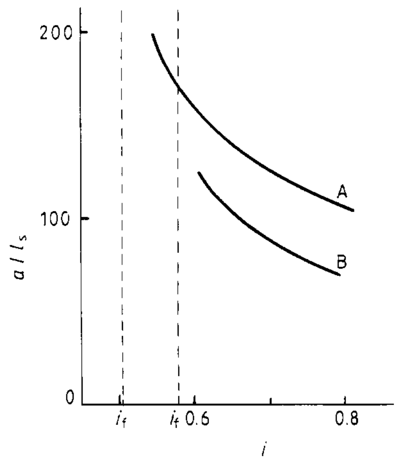

Figure 10. Function $a = a(i)$ for a TRC with parameters: A, $l_n/l_s = 50$, $l_i/l_s = 100$ and $h = 0.05$; B, $l_n/l_s = 33$, $l_i/l_s = 75$ and $h = 0.2$.

Obviously, the period a of the domain structure formed as a result of the 'self-organisation' as described above depends on the value of the transport current density. This dependence is shown for the TRC in figure 10 and for the FRC in figure 2. In the case of $0 < i - i_f \ll i_f$ we meet with considerable computational difficulties when calculating the domain structure because a grows indefinitely ($a \to \infty$) as $i \to i_f$. Figure 2(a) also shows that taking the thermal coupling of the normal and superconducting components of the FRC into account results in a certain decrease of the transport current density i_f.

5. Effect of boundary conditions on the propagation of the domain structure

In order to study the effect of specimen boundaries on the processes of formation and propagation of resistive domains, we have analysed the situation in which the temperature θ of the normal and superconducting components of the composite is assumed to be zero ($\theta_n = \theta_s = 0$) for $|x| > L$. Calculations were carried out for a TRC with the following set of parameters: $l_n/l_s = 50, l_i/l_s = 100, h = 0.05$ and $L = 2.5l_i$, for four values of the initial transport current density: $i = 0.55$, $i = 0.60$, $i = 0.65$ and $i = 0.70$. The thermal pulse that triggered the formation of the initial non-stationary domain was localised in the superconducting component ($\theta_n(x, 0) \equiv 0$ and $\theta_s(x, 0) = 1$ if $|x| \leqslant \frac{1}{2}l_s$; $\theta_s(x, 0) = 0$ if $|x| > \frac{1}{2}l_s$).

The main results of the analysis are as follows. After the formation of four domains (as a result of the appropriate number of splittings) in the $i = 0.55$ and $i = 0.60$ modes,

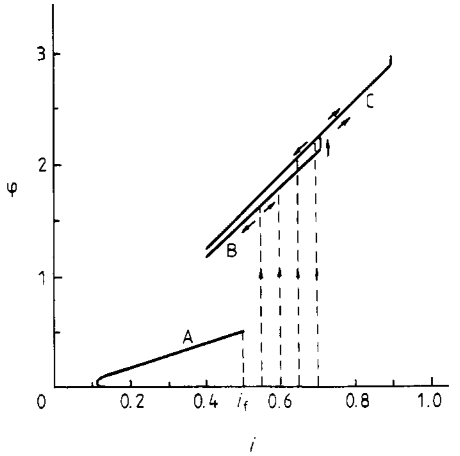

Figure 11. Current–voltage characteristic of a specimen with resistive domains (a TRC with parameters $l_n/l_s = 50$, $l_i/l_s = 100$ and $h = 0.05$). The specimen contains a single domain (A), four domains (B) and six domains (C).

and after the formation of six domains in the $i = 0.65$ and $i = 0.70$ modes, the leading domains reach the boundaries of the specimen ($|x| = L$) and the temperature and current distributions $\theta_n = \theta_n(x)$, $\theta_s = \theta_s(x)$ and $i_n = i_n(x)$ cease to change. The transport current density i was then varied slowly (in comparison with the relaxation rates of θ_n and θ_s) and the potential difference φ was calculated for each value of i. The current–voltage characteristics thus obtained, the $\varphi = \varphi(i)$, are shown in figure 11 and the corresponding stationary positions of the resistive domains for several different values of i are shown schematically in figure 12. The figures demonstrate that when the transport current density i increases from $i = 0.55$ to $i = 0.71$, the potential difference φ grows linearly. The change in temperature distribution $\theta_s = \theta_s(x)$ reduces to only a negligible displacement of the inner resistive domains (domain 1 in figures 12(a) and (b)). At $i = 0.71$ these domains split and their positions in the specimen change drastically. As can be seen in figure 11, the decrease in transport current density i is accompanied by a strong hysteresis (see also figure 12(d) where the distribution of resistive domains is shown for the initial

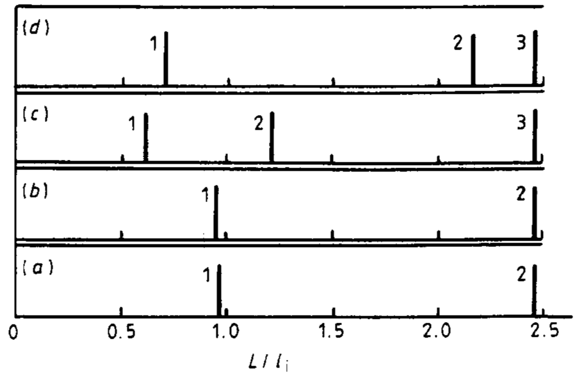

Figure 12. Schematic arrangement of resistive domains at different values of the transport current density i.

value $i = 0.65$) because the number of resistive domains and their positions within the specimen remain unaltered. Furthermore, the multi-domain structures generated at $i > i_f$ exist when i diminishes to below i_f (see figure 11). The results of the calculation also show that an increase in the number of resistive domains in a finite-sized specimen does not lead to a proportional growth of the potential φ. For a fixed number of domains and a given transport current density the value of φ is practically independent of the distribution of domains within the composite.

In order to analyse the multi-domain structures in a finite-sized specimen when $i < i_f$ for stability, we calculated the time of coalescence τ_f of two resistive domains as a function of the initial separation b between them. These domains were initiated in the superconducting components of a TRC with parameters $l_n/l_s = 50$, $l_i/l_s = 100$ and $h = 0.05$ at $i = 0.4$ and $i_f = 0.5$ by an initial thermal pulse such that $\theta_n(x, 0) \equiv 0$ and $\theta_s(x, 0) = 1$ if $|b - x| < \frac{1}{2}l_s$ and $|b + x| < \frac{1}{2}l_s$; $\theta_s(x, 0) = 0$ if $|b + x| > \frac{1}{2}l_s$ and $|b - x| > \frac{1}{2}l_s$. Figure 13 plots the function $\tau_f = \tau_f(b)$. Clearly, $\tau_f \sim \tau_s$ at $b \sim l_s$. However, the time τ_f rapidly increases as b increases. If we recall that the distance between resistive domains in multi-domain structures (in finite-sized specimens when $i < i_f$) is of the order of $l_i \gg l_s$, it becomes clear that these structures are metastable. Having been created once, in some manner, they exist virtually indefinitely (for constant i) and disappear jumpwise if i drops to $i \sim i_r$.

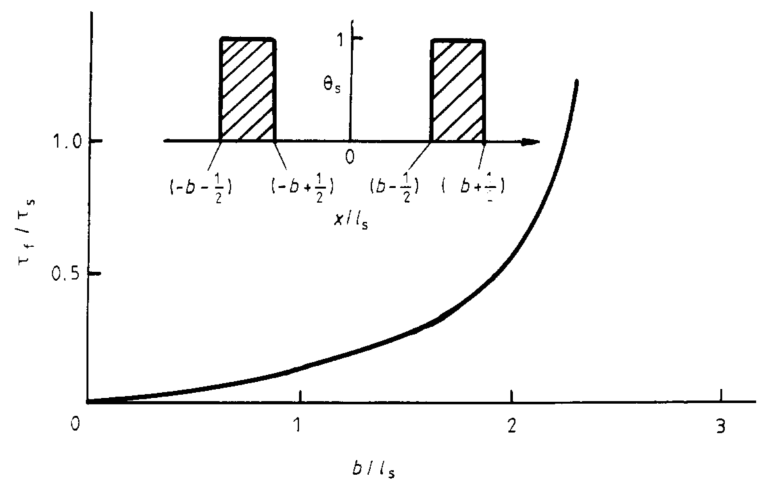

Figure 13. Time of coalescence τ_f of two resistive domains as a function of initial separation between them.

6. Conclusions

It is shown that the normal zone initiated by a thermal pulse in composite superconductors with high thermal resistance and high electrical contact resistance can propagate via multiple splitting of resistive domains. The minimum current I_f at which this process starts is less than, or of the order of, the minimum normal zone existence current I_m. The formation of resistive domains in a superconductor results in various hysteresis effects in quenching and recovery of superconductivity.

References

Al'tov V A, Zenkevich V B, Kremlev M G and Sychev V V 1975 *Stabilization of Superconducting Magnetic Systems* (Moscow: Energiya) (in Russian)
Akhmetov A A and Baev V P 1984 *Cryogenics* **24** 67–72
Akhmetov A A, Baev V P and Mints R G 1983 *Pis'ma v Zh. Tekh. Fiz.* **9** 1188–91
Akhmetov A A and Mints R G 1982 *Pis'ma v Zh. Tekh. Fiz.* **8** 1093–7
—— 1983a *J. Phys. D: Appl. Phys.* **16** 2505–18
—— 1983b *Pis'ma v Zh. Tekh. Fiz.* **9** 1306–10
Keilin V E and Kruglov S L 1984 *Cryogenics* **24** 525–30
Keilin V E and Ozhogina V K 1977 *Superconductivity* vol 4 (Moscow: Atomizdat) pp 170–3 (in Russian)
Kremlev M G 1980 *Izv. Akad. Nauk* Energetika i Transport no 4 10–6
Nicolis G and Prigogine I 1977 *Self-Organization in Nonequilibrium Systems* (New York: Wiley)