# Nonlinear waves under conditions of the quantum Hall effect

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It is shown that the quantum Hall effect leads to the appearance of nonlinear waves produced due to electronic overheating, travelling switching waves, thermoelectric domains, as well as stationary localized structures.

The quantum Hall effect[1] almost "nondissipative" states which are destroyed in a sufficiently strong electric field $E$ due to electronic overheating,[2,3] leading to a thermal instability.[3] In this paper we examine the nonuniform states arising from such instability.

  

We shall write the equation for the electronic temperature $\theta$

$$\nu \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \kappa \frac{\partial \theta}{\partial x} - j \Pi \frac{\partial \theta}{\partial x} + \rho(\theta) j^2 - W(\theta), \tag{1}$$

where $\nu$ and $\kappa$ are the electronic heat capacity and thermal conductivity, $j$ is the current density ($j \| x$), $\Pi = \theta(\partial\alpha/\partial\theta)$, $\alpha$ is Seebeck's constant, $\rho$ is the longitudinal resistivity, and $W(\theta)$ is the intensity of energy transfer from electrons to the lattice. At low temperatures ($\theta \lesssim \Gamma$, $\Gamma$ is the width of the Landau levels) $W(\theta)$ can be written in the form

$$W(\theta) = \frac{\pi^2 N_F \theta(\theta - T_0)}{3\tau(\theta, T_0)}, \tag{2}$$

where $T_0$ is the temperature of the lattice, $N_F$ is the density of states at the Fermi level, and $\tau$ is the energy relaxation time.

On the plateau of the Hall resistance, in the region $T_0 < \theta \lesssim \theta_c \sim \hbar\omega_c \equiv e\hbar H/mc$, the liberation of heat $\rho(\theta) j^2$ increases with increasing temperature by several orders of magnitude, increasing much more rapidly than the heat removal $W(\theta)$.[2] For $\theta \gtrsim \theta_c$, the situation changes and the growth of $\rho(\theta) j^2$ slows down sharply.[3-5] If in this region $\partial W/\partial\theta > j^2 \partial\rho/\partial\theta$, then the energy balance $W(\theta) = \rho(\theta) j^2$ in the range of currents $j_{c_1} < j < j_{c_2}$ is satisfied for three values of $\theta$ (see Fig. 1), of which $\theta = \theta_1$ and $\theta = \theta_3$ are stable, uniform states of the electronic system. We note that points 2 and 3 combine at $j = j_{c_1}$ and points 1 and 2 combine at $j = j_{c_2}$.

Since $\theta_3 \gtrsim \theta_c$, the value of $j_{c_1}$ can be estimated by using Eq. (2) and by assuming that $N_F \sim m/\hbar^2$ and that $\rho(\theta)$ and $\tau(\theta)$ depend weakly on $\theta$. From the condition $\rho j_{c_1}^2 \sim W(\theta_c)$ we then find

$$j_{c_1} \sim \frac{\theta_c}{\hbar} \left( \frac{m}{\tau\rho} \right)^{1/2} \Bigg|_{\theta = \theta_c}. \tag{3}$$

At $j = j_{c_2}$ we have $\theta_1 = \theta_2 \sim \theta_c$. Therefore, in order to estimate $j_{c_2}$ we can use Eq. (2) with $N_F \sim m/\hbar^2$. Assuming that $\rho = \rho_0 \exp(-U/\theta)$,[4,5] where $\rho_0 = \text{const}$, and

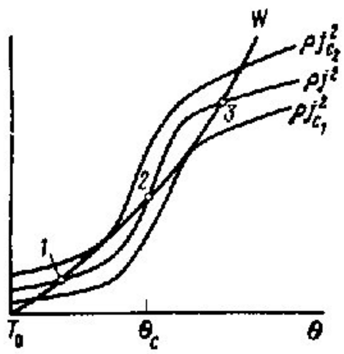


FIG. 1.

$U \sim \theta_c$, we obtain

$$j_{c_2} \sim \frac{U}{\hbar}\sqrt{\frac{m}{\tau\rho_0}}\ . \tag{4}$$

As is evident from (4), the dependence $j_{c_2}(H)$ is determined primarily by the dependence $U(H)$, consistent with experiment.[3]

An increase of the current density (from the region $j < j_{c_2}$) leads to the development of a thermal instability at $j = j_{c_2}$.[3] If $j > j_{c_2}$, then the "phases" with $\theta = \theta_1$ do not exist. At $j_{c_1} < j < j_{c_2}$ stratification into "phases" with $\theta = \theta_1$ and $\theta = \theta_3$ can occur and self-similar waves $[\theta = \theta(x - vt)]$ produced as a result of switching of the system from the state with $\theta = \theta_1$ to the state with $\theta = \theta_3$ and back again can arise, as well as solitary waves: thermoelectric domains (see, for example, Refs. 6 and 7).

In the fixed-current regime, only the switching wave is stable and its velocity $v$ changes sign at $j = j_p$. Judging from the results in Ref. 8, Thomson's heat is small here compared to $\rho j^2$, so that we shall first examine the case $\Pi = 0$. The value of $j_p$ can then be determined from the equation $S(\theta_3, j_p) = 0$, where

$$S(\theta, j) = \int_{\theta_1}^{\theta} \kappa(\theta)[W(\theta) - \rho(\theta)j^2]\,d\theta\ . \tag{5}$$

If $j > j_p$, then the "phase" with $\theta = \theta_3$ displaces the "phase" with $\theta = \theta_1$ $(j_{c_1} < j_p < j_{c_2})$. In the region $|j - j_p| \ll j_p$ we have $v = v_0(j/j_p - 1)$, where $v_0 \simeq L/\tau$ and $L \sim \sqrt{\kappa\tau/\nu}$ is the width of the wave front. At $\Pi \neq 0$, the velocity of the switching wave in the direction of the current $(v_+)$ and opposite to it $(v_-)$ differ by $v_+ - v_- \sim j\Pi/\nu$.

In the fixed-voltage regime the thermoelectric domain is stable, and the maximum temperature in it $\theta_m$ is determined from the equation $S(\theta_m, j) = 0$. If the length of the domain is $D \gg L$, then $\theta_m = \theta_3$,

$$D = [V - \rho(\theta_1)j_p L_0]/[\rho(\theta_3) - \rho(\theta_1)]j_p\ , \tag{6}$$

where $L_0$ is the length of the specimen, and $V$ is the applied voltage. The thermoelectric effect leads to motion of the domain in a uniform specimen at the velocity[7]

$$v_d = j\int_{\theta_1}^{\theta_m} \Pi S^{1/2}\,d\theta \Big/ \int_{\theta_1}^{\theta_m} \nu S^{1/2}\,d\theta, \tag{7}$$

which can be accompanied by oscillatory phenomena, analogous to the Gunn effect.[6]

In nonuniform specimens with $j \simeq j_p$, the formation of different stationary structures consisting of domains localized on inhomogeneities and switching waves is possible.[7] A change in $j$ or $H$ leads to a jump-like restructuring of such structures either due to the delocalization of some waves or because the system leaves the region of the plateau in the Hall resistance. This mechanism can account for the stepped nature of the destruction of the state with high conductivity observed in Ref. 3 when the current in a number of specimens was increased.

The appearance of an electric-field domain, localized between Hall junctions, was observed recently by Cage *et al.*[9] We shall estimate the parameters of such a domain, under the assumption that it arises due to electronic overheating. For GaAs with $H = 60$ kOe, $U \simeq (\hbar\omega_c/2) - \Gamma$ is of the order of 40–50 K.[4,5] Taking $\tau = 5\times10^{-9}$ s (Ref. 2) and setting for the estimate $N_F = m/\pi\hbar^2$, we find that under the conditions of Ref. 9 ($\rho = 243\ \Omega$, $j = 10^{-2}$ A/cm)$\theta_m \sim U$. Equation (4) gives in this case $j_{c_2} \sim 10^{-2}$ A/cm, i.e., a result which agrees well with the data in Refs. 3 and 9. Estimating $\kappa$ with the help of the Wiedemann–Franz law, we obtain $L \sim 10^{-4}$ cm and $v_0 \sim 10^4$–$10^5$ cm/s. The dependence $\Pi(\theta)$ is presently unknown for GaAs heterostructures, which makes it impossible to estimate the velocity of the domain $v_d$ with sufficient accuracy. If, however, $\theta_m \gg \hbar\omega_c$, then $v_d$ would be of the order of the drift velocity of electrons.

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