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Current–voltage characteristics and superconducting state stability in composites

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Abstract. The influence of the electric field dependence of the differential conductivity on superconducting state stability in multifilamentary superconducting composites is studied theoretically. The obtained stability criterion allows one to determine the upper limit of the values of transport current and background electric field in the superconductor. To illustrate the theory, the superconducting state stability is studied in a wire in a range of sufficiently high magnetic fields.

1. Introduction

Theoretical investigations of the superconducting state stability in multifilamentary superconducting composites have been made by several authors (Hart 1969a, b, Duchateau and Turk 1975, Kremlev *et al* 1976, 1977). In these papers the thermal and electromagnetic processes, whose development leads to the flux jump, have been treated in details and the stability criteria have been obtained for the case when the initial disturbances leading to the instability are large enough and the flux jump arises in the viscous flux flow mode. Since in the viscous flux flow mode, the differential conductivity $\sigma = \mathrm{d}j/\mathrm{d}E$ is independent of the electric field E (j is the current density), the stability criteria are independent of the background electric field in the sample and, in particular, of the rate of change of the external magnetic field $\dot{H}_a$ and transport current $\dot{I}$. However, at sufficiently low electric fields the current–voltage curve of the superconductor is nonlinear and differential conductivity σ is a function of the field E.

Recent experiments by Dorofeev *et al* (1978, 1980), Gentile *et al* (1980), Andrianov *et al* (1981, 1982) have demonstrated that the differential conductivity of the composites depends on E in a wide range of electric fields ($E \leqslant 10^{-5}$ V cm^{-1}). Moreover, the value of $\sigma = \sigma(E)$ may be much greater than σ_n, the conductivity of the normal (say, copper) matrix, at $E \leqslant 10^{-6}$–10^{-5} V cm^{-1}. The high value of $\sigma(E)$ at small E may considerably affect the superconducting (critical) state stability. As was shown in the previous studies, the stability increases with growth of σ. Consequently, the stability increases with the decrease of the value of background electric field in the sample.

In this paper we report a theoretical study of the superconducting state stability in multifilament composite conductors, taking into account the real nonlinear I–V characteristics of the material. The inhomogeneous composite conductor is considered here as a uniform anisotropic superconducting medium. The physical properties of such a medium are defined by superconducting filaments and normal matrix characteristics averaged over the cross-section of the composite. The applicability of such an approach has already been discussed in detail (Kremlev *et al* 1977, Mints and Rakhmanov 1977, 1981).

2. Dynamic stability criterion

As it is easy to find, the heat diffusivity $D_t = \kappa/\nu$ is much greater than the magnetic diffusivity $D_m = c^2/4\pi\sigma$ in superconducting composites, and

$$\tau = \frac{D_t}{D_m} = \frac{4\pi\sigma\kappa}{c^2\nu} \gg 1$$

where κ is the transverse (with respect to the filaments) heat conductivity, σ is the longitudinal (differential) electric conductivity and ν is the heat capacity. Let us denote the dimensionless parameter of the external cooling as follows:

$$W = W_0 S/\kappa P$$

where W_0 is the heat transfer coefficient and P and S are the cross-section perimeter and area. For composite superconductors one can assume that $W < 1$ in any practical situation.

Hart (1969a) has found the stability criterion in the first approximation with respect to $\tau \gg 1$ (dynamic stability criterion) which may be written in the form

$$I < j_m S = \frac{W_0 P \sigma}{|dj_S/dT|} \tag{1}$$

where I is the transport current and j_S is the critical current density in the composite. The criterion (1) is valid if $W < 1$ and the values j_S and σ are uniform. The accuracy of the criterion (1) is of the order of $(W\tau)^{-1/3}$ if $\tau \gg 1$ and $W\tau \gg 1$ (Kremlev *et al* 1977). In some cases the next term with respect to W/τ may be considerable and the maximum current density j_m may be higher than the value given by equation (1). Nevertheless, equation (1) determines j_m correctly within an order of magnitude up to $W\tau \sim 1$ (Kremlev *et al* 1977).

The value j_m calculated from equation (1) is hard to compare quantitatively with experiment in many cases (Mints and Rakhmanov 1977, 1981). This situation is mainly due to the lack of data necessary to calculate j_m, and one has to estimate the values $j_S(T)$, σ and W_0 or to use some adjusting parameters. The most unreliable are the estimations of the conductivity of the composite σ, as under the experimental conditions σ may be a function of the background electric field and consequently of $\dot{H}_a$, $\dot{I}$ and $\dot{T}$ (Mints and Rakhmanov 1979, 1981). It was assumed in the papers (Hart 1969a, b, Duchateau and Turk 1975, Kremlev *et al* 1976, 1977) that the priming disturbances giving rise to the flux jump are high enough to assume the composite to be in the viscous flux flow mode where

$$\sigma \simeq x_n \sigma_n$$

(x_n is the normal metal concentration). Recently Andrianov *et al* (1981, 1982), have reported the investigations of the superconducting state stability in composites, along with the measurements of W_0, j and $\sigma(E)$. The analysis of these experiments leads to the conclusion that the approximation

$$\sigma = x_n \sigma_n$$

is not adequate in many cases of practical interest. The flux jumps as a rule nucleate in the composites at relatively small E where

$$\sigma(E) \gg \sigma_n .$$

On the basis of this statement one can explain a number of experimental results, for example, the dependence of j_m on H_a, $\dot{H}_a$, W_0 and sample diameter, the inequality $j_m \gg j_p$, where j_p is the density of the normal zone propagation current (Andrianov *et al* 1981, 1982).

3. *I–V* characteristics of superconducting composites and stability

The *I–V* curves of the superconducting composites have been studied in a number of experiments. Dorofeev *et al* (1978, 1980) have studied in detail the function $j = j(E)$ for multifilamentary composites with copper matrix and Nb–Ti, Nb_3Sn or Nb–Zr filaments. Gentile *et al* (1980) have investigated *I–V* curves of single-core Nb–Ti–Cu conductors. Andrianov *et al* (1981, 1982) have measured the *I–V* curves of multifilament Nb–Ti–Cu conductors. These experiments have shown that in a wide range of electric (10^{-11} V cm^{-1} < E < 10^{-7}–10^{-5} V cm^{-1}) and magnetic ($0 < H_a < 7$ T) fields and temperatures (1.6 K $< T < 8.3$ K) the field E depends on j exponentially, i.e. $E \propto \exp(\eta \cdot j)$ and the longitudinal differential conductivity $\sigma(E)$ may be therefore expressed as follows:

$$\frac{dj}{dE} = \sigma(E) = \frac{j_1}{E} \qquad (2)$$

where $\partial j_1/\partial H < 0$ and in the range of fields $0 < H < 7$ T the value of j_1 is of the order of 10^3 A cm^{-2}. It is interesting to note that j_1 is independent of T as it was obtained by Gentile *et al* (1980).

The dependence $\sigma \propto E^{-1}$ was observed for different superconductors produced by different technologies. It allows one to suppose that the *I–V* characteristics of hard and composite superconductors is described by equation (2) over a wide range of parameters. In the present paper the dependence (2) will be used to find the explicit form of the respective stability criteria.

To evaluate the electric field E_0 at which $\sigma(E_0) = \sigma_n$ let us assume that

$$j_1 = 2 \times 10^3 \text{ A cm}^{-2}$$

and

$$\sigma_n = 3 \times 10^{19} \text{ S}^{-1} (3.3 \times 10^7 \,\Omega^{-1} \text{cm}^{-1}).$$

We find then that $E_0 = 6 \times 10^{-5}$ V cm^{-1}. The background electric field $E \sim 10^{-5}$–10^{-4} V cm^{-1} may be induced in the sample of radius $R = 10^{-1}$ cm, for example, by the external magnetic field varied at the rate of $\dot{H}_a \sim 10^4$–10^5 Oe S^{-1}. Thus the limit $\sigma = x_n \sigma_n$ is achieved at a rather high level of the external perturbations.

The criterion (1) can be used to calculate j_m at $E \ll E_0$ in some simple cases, where the background electric field E is uniform. Consider for example a wire of the radius R with a transport current I and assume that

$$0 < I - I_S \ll I_S = \pi R^2 j_S$$

(j_S is the critical current density). In this case one can readily find that the distribution of the background electric field is uniform. If the conductivity $\sigma(E)$ is described by the dependence (2) and

$$\tau(E) = \frac{4\pi\kappa j_1}{c^2 \nu E}$$

then the stability criterion has the form of equation (1) with $\sigma(E) = j_1/E$. Assuming for simplicity that

$$\frac{\mathrm{d}j_S}{\mathrm{d}T} = -\frac{j_S(T_0)}{T_c - T_0} \tag{3}$$

we have from equation (1) the stability criterion of interest:

$$R < R_m = \frac{2W_0(T_c - T_0)j_1}{j_S^2 E}. \tag{4}$$

Thus if $R < R_m(E)$ then the resistive transition is not accompanied by a flux jump. Let us now evaluate R_m numerically. The resistive transition is usually determined at $E = 10^{-6}\ \mathrm{V\,cm^{-1}}$. Then by making use of the values

$$j_1 = 2 \times 10^3\ \mathrm{A\,cm^{-2}}$$

$$j_S = 10^5\ \mathrm{A\,cm^{-2}}$$

$$E = 10^{-6}\ \mathrm{V\,cm^{-1}}$$

$$W_0 = 5 \times 10^5\ \mathrm{erg\,s^{-1}cm^{-2}K^{-1}}\ (5 \times 10^{-2}\ \mathrm{W\,cm^{-2}K^{-1}})$$

one can evaluate R_m as $\sim 10^{-1}$ cm. Note that $R_m \sim 10^{-3}$ cm at $\sigma \sim \sigma_n \sim 3 \times 10^{19}$ $\mathrm{S^{-1}}(3.3 \times 10^7\ \Omega^{-1}\mathrm{cm^{-1}})$.

The criterion (4) may be also treated as the upper limit to the value of the background electric field E and the inequality (4) may be rewritten as

$$E < E_m = \frac{2W_0(T_c - T_0)j_1}{R j_S^2}. \tag{5}$$

So, the superconducting state is stable in a composite with given properties at fixed external cooling W_0 if $E < E_m$. It must be emphasised that the criterion (5) allows one to calculate the upper limit of any priming perturbation giving rise to the existence of the background electric field in the superconductor. Let us now consider the situation in the general case.

4. Basic equations

The values E, H and T are described by the Maxwell and heat equations:

$$\left.\begin{aligned} \nu\frac{\partial T}{\partial t} &= \nabla(\kappa\nabla T) + \boldsymbol{j}\cdot\boldsymbol{E} \\ \operatorname{curl}\operatorname{curl}\boldsymbol{E} &= -\frac{4\pi}{c^2}\frac{\partial \boldsymbol{j}}{\partial t} \end{aligned}\right\}. \tag{6}$$

To find the stability criterion with respect to small disturbances, it is convenient to seek the solution of equations (6) in the form:

$$\left.\begin{aligned} \boldsymbol{E} &= \boldsymbol{E}_b + \frac{\kappa(T_b)T_b}{j_S(T_b)R^2}\,\boldsymbol{\varepsilon}\exp(\lambda t/t_\kappa) \\ \boldsymbol{H} &= \boldsymbol{H}_b + \frac{\kappa(T_b)T_b}{j_S(T_b)R^2}\,\boldsymbol{h}\exp(\lambda t/t_\kappa) \\ T &= T_b[1 + \theta\exp(\lambda t/t_\kappa)]. \end{aligned}\right\} \tag{7}$$

Where E_b, H_b, T_b are the background values of electric and magnetic fields and temperature, $\varepsilon, h, \theta \ll 1$ are the corresponding perturbations,

$$t_\kappa = R^2\nu(T_b)/\kappa(T_b)$$

is the time of heat diffusion and λ is the eigenvalue of the problem to be found. It is natural to suppose that $E_b(t)$, $H_b(t)$ and $T_b(t)$ vary slowly compared with $\varepsilon(t)$, $h(t)$ and $\theta(t)$.

By making use of the conditions $W < 1$ and $\dot{T}_b, \dot{E}_b \simeq 0$ one can easily find from (7)

$$\left.\begin{aligned} T_b &= T_0 + \frac{1}{W_0 P}\int \mathrm{d}S(\boldsymbol{j}_S \cdot \boldsymbol{E}_b)_{T_b} \\ &\text{curl curl}\,\boldsymbol{E}_b = 0 \end{aligned}\right\} \tag{8}$$

where the integral is taken over the cross-section of the sample.

The magnetic field H_b may be obtained from the equation:

$$\text{curl}\,\boldsymbol{H}_b = (4\pi/c)\,\boldsymbol{j}(T_b, E_b, H_b).$$

Inserting the expressions (7) in equations (6) it is easy to find

$$\left.\begin{aligned} (\lambda + \varepsilon_b)\theta &= \nabla^2\theta + \left(1 + \frac{j_1}{j_S(T_b)}\right)\varepsilon + \frac{\alpha_b}{\lambda\tau_b}\frac{j_1}{j_S(T_b)}(\boldsymbol{e}\cdot\text{curl}\,\boldsymbol{\varepsilon}) \\ (\boldsymbol{e}\cdot\text{curl curl}\,\boldsymbol{\varepsilon}) &= \lambda\beta_b\theta - \alpha_b(\boldsymbol{e}\,\text{curl}\,\boldsymbol{\varepsilon}) - \lambda\tau_b\varepsilon \end{aligned}\right\} \tag{9}$$

where

$$\boldsymbol{e} = \boldsymbol{\varepsilon}/\varepsilon \qquad \alpha_b = \frac{4\pi R}{c}\left|\frac{\partial j}{\partial H}\right|_{T_b, E_b, H_b}$$

$$\varepsilon_b = \frac{R^2 E_b}{\kappa(T_b)}\left|\frac{\partial j}{\partial T}\right|_{T_b, E_b, H_b} \qquad \tau_b = \tau(T_b, E_b, H_b)$$

$$\beta_b = \frac{4\pi R^2}{c^2}\left(\frac{j}{\nu}\left|\frac{\partial j}{\partial T}\right|\right)_{T_b, E_b, H_b}$$

and the space derivatives are taken with respect to the dimensionless coordinates $\boldsymbol{x}/R$.

To determine the eigenvalues λ, boundary conditions must be imposed on equations (9). If, for the given values of parameters, there exists some $\lambda > 0$ for which the equations have a solution, instability occurs.

As it can be readily seen this procedure is rather complicated. But in many cases of practical interest it may be considerably reduced. In the range of electric fields $E \leqslant 10^{-5}$ V cm^{-1} and at $W_0 \geqslant 10^5$ erg cm^{-2} s^{-1} K^{-1} the values τ_b and $W\tau_b$ are large enough $\tau_b \sim 10^4$–10^5, $W\tau_b \sim 10^2$–$10^3 \gg 1$. Analogously with the results of Kremlev *et al* (1976, 1977), to find the stability criterion under these conditions, one may reduce the fourth-order system (9) to the second-order one by putting $|\lambda|\tau_b, |\lambda|\beta_b \to \infty$, but $|\lambda| \ll 1$.

As $\sigma(E_b)E_b = j_1 \ll j_S$ we find from the equations (9):

$$\nabla^2\theta + (\beta_b/\tau_b - \lambda)\theta = 0. \tag{10}$$

The usual thermal boundary conditions on the external surface of the sample A should be imposed on equations (10):

$$\left(\boldsymbol{n}\frac{\partial\theta}{\partial\boldsymbol{n}} + \frac{RW_0}{\kappa(T_b)}\right)_A = 0 \tag{11}$$

where $\boldsymbol{n}$ is the outward normal to the surface A. Note that in general case the ratio β_b/τ_b depends on the coordinates.

The second simplification follows from the experimental fact, that

$$j = j(T_b, H_b, E_b) \simeq j_S = I_S/A$$

(the critical state model).

Thus on the basis of equations (10) and (11) one can find the stability criterion in the first approximation with respect to $(W\tau_b)^{-1} \ll 1$. Note that the dynamics of the flux jump evolution (i.e. the eigenvalue spectrum $\lambda = \lambda(\tau_b, \beta_b, W)$) could not be found by this method.

To solve equation (10) one can use the WKBJ method (Mints and Rakhmanov 1976, 1981). However, the stability criterion may be found by means of the following simple procedure in the case $W < 1$ which is of practical interest.

By integrating the equation (10) over the sample volume V, and then using Gauss's theorem and the boundary condition (11), it may be found (in the first with respect to the $(W\tau_b)^{-1} \ll 1$ approximation):

$$\lambda = \left(\int_V \frac{\beta_b}{\tau_b}\theta\,\mathrm{d}V - \frac{W_0R}{\kappa(T_b)}\int_A \theta\,\mathrm{d}A\right)\cdot\left(\int_V \theta\,\mathrm{d}V\right)^{-1}. \tag{12}$$

In these terms the stability criterion has the form $\lambda < 0$ or

$$\int_V \frac{\beta_b}{\tau_b}\theta\,\mathrm{d}V < \frac{W_0R}{\kappa(T_b)}\int_A \theta\,\mathrm{d}A.$$

Under the condition $W < 1$ the temperature is practically uniform in the sample during the instability development (Mints and Rakhmanov 1981) and one can look for the solution of equation (10) in the form:

$$\theta = \theta_0 + W\theta_1 + W^2\theta_2 + \ldots$$

where $\theta_0 = \text{const}$, $\theta_1 \sim \theta_2 \sim \ldots \sim \theta_0$. In the first approximation it is easy to find that

$$\int_S \frac{\beta_b}{\tau_b}\,\mathrm{d}S < \frac{W_0P}{\kappa(T_b)}. \tag{13}$$

The corrections to the criterion (13) are of the order of $W^2 \ll 1$.

The parameter τ_b depends on the value of the background electric field E_b as it was stated before. Consider now, for example, the case when j_S and j_1 are independent of the coordinates. Using the explicit expressions for τ_b, β_b, W and the equation (2) one may find the criterion of interest in the final form

$$\langle E_b\rangle = \frac{1}{S}\int_S E_b\,dS < E_m = \frac{W_0 P}{S}\left(\frac{j_1}{j_S\,|\partial j_S/\partial T|}\right)_{T_b}. \tag{14}$$

Thus, the superconducting state in the composite is stable if the background electric field, averaged over the conductor cross-section, does not exceed the value E_m. Note that if the electric field E is uniform the criterion (14) coincides with the inequality (5).

We have to emphasise that equation (12) leads to the fact that the instability occurs at $\lambda = \lambda_c \rightarrow 0$. However, the equation (12) is correct at $W\tau_b \rightarrow \infty$ and $\lambda_c\tau_b \rightarrow \infty$ at $\tau_b \rightarrow \infty$. There is not any contradiction between the two conditions $\lambda_c \rightarrow 0$ and $\lambda_c\tau_b \rightarrow \infty$. Both of them can be obtained on the basis of more accurate treatment of the problem using equations (9) and appropriate boundary conditions (see for example Mints and Rakhmanov 1981).

To illustrate the results obtained here we shall study below two specific examples.

5. The wire with a transport current $I \geqslant I_S$

Let us consider a wire of radius R placed in a transverse magnetic field $H_a = H_a(t)$. The transport current in the wire I = const and

$$0 < I - I_S \ll I_s.$$

The electric field E does not change its direction in the sample if the derivative $\partial H_a/\partial t$ is not too large (see figure 1). The conditions of applicability of such an assumption will be found below.

To simplify the calculations we shall investigate here only the case of sufficiently high magnetic fields, where one can suppose that j_1 and j_S are practically independent of the local value of H and

$$j_1 \simeq j_1(H_a) \qquad j_S \simeq j_S(H_a) \qquad |H_a - H| \ll H_a.$$

In the case under consideration we have for E_b:

$$\text{curl}\,\boldsymbol{E}_b = -(1/c)\,\dot{\boldsymbol{H}}_a$$

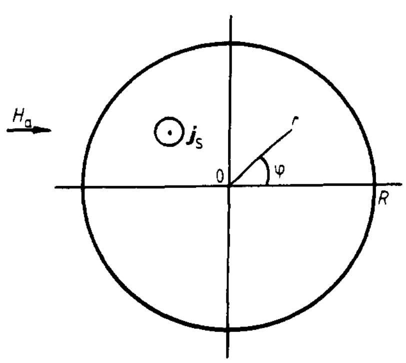

Figure 1. The wire with a current $I \geqslant I_S$.

and

$$E_b = \frac{R\dot{H}_a}{c} r \sin\varphi + E_1 \tag{15}$$

here $E_1 = \text{const}$ and may be found from the condition

$$I = R^2 \int_0^{2\pi} d\varphi \int_0^1 r j(E_b)\, dr. \tag{16}$$

As the differential conductivity σ is proportional to E^{-1} then the current density $j(E)$ may be written in the form (Dorofeev *et al* 1978)

$$j(E) = j_0 + j_1 \ln(E/E_0) \tag{17}$$

where $j_0 = I/\pi R^2$, and the value of E_0 is defined from the condition $j(E_0) = j_0$. Substituting $j = j(E)$ in the form (17) into the equation (16) one can obtain after integration:

$$\ln\left(\frac{E_1 + [E_1^2 - (R\dot{H}_a/c)^2]^{1/2}}{2E_0}\right) + \left(\frac{E_1 c}{R\dot{H}_a}\right)^2 - \frac{E_1 c}{R\dot{H}_a}\left[\left(\frac{E_1 c}{R\dot{H}_a}\right)^2 - 1\right]^{1/2} = \frac{1}{2}. \tag{18}$$

The value $\langle E_b \rangle = E_1$ is easy to find in two extreme cases $\dot{H}_a = 0$ and $\dot{H}_a = cE_1/R$: we obtain

$$E_1 = E_0 (\dot{H}_a = 0)$$

and

$$E_1 = 1.21\, E_0 (\dot{H}_a = cE_1/R).$$

The electric field does not change its direction if $\dot{H}_a < cE_1/R$, then $0 < \dot{H}_a < 1.21\, E_0 c/R$. In this range of $\dot{H}_a$ the stability criterion has obviously the form

$$\langle E_b \rangle = E_1 \leqslant 1.21 E_0 < \frac{2W_0}{R}\left(\frac{j_1}{j_S\, |\partial j_S/\partial T|}\right)_{T_b} \tag{19}$$

where $j(E_0) = I/\pi R^2$ and the temperature T_b is equal to

$$T_b = T_0 + \frac{IE_1}{2\pi R W_0}. \tag{20}$$

6. Maximum transport current in the wire

In this section we shall consider a wire of radius R placed in the constant external transverse magnetic field H_a. The transport current $I = 0$ at $t = 0$. Then the current I increases at constant rate $\dot{I}$, $I < I_S$ (see figure 2). A current of density $j \simeq j_S$ flows in the region $\delta < r < 1$, where $\delta = \delta(t)$ is defined by the equation

$$I = R^2 \int_0^{2\pi} d\varphi \int_\delta^1 r j_S\, dr. \tag{21}$$

Here we shall find the maximum transport current I_m at which the superconducting state becomes unstable.

By means of Maxwell's equations and appropriate boundary conditions it is easy to

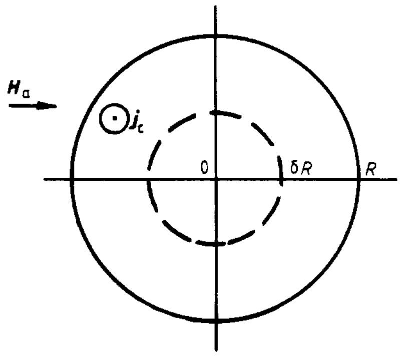

Figure 2. The wire with a current $I < I_S$.

obtain

$$E_b = (2\dot{I}/c^2)\ln(r/\delta). \tag{22}$$

Assuming as in the previous section that H_a is high enough ($|H_a - H(r)| \ll H_a$) let us suppose that $j_1 = j_1(H_a)$, $j_S = j_S(H_a)$. Then using criterion (14) one gets

$$\int_\delta^1 r\ln(r/\delta)\,\mathrm{d}r < \left(\frac{c^2 j_1 W_0}{2j_S\,|\partial j_S/\partial T|\,R\dot{I}}\right)_{T_b} \tag{23}$$

where $\delta = (1 - I/I_S)^{1/2}$. After integration the equation to determine I_m has the following form:

$$\left.\begin{aligned} &i_m = I_m/I_S \\ &i_m + \ln(1 - i_m) + \left(\frac{2W_0c^2 j_1}{R\dot{I}j_S\,|\partial j_S/\partial T|}\right)_{T_b} = 0. \end{aligned}\right\} \tag{24}$$

The equation (24) coincides with an analogous one derived in the paper by Andrianov *et al* (1981) on the basis of a qualitative treatment.

The value of I_m obtained from equation (24) is in good quantitative agreement with experiment (Andrianov *et al* 1981, 1982). Note that the dependence of I_m on H_a, W_0, R and $\dot{I}$ differs considerably from the results given by the dynamic criterion (1) at $\sigma = x_n\sigma_n$.

7. Conclusions

It has been shown that the nonlinear dependence of resistive current on electric field affects considerably the superconducting state stability in composites. The stability criterion (13) allows one to determine the upper limit of the transport current in the sample as well as the rate of change of external parameters. The criterion (13) allows one to find the maximum level of external perturbations giving rise to the existence of the background electric field in the sample. Such perturbations may be thermal, electromagnetic or mechanical in nature.

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