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ON THE THEORY OF NORMAL ZONE PROPAGATION IN SUPERCONDUCTORS

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This paper contains a study of properties of stable normal (N) regions of finite size - resistive domains (R.D.), in superconductors (S) with transport current I. It is demonstrated that in homogeneous superconductors R.D. are moving due to thermoelectric effect (Thomson heat) while the rate of R.D., v_d, for different materials ranges from 1 to 10^2cm/s. It is also shown that the thermoelectric effect leads to asymmetry in the rate of the N-S boundary Δv, relative to the direction of I, with $\Delta v \sim v_d$. The conditions for localizing R.D. in an inhomogeneous superconductor have been obtained, as well as the I-V characteristics of a sample with R.D. Hysteresis effects are discussed associated with the localization of R.D and the thermoelectric effect.

1. Introduction

The propagation of N-zone in superconductors with transport current has been repeatedly discussed in literature (cf., Refs.[1,2]) . This however, mainly involved the study of a single N-S boundary movement due to Joule heat release in the N-zone. The problem of the existence, under the same conditions, of limited stable N-regions such as R.D. has been considerably less studied. On the other hand, the behaviour of such R.D. in real superconducting systems may be of considerable practical interest.

2 . Basic equations

For simplicity, let us consider the case when the phase distribution in a superconductor is described by the solution of the one-dimensional heat equation

$$\nu \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} æ \frac{\partial T}{\partial x} - jq\frac{\partial T}{\partial x} - f(T, x) \quad (1)$$

where f = W - Q, W = h(T) $(T-T_o)/d$ is heat transfer to a cooler with a temperature T_o, h(T) is heat transfer coefficient, d = A/P, A is the area P is the cross sectional perimeter of the sample, q(T, |j|) is the thermoelectric constant (Thomson constant), j=I/A, ν is heat capacity, æ is heat conductivity, Q is Joule heat release per unit volume which can be written as:

$$Q = \frac{j^2}{\sigma}\eta \quad (2)$$

$$\eta = \begin{cases} 0, & T < T_r \\ 1 - j_c/|j|, & T > T_r \end{cases} \quad (3)$$

where σ is conductivity, j_c is the critical current density, T_r is the root of the equation $j = j_c(T_r)$. The function $\eta(T)$ defines the state of superconductor, i.e., normal state at $T > T_c$, resistive state at $T_c > T > T_r$, superconducting state at $T < T_r$ (T_c is the critical temperature of superconductor). In an inhomogeneous sample, f, æ, q and ν depend explicitly upon both T and x.

Consider first the movement of a single N-S boundary in a homogeneous superconductor at a constant rate v. Clearly, the inclusion of Thomson heat $Q_T = -jq\frac{\partial T}{\partial x}$ will lead to asymmetry in the rate of the N-S boundary relative to the direction of I inasmuch as Q is independent of the sign of j while Q_T varies the sign upon the change of j to -j (at real values of parameters $\varepsilon = Q_T/Q \sim jq\left(\frac{d}{hæ}\right)^{1/2} \ll 1$).

The rate v can be found in the general case only numerically, or for models in which Eq.(1) is solved accurately (the procedure for finding v is described, for example, in Refs.[1,2]). We shall demonstrate here the influence of thermoelectric effect in the case where v is small, i.e., $\delta = (I - I_p)/I_p \ll 1$ where I_p is the minimum propagation current of the N-zone[1]. The solution to Eq.(1), satisfying the boundary conditions $T(\infty) = T_o$, $T(-\infty) = T_N$, $\frac{\partial T}{\partial x}\big|_{\pm\infty} = 0$ in a system of coordinates moving at a rate v, at $\varepsilon \ll 1$, $\delta \ll 1$ has the form:

$$x = \frac{1}{\sqrt{2}} \int_T^{T_c} \frac{æ\, dT}{S^{1/2}} \quad (4)$$

$$S(T) = \int_{T_o}^{T} (W - Q)\, æ\, dT \quad (5)$$

The formulae (4), (5) describe the static N-S boundary having a width on the order of $L = (d\varkappa/h)^{1/2}$, with $T(0) = T_c$, and T_N and I_p being defined by the "equal area theorem"[1]: $S(I_p, T_N) = 0$, $W(I_p, T_N) = Q(I_p, T_N)$. Using Eqs.(4) (5), we find that $v_{\pm} = v_0 \pm \Delta v$ where $v_{\pm}$ are the rates of N-S boundary parallel (v_+) and antiparallel (v_-) the current, v_0 and Δv being defined to an accuracy up to max $(\varepsilon, \delta) \ll 1$ by the expressions:

$$v_0 = \gamma \frac{|I| - I_p}{\int_{T_0}^{T_N} \nu S^{1/2} dT} \qquad (6)$$

$$\Delta v = j_p \frac{\int_{T_r}^{T_N} q S^{1/2} dT}{\int_{T_0}^{T_N} \nu S^{1/2} dT} \qquad (7)$$

where $j_p = I_p/A$, $\gamma = \sqrt{2}\frac{\partial}{\partial I} S(I, T_N)\Big|_{I=I_p}$. Using the formulae (6), (7), we find the currents I_p^+ and I_p^- at which $v_+=0$ and $v_- = 0$, respectively. As a result, $I_p^+ = I_p - \Delta I$, $I_p^- = I_p + \Delta I$, where

$$\Delta I = \frac{j_p}{\gamma} \int_{T_r}^{T_N} q S^{1/2} dT \qquad (8)$$

Note that ΔI and Δv depend on the value of $q(T)$ in the N (resistive) regions of the sample inasmuch as the thermoelectric effects in the S-regions are negligibly small. In particular, for a composite superconductor, ΔI and Δv only depend on the thermoelectric constants of normal metal in the composite inasmuch as $\sigma_s/\sigma_n \ll 1$ where σ_n and σ_s are the conductivities of normal metal and superconductor in the composite, respectively. From Eq. (6) we derive that $\Delta v \sim jq/\nu$ (for example, we shall assume $q > 0$). Assuming $\nu = 10^4$ erg/cm^{3}K, $j = 5\cdot10^4$ A/cm^2, $q = T\cdot10^{-8}$ V/K and $T_N = 20$ K, we find $\Delta v \sim 10$ cm/s which is in qualitative agreement with the data of Ref.[3].

It follows from the formulae (6),(7) that: 1) at $I_p^+ < I < I_p^-$ we obtain $v_+ > 0$, $v_- < 0$. In this region both N and S-states of the sample with current are metastable. Indeed, a relatively strong fluctuation of T at the sample edge transforms the sample from N-state to S-state ($v_- < 0$) and, vice versa, from the S-state to N-state ($v_+ > 0$). Therefore, the thermoelectric effect leads, at $I_p^+ < I < I_p^-$, to hysteresis upon the destruction (restoration) of superconductivity with current; 2) at $I = I_p$ we obtain $v_+ = -v_- = \Delta v$. In this case, if the length of the N-zone $\mathcal{L} \gg L$, $\mathcal{L}$ does not vary with time, i.e., there occurs a R.D. moving at a rate of $v_d = \Delta v$. Therefore, v_d depends on the presence of the thermoelectric effect.

3. R.D. in a homogeneous superconductor

The temperature distribution in R.D. may be obtained at an arbitrary ratio of $\mathcal{L}$ and L, i.e , at $I \neq I_p$. The corresponding solution to Eq.(1), satisfying the boundary conditions $T(\pm\infty) = T_0$, $\frac{\partial T}{\partial x}\Big|_{\pm\infty} = 0$ in a system of coordinates moving at a rate of v_d has, at $\varepsilon \ll 1$, the form:

$$|x| = 2^{-1/2} \int_T^{T_m} \varkappa S^{-1/2} dT + 0(\varepsilon) \qquad (9)$$

Here, T_m is found from the equation $S(I, T_m) = 0$ having nontrivial ($T_m \neq T_0$) solutions only at $I > I_p$. The rate $v_d = \Delta v$, where Δv is defined by the formula (7) with $T_n = T_m$. Using Eq.(9), the I-V characteristic of R.D. $V = V(j)$ in an infinite superconductor can be derived in the general form:

$$V = 2^{1/2} \int_{T_r}^{T_m} \varkappa\, \sigma^{-1}(j - j_c)\, S^{-1/2} dT \qquad (10)$$

An analogous stationary R.D. was considered by the authors of Ref.[4]. Without taking into account the boiling crisis of the cooler, the $V = V(j)$ dependence is shown qualitatively in Fig.1 (curve 1). The dropping character of I-V characteristic is indicative of the R.D. instability in the fixed current regime. The stabilization of R.D. can be effected by shunting the sample with a resistance $r < \left|\frac{\partial V}{\partial I}\right|$ where $V(I)$ is found with the aid of the formula (10).

The movement of R.D. in a homogeneous superconductor may lead to the generation of variable electric field and current[5], analogously with Gann effect in semiconductors[6]. Such a generator can be visualized as follows. The sample is divided into region 1 where $I > I_p$ and region 2 where $I < I_p$ (for example, due to different values of heat transfer, h_1 and h_2, in the regions 1 and 2). Then, R.D. can only exist in region 1 because it disappears on getting to the "cooler" region 2. R.D. may

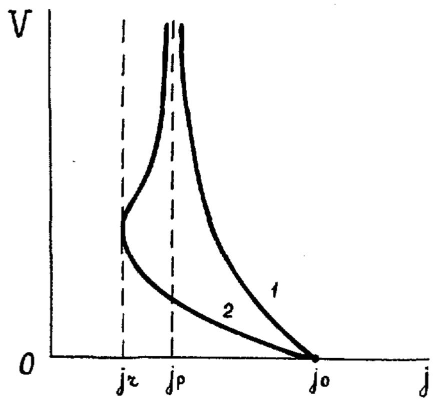

Fig.1

emerge either due to external heat pulses or because of inhomogeneities in the superconductor properties, for example, in a "weak" link featuring a reduced critical current[5]. In so doing, the generation frequency is equal to v_d/L_1 where L_1 is the distance from the point of R D. formation to the boundary of the regions 1 and 2.

4. R.D. in an inhomogeneous superconductor

In an inhomogeneous superconductor, R.D. may localize on the inhomogeneity[7]. Consider now a static localized R.D., assuming the inhomogeneity to be in the $-l < x < l$ region, with $\ell \ll L$ where L is of the order of the R.D. length. For simplicity assume $q = 0$. Then, at $\ell \ll L$, Eq (1) within the inhomogeneity can be solved in the general form. The continuity conditions $T(\pm\ell)$ and $\left.\frac{\partial T}{\partial x}\right|_{\pm\ell}$ for the solution at $|x| < \ell$ and for the solution at $|x| > \ell$ (cg., formula (9)) yield the following equation for determining the maximum temperature in the domain, $T_m = T(0)$:

$$S(T_m) = \frac{1}{8} F^2 (T_m) \tag{11}$$

where

$$F^2(T_m) = \left[\int_{-\ell}^{\ell} f(T_m, x)\, dx\right]^2 - 8\,\text{æ}_m f_m \int_0^{\ell} \frac{dx}{\text{æ}(T_m, x)} \int_0^{x} f(T_m, x')\, dx' \tag{12}$$

f_m and æ_m are the values of f and æ in the homogeneous portion of the superconductor at $T = T_m$. If f and æ do not explicitly depend on x, $F \equiv 0$. The existence of non-trivial solutions to Eqs.(11), (12) points to the possibility of existence of localized R.D. on the homogeneity. From the physical standpoint, such localization is associated with increased heat release on the inhomogeneity. In this sense, the condition $\ell \ll L$ enables one to regard the inhomogeneity in Eq.(1) as a point heat source, $F(T, j) \cdot \delta(x)$. It should be stressed that the value of F is not a constant here but is found selfconsistently from Eqs.(11),(12). The temperature distribution and I-V characteristic of localized R.D. are provided by the formulae (9),(10) where T_m is found from Eqs.(11),(12). Eqs.(11),(12) may have several roots, which points to the possibility of localizing R.D. of several types differing by the value of T_m. R.D. of each type has on the I-V curve its branch defined by Eq.(10).

As an example illustrative of solutions to Eqs.(11), (12), we shall consider a case when h is independent of T while the inhomogeneity is due to a decrease of æ and σ at $|x| < \ell$, with æ and σ being related to each other by means of Wiedemann-Franz law. In this case, $F^2/F_\circ^2 = 4\alpha i^2 \theta \eta \Gamma^2$ where $i = j/j_\circ$, $j_\circ = j_c(T_\circ)$, $F_\circ = (T_c - T_\circ)h/d$, $\theta = (T - T_\circ)/(T_c - T_\circ)$, $\alpha = d j_\circ^2/(T_c - T_\circ)\sigma h$ is Stekly parameter, $\Gamma = \frac{\ell}{L}\frac{\Delta R}{R}$, R is the resistance of a homogeneous N-portion having a length of 2ℓ at $T = T_m$, ΔR is excess, as compared to R, resistance of inhomogeneity, and $\Delta R \gg R$. Note that $F = 0$ at $T < T_r$. The I-V characteristic of an R.D. localized on such an inhomogeneity is shown in Fig.1 (curve 2). The most important is the existence of R.D. in the $j_r < j < j_p$ region where the I-V characteristic is two-valued while one of its branches is rising. At $j = j_r$ (cf., Fig.1), R.D. on the inhomogeneity disappears in a jump. The expression for $i_r = j_r/j_\circ$ in the case when $j_c = j_\circ(1 - \theta)$ while æ and σ are independent of T has the form:

$$i_r = \left(1/(4\alpha_1^2) + 2/\alpha_1\right)^{1/2} - 1/(2\alpha_1) \tag{13}$$

where $\alpha_1 = (1 + \Gamma^2)\alpha$, $\Gamma^2 \ll 1$. The formula for j_p is derived from Eq.(13) by the substitution $\alpha_1 \to \alpha$ [2]. Therefore, the difference between

j_r and j_p will only be considerable in the case of a strong inhomogeneity $\Delta R/R \sim L/\ell$. It can be shown that the effect of other inhomogeneities (for example, in j_c, h, etc.) is relatively small inasmuch as any inhomogeneity, with the exception of inhomogeneity in σ and A, only affects heat release in proportion to ℓ/L. A study of the stability of localized R.D. shows R D. with a rising I-V characteristic to be stable upon any connection of the sample in the circuit. R.D. with a dropping I-V characteristic can be stabilized by shunting.

The presence of localized R.D. leads to a number of hysteresis effects. In particular, the current $I_r = j_r \cdot A$ is in this case the superconductivity recovery current, with $I_r < I_p$. In case several strong inhomogeneities (for example, junctions) are present in the sample a series of steps emerge on the I-V characteristic associated with successive disappearance of R.D upon the reduction of current ($I < I_r^{(n)}$ where n is the inhomogeneity number). If the distance between the inhomogeneities is greater than the length of R.D., no such steps upon the increase of current inasmuch as the formation of new R.D. calls for a fluctuation $\delta T \sim T_r - T_0$ over a length on the order of L

5. Conclusions

It has been shown that the propagation of N-zone in a superconductor is asymmetric with respect to the direction of transport current, which is associated with thermoelectric effect. This effect also leads to the movement of R.D. and to hysteresis upon the destruction (restoration) of superconductivity with current. The movement of R.D. leads, under certain conditions to the generation of variable electric field and current.

In an inhomogeneous superconductor, R.D. is localized on the inhomogeneity, with the existence of R.D. of several types being possible at $j_r < j < j_p$. The value of superconductivity recovery current depends in this case upon the value of inhomogeneity.

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