The thermoelectric mechanism of the asymmetrical velocity of normal zone propagation in superconductors, relative to the direction of the transport current is suggested. The difference of the normal-superconducting interface velocities parallel and anti-parallel to the current direction due to this mechanism is of the order of 1 - 50 cm s^{-1} for different metals. The thermoelectric effect (Thomson heat) has a hysteresis which occurs opon destruction (or restoration) of superconductivity in the sample with respect to the transport current.

Asymmetry of the normal zone propagation velocity in superconductors

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Bartlett et al.[1] had observed the asymmetry of the normal zone propagation velocity in a multifilamentary composite superconductor with respect to the direction of transport current, I. In this paper we shall consider a possible mechanism causing the asymmetry. The results are valid for the composite superconductors and thin superconducting films.

The propagation of the interface between the normal, N, and superconducting, S, regions is defined by the heat flow equation. This equation takes into account the heat, Q, generated in the normal and resistive zones of the sample, the heat, $W(T)$, transferred to the coolant, and the heat flux from the N to S-zone.[2] The heat, Q, is usually assumed to be equal to Joule heat, Q_J, which is independent of the current direction. In this case the interface propagation velocities parallel (v'_+) and antiparallel (v'_-) to the current direction are the same. However, apart from Q_J, the heat Q is also contributed to by the thermoelectric effects (Thomson heat) which reverse their sign with the reversal of current direction.[3] This effect leads to the asymmetry of the N-S interface velocity ($v'_+ \neq v'_-$). Let us now consider this phenomenon in more detail.

Basic equations

Consider the N-S interface, Fig. 1a, propagating along the sample (x-axis) with the rate v'. The temperature distribution in the sample, $T(x)$, is described by the heat flow equation. This equation is one-dimensional when $hd/\kappa_\perp \ll 1$, where $h(T)$ is the coefficient of heat transfer to the coolant, $\kappa_\perp$ is the transverse heat conductivity, $d = A/P$, where A is the area, P is the perimeter of the cross-section. Hence the heat flow equation may be written in the co-ordinate system moving with the N-S interface in the form:

$$\frac{\mathrm{d}}{\mathrm{d}x}\,\kappa\,\frac{\mathrm{d}T}{\mathrm{d}x} + (\nu v' - qj)\,\frac{\mathrm{d}T}{\mathrm{d}x} + Q_J - W = 0 \qquad (1)$$

where ν and κ are the cross-section-averaged heat capacity and heat conductivity, respectively; $Q_T = -qj\,\mathrm{d}T/\mathrm{d}x$ is the Thomson heat[3]; $W = h(T)(T - T_0)/d$; T_0 is coolant temperature; Q_J is given by

$$Q_J = \frac{j^2}{\sigma}\,\eta \qquad (2)$$

$$\eta = \begin{cases} 0, & T < T_r \\ 1 - j_c/|j|, & T > T_r \end{cases} \qquad (3)$$

where σ and j_c are the cross-section averaged electrical conductivity and critical current density, respectively; $j = I/A$. Temperature T_r is found from $j = j_c(T_r)$. The function $\eta(T)$ determines the state of the superconductor: normal when $T > T_c$, resistive when $T_r < T < T_c$, superconducting when $T < T_r$ (T_c is critical temperature of the superconductor).

To find the velocity v' we shall multiply (1) by $\kappa\,\mathrm{d}T/\mathrm{d}x$ and integrate with respect to x from $-\infty$ to $+\infty$:

$$V' = \frac{S(T_m) + j\displaystyle\int_{T_r}^{T_m} q\kappa\,\frac{\mathrm{d}T}{\mathrm{d}x}\,\mathrm{d}T}{\displaystyle\int_{T_0}^{T_m} \nu\kappa\,\frac{\mathrm{d}T}{\mathrm{d}x}\,\mathrm{d}T} \qquad (4)$$

$$S(T) = \int_{T_0}^{T} (W - Q_J)\,\kappa\,\mathrm{d}T \qquad (5)$$

where T_m is temperature of the N-zone determined by $Q(T_m) = W(T_m)$, moreover the stability condition $\partial^2 S/\partial T^2|\,T = T_m > 0$ must be satisfied. We shall now estimate the relation between Q_T and Q_J. The characteristic values of Q_T are of the order of $(T_m - T_0)\,jq/L$, where $L = (\mathrm{d}\kappa/h)^{1/2}$ is the thermal correlation length (which is of

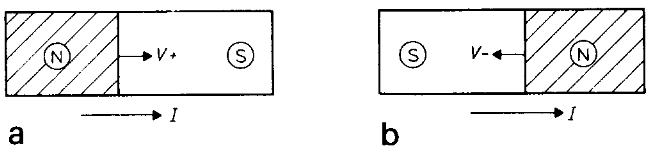

Fig. 1 The normal superconducting interface propagating along the samples x - axis

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the order of the N-S boundary width).
Since $T_m - T_0 \sim Q_J d/h$, then

$$Q_T \sim \frac{jq}{L}\frac{Q_J d}{h} = \epsilon Q_J;\ \epsilon = jq(d/h\kappa)^{1/2}$$

For the characteristic values of the parameters one finds that $\epsilon \ll 1$ (for the qualitative evaluations see below) and $\epsilon(j) = -\epsilon(-j)$. The condition $\epsilon \ll 1$ allows the expressions for $T(x)$ and $v'(j)$ to be represented in the form: $T(x,j) = T_1(x,j) + \epsilon T_2(x,j)\ v'(j) = v'_0(j) + \epsilon v'_1(j)$, where $T_1(x,j) = T_1(x,-j)$ is the solution of (1) for $q = 0$, $T_2(x,j) = T_2(x,-j)$, $v'_0(j) = v'_0(-j)$, $v'_1(j) = v'_1(-j)$. Function $v'_1(j)$ may be easily expressed through $T_2(x,j)$ from (4). Thus $\Delta v' = v'_+ - v'_- = 2\epsilon v'_1$. Using (4) and (5), the values $v_\pm$ may be found numerically for every solution of (1).

Special cases

It is impossible to obtain an explicit expression for $v'(j)$ at arbitrary j in the general form. First, we shall consider the case when the velocities $v'_\pm$ are small. If $v'_\pm = q = 0$, then (1) can be solved exactly. In this case the solution of describing the N-S interface may be written in the form

$$x = \frac{1}{\sqrt{2}}\int_T^{T_c} \frac{\kappa\,dT'}{S^{1/2}} \tag{6}$$

where $T(0) = T_c$, $T(-\infty) = T_m$, $T(\infty) = T_0$. The velocity v' may be found now by taking into account the term $(vv' - qj)\,dT/dx$ in (1) as a perturbation. In this case the derivative dT/dx in (1) may be calculated using the solution (6). As a result one obtains

$$v' = \frac{|\gamma|(|I| - I_p) + j\int_{T_r}^{T_m} q\,S^{1/2}dT}{\int_{T_0}^{T_m} vS^{1/2}dT} \tag{7}$$

where I_p is the root of the equation $S(T_m, I_p) = 0$; $\gamma = \sqrt{2}\{\partial S(T_m, I)/\partial I\}_{I_p}$. The current I_p is the minimum current for the propagation of a normal zone at $q = 0$.[2] Equation (7) is valid when the condition $\max(\epsilon, (I - I_p)/I_p) \ll 1$ is satisfied. From (7) it is seen that $v'_+ \neq v'_-$, and also $v'_+ = v'_0 + (1/2)\Delta v'$; $v'_- = v'_0 - (1/2)\Delta v'$; where:

$$v'_0 = \frac{|\gamma|(|I| - I_p)}{\int_{T_0}^{T_m} vS^{1/2}dT},\quad \Delta v' = 2j_p\frac{\int_{T_r}^{T_m} qS^{1/2}dT}{\int_{T_0}^{T_m} vS^{1/2}dT} \tag{8}$$

Note that at $I = I_p$ the velocities v'_+ and v'_- are the same in magnitude but have opposite signs. In Fig. 2a this situation corresponds to both N-S interfaces moving in the same direction.

From (7) it is easy to find the currents I_p^+ and I_p^- for which $v_+ = 0$ and $v_- = 0$, respectively. We find that $I_p^+ = I_p - \Delta I$; $I_p^- = I_p + \Delta I$, where

$$\Delta I = \frac{j_p}{|\gamma|}\int_{T_r}^{T_m} qS^{1/2}dT \tag{9}$$

Here $j_p = I_p/A$. Thus when we take into account the thermoelectric effects $I_p^+ \neq I_p^-$. In particular, for $q > 0$, we have $I_p^+ < I_p^-$, since $\partial S/\partial I\ (T_m, I) < 0$

To find $v_0(j)$ and $\Delta v(j)$ at arbitrary j let us consider a simple model, where h, κ, σ, v are independent of T. It is convenient now to use dimensionless temperature $\theta = (T - T_0)/(T_c - T_0)$. For the functions $Q(\theta)$ and $q(\theta)$ we shall assume the simplest approximations: $Q_J = j^2/\sigma$; $q(\theta) = q_0$ at $\theta > \theta_r$; $Q(\theta) = q(\theta) = 0$ at $\theta < \theta_r$, q_0 = const. For composite superconductors it may be assumed that $j_c = (1 - \theta)j_0$ and $\theta_r = 1 - i$, where $i = j/j_0$. For thin superconducting films $j_c = (1 - \theta)^{3/2} j_0$,[4] $\theta_r = (1 - i)^{2/3}$. Solving (1), we have for $\Delta v'$:

$$\Delta v' = jq_0/v \tag{10}$$

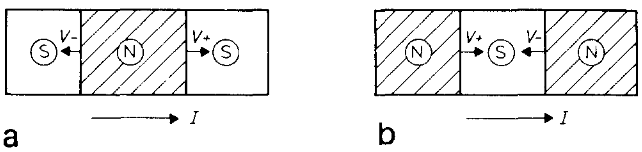
Value of v'_0 is given by[5]:

$$v'_0 = \frac{1}{v}\left(\frac{h\kappa}{d}\right)^{1/2}\frac{ai^2 - 2\theta_r}{[(ai^2 - \theta_r)\theta_r]^{1/2}} \tag{11}$$

where $a = j_0^2 d/(T_c - T_0)\sigma h$ is Stekly's parameter. In the model in question, $\Delta v(j)$ lineary increases with the current. The velocity asymmetry of the N-S interface in this case is connected with the thermoelectric effect only in the N-zone of the sample. Note that this model describes qualitatively the situation in thin films, since the thermoelectric effects are vanished in type-I superconductors. In composite superconductors $jq = jx_n q_n + jx_s q_s$, where x_n and x_s are the shares of the normal metal (n) and superconductor (s), $x_n + x_s = 1$. Then, $jq_n = T\,\partial/\partial T\,(\pi_n j_n)$;[3] $jq_s = j_s T\eta_s^2\,\partial\pi_s/\partial T$,[6] where j_n and j_s are current densities through the normal metal and superconductor; π_n and π_s are thermoelectric constants[3]; $\eta_s = 1 - j_c/(x_s|j_s|)$ when $T > T_r$, $\eta_s = 0$ when $T < T_r$. The thermoelectric currents usually are much smaller than the ohmic current ($\epsilon \ll$ i), therefore: $j_s = j_c/x_s + j\sigma_s\eta/\sigma$, $j_n = j\sigma_n\eta/\sigma$, $\sigma = \sigma_n x_n + \sigma_s x_s$. For a normal metal with high conductivity ($\sigma_n \gg \sigma_s$) and $x_n \gg \sigma_s/\sigma_n$ the value of q is determined only by the thermoelectric constants of the normal metal in the composite, since $\pi_n \sim \pi_s$. Therefore, for the composite superconductor:

$$q = x_n T\frac{\partial}{\partial T}(\pi_n\eta) \tag{12}$$

We will now evaluate the value of $\Delta v'$. Taking $v = 10^4$ erg cm^{-3} K^{-1}, $j \sim j_c \sim 5 \times 10^4$ A cm^{-2}, $T_m = 20$ K, $x_n \sim x_s$, $\pi \simeq x_n T \cdot (10^{-8} - 10^{-7})$ V K^{-1}, one finds $\Delta v' \sim 10$ cm s^{-1}. This is in qualitative agreement with the experimental results of.[1] When $I \simeq I_p$, the general expression, (8), gives the same orders of magnitude. From (9) and (10) we get at $a \gg 1$ then $\Delta I/I_0 = j_0 q_0/a\,(d/h\kappa)^{1/2} \sim \epsilon/a$, where $I_0 = Aj_0$. For $\sigma = 10^{19}$ s^{-1}, $h = 10^7$ erg s^{-1} cm^{-2} K^{-1}, $\kappa = 10^7$ erg



a b

Fig. 2 Diagram to show normal and superconducting interface propagating in the same direction

s^{-1} cm^{-1} K^{-1}, $d = 10^{-1}$ cm we find $a \sim 3$, $\epsilon \simeq 3 \cdot (10^{-3} - 10^{-2})$, $\Delta I/I_0 \simeq 10^{-3} - 10^{-2}$.

Discussion of the results

Apart from the velocity asymmetry, the thermoelectric effects lead to a number of hysteresis phenomena occurring when the superconductivity is disturbed by current I. These phenomena are due to the difference of the currents I_p^+ and I_p^-.

Consider the case of $q > 0$ ($I_p^+ < I_p^-$). Let a normal phase zone appear in a superconductor (Fig. 2a), (the length of N-zone is assumed to be much larger than L). Then depending on the magnitude of I five cases are possible: one, $I < I_p^+$. Here $v'_+ < 0$, $v'_- < 0$. The N-zone diminishes and finally disappears. Two, $I_p^+ < I < I_p$. Here $v'_+ > 0$, $v'_- < 0$, $|v'_-| > |v'_+|$. The N-zone diminishes and moves towards the right-hand side of the sample (Fig. 2a). The N-zone may finally vanish either in the bulk or at the right-hand of the sample. Three, $I = I_p$. Here $v'_+ > 0$, $v'_- < 0$, $|v'_+| = |v'_-|$. The N-zone moves towards the right-hand end of the sample without changing its length. At the boundary the N-zone disappears. This regime is closely associated with the existence of moving resistive domains in superconductors.[6]

Four, $I_p < I < I_p^-$. Here $v_+ > 0$, $v_- < 0$, $|v_+| > |v_-|$. The N-zone expands as it moves to the right-hand end of the sample. After reaching the boundary it begins to diminish in length, then it disappears and the sample stays fully superconducting. Five, $I > I_p^-$. Here $v_- > 0$, $v_+ > 0$. The N-zone gradually expands soon spanning the whole sample. The sample becomes fully normal.

Hence I_p^- is the minimum superconductivity destroying current, provided that an N-zone has appeared within the superconductor bulk. The evolution of the S-zone in a sample transformed by current to the normal state may be treated in a similar manner (Fig. 2b). In this case the maximum current at which the superconductivity would recover is I_p^+ ($I_p^+ < I_p^-$).

Strictly speaking, both superconducting and normal states are metastable when $I_p^+ < I < I_p^-$. In this case $v_+ > 0$, and the N-zone that will have appeared at the left-hand boundary of sample (Fig. 1a) will expand, converting the sample into an N-metastable state. However, the resistive state at $I_p^+ < I < I_p^-$ is also metastable as $v_- < 0$ and S-zone arising on the left-hand boundary of the sample (Fig. 2a) will expand. Therefore, I_p^- is the minimum propagating current of the N-zone, and I_p^+ is the maximum propagating current of the S-zone only in the absence of thermal perturbations at the sample ends.

In some metals it is possible that $q < 0$.[3] Here, the maximum propagating current of the S-zone is I_p^-, and the minimum propagating current of the N-zone is I_p^+.

The above-considered mechanism of the asymmetry of $v_\pm$ gives a qualitative description of the experimental results.[1] It is, however, impossible to compare the present theory with the experiment[1] numerically because of the lack of data concerning the magnitude and temperature dependence of q in the sample used in.[1]

Conclusions

When the thermoelectric effect is taken into account it leads to the following: asymmetry of the N-S interface velocity arises relative to the direction of the transport current. The values of $\Delta v' = v'_+ - v'_-$ are of the order of $1 - 50$ cm s^{-1} for different materials. In composite superconductors the difference ($v'_+ - v'_-$) is determined by the thermoelectric constant of the normal metal. In superconducting films the asymmetry of v' is due to the thermoelectric effect in the N-zone.

Hysteresis effects take place when superconductivity is destroyed (or restored) in the sample by the transport current. The minimum propagating current of the N-zone becomes larger than the maximum propagating current of the S-zone.

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