This paper is concerned with the stability against flux jumpes of current and temperature distribution in a composite superconductor which has internal cooling channels. The stability and perturbation increments have been found.

Flux jumps in internally cooled composite superconductors

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It is known[1,2] that thermomagnetic instabilities, flux jumps, may arise in hard superconductors or superconducting composites. In the case of composites, the stability criterion strongly depends on the external cooling. For this reason, the cooling channels in conductors consisting of composite wires are usually provided in the bulk of the material. The present paper is concerned with the stability of such conductors with respect to small perturbations covering a large portion of the sample volume. Although similar, this problem is different from the cryostatic stability which has been discussed in the literature[3].

To initiate the instability in question, there must be an initial perturbation covering the entire sample cross-section and having a longitudinal dimension greater than the cross-section perimeter[2]. Such a perturbation may be brought about by a flux jump in individual sample elements caused by current input and output, displacement of coil turns, etc.

Note that this presentation of the problem eliminates the effect of twisting on the stability[1,4,5]

To find the stability criterion one has to derive the equations describing the evolution of small perturbations of the electric field E and the temperature T. Following the methods used in[4,5], the inhomogeneous conductor with cooling channels is considered here as the continuous medium with parameters averaged over a small volume. The feasibility of such an approach has already been discussed in detail[5].

The averaged Maxwell equation has the following standard form:

$$\text{curl curl}\,\vec{E} = -\frac{4\pi}{c^2}\frac{\partial \vec{j}}{\partial t} \tag{1}$$

where the current density j is given by:

$$j_i = j_{si}(T, H) + \Sigma_k\, \sigma_{ik} E_k$$

j_s and σ_{ik} are the averaged density of the supercurrent and conductivity tensor, respectively; i and k are tensor indices $(x_1 = x, x_2 = y, x_3 = z)$.

Analogous to, for example, reference five, one may find in the linear approximation with respect to the temperature disturbance $\theta = T\text{-}T_0$.

$$\nu\frac{\partial\theta}{\partial t} = \Sigma_{i,k}\kappa_{ik}\frac{\partial^2\theta}{\partial x_i\,\partial x_x} + j_s E - \frac{2\eta_H}{r_0}q_H \tag{2}$$

Where ν and κ_{ik} are averaged heat capacity (without allowance of the helium circulating in the channels) and thermal conductivity tensor, η_H is the portion of the cross-section occupied by the channels, r_0 is the channel radius, q_H is the heat flux to the helium from the unit cooling channel surface.

Similarly for helium temperature T_H we have:

$$\nu_H\frac{\partial\theta_H}{\partial t} = \frac{2q_H}{r_0} - u\nu_H\frac{\partial\theta_H}{\partial z}$$

$$\theta_H = T_H - T_0 \tag{3}$$

where ν_H is the helium heat capacity, and u the helium velocity. For simplicity it is assumed that the cooling channels are oriented along the axis oz.

The averaged values of j_s, ν, σ, κ appearing in (1) - (3) have been discussed[4,5]. To determine the value q_H exactly is extremely difficult since convection and boiling of helium and the specific structure of the composite must be taken into account.

If the helium flow rate u is small the helium has enough time to receive heat so that $\theta_H = \theta$ and then $q_H = \nu_H\, 2/r_0\dot{\theta}$. It is easily seen that the perturbation equations will in this case be identical with those for a composite without channels[4,5], provided that the corrections in j_s, ν, σ, κ have been properly taken into account. The values of σ, κ, j_s decrease and the effective heat capacity $\nu + \nu_H\,\eta_H$ increases when cooling channels are provided. However, in the first approximation the stability criterion for the composites is independent of the heat capacity[4,5]. Therefore, the internal cooling will fail to be effective if the helium velocity is small. Note, that for hard superconductors without normal metal, the situation is quite different: the stability increases with heat capacity[2].

When helium flows rapidly, its heating is insignificant: $\theta \gg \theta_H$ Then we may use the conventional empirical equation for q_H: $q_H = W_0\,(T - T_H)$, where $T_H \simeq T_0$, W_0 is the heat transfer coefficient. Then from (3) it follows that:

$$\theta \sim u\,\frac{\nu_H r_0}{W_0 l}\theta_H$$

Here, we have used the estimate: $\partial\theta_H/\partial z \sim \theta_H/l$ and l is the

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longitudinal dimension of the perturbed zone. Consequently, the approximation $\theta_H << \theta$ is valid if:

$$u >> u_c = \frac{W_0 l}{\nu_H r_0} \qquad (4)$$

Approximation $\theta \simeq \theta_H$ may be used at $u << u_c$.

If P is the cross-section perimeter, then $l \gtrsim P$ and $u_c \gtrsim W_0 P/\nu_H r_0$. For an estimate we shall take $W_0 = 10^6$ erg cm^{-2} K^{-1} s^{-1}, $r_0 = 10^{-1}$ cm $\nu_H = 10^7$ erg cm^{-3} K^{-1}; $P = 1$ cm. Then $u_c \gtrsim 1$ cm s^{-1}.

Let $u >> u_c$ and consider a sample with geometry shown in Fig. 1. For simplicity it will be assumed that j_s is independent of the local magnetic field (Bean's critical state model). The solution of (1), (2) will be as usual[2] sought in the form:

$$\theta = \theta_0(x/b) \exp \{\lambda t \kappa_\perp/\nu b^2\}$$

$$E = E_0(x/b) \exp \{\lambda t \kappa_\perp/\nu b^2\}$$

where $\kappa_\perp$ is the heat conductivity across the oz axis, and λ is an eigen value to be determined. Then, eliminating E_0 we have for $l, L >> b$:

$$\theta_0^{\text{IV}} - [\lambda(1+\tau) + W_f]\,\theta_0'' - \lambda[\beta - \tau(\lambda + W_f)]\,\theta_0 = 0$$

$$E_0 = \frac{\kappa_\perp}{j_s b^2}\,[(\lambda + W_f)\,\theta_0 - \theta_0^{11}]$$

$$\beta = \frac{4\pi b^2 j_s}{c^2 \nu}\left|\frac{\mathrm{d}j_s(T)}{\mathrm{d}T}\right|; \tau = \frac{4\pi\sigma_{11}\kappa_\perp}{\nu c^2}; \; W_f = \frac{2\eta_H W_0 b^2}{\kappa_\perp r_0} \qquad (5)$$

where σ_{11} is the conductivity along the oz axis. The differentiation is performed with respect to the dimensionless coordinate x/b. For a superconducting composite, the characteristic value of τ is usually high: $\tau = 10 - 10^3$. The value of W_f may lie between 10^{-1} and 10.

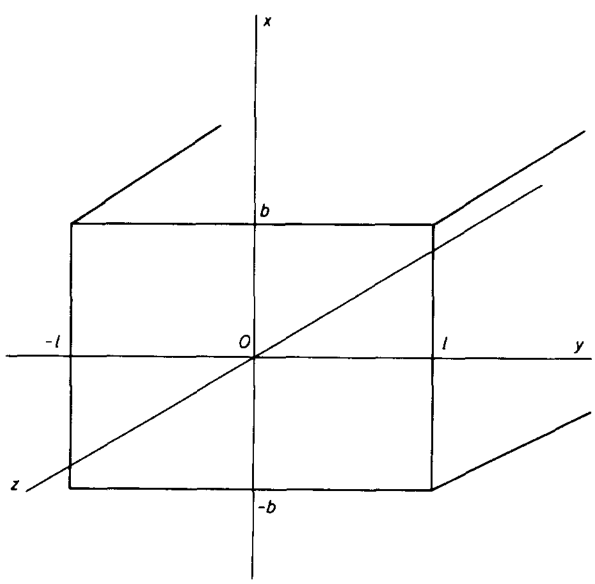

Fig. 1 The sample geometry

To determine the stability criterion, boundary conditions must be imposed on (5). The instability region corresponds to the parameters for which there exists a non-zero solution of (5) with $\lambda > 0$.

To begin with we shall consider the simplest case. Assume that there is no transport current and surface cooling, and that the magnetic flux totally penetrates into the sample (Fig. 2). In this case the boundary conditions are: $E_0 = 0$, $\theta_0^1 = 0$ at $x = 0$; $E_0^1 = 0$ $\theta_0^1 = 0$ at $x = b$. The curve $\lambda = \lambda(\beta)$ is shown qualitatively in Fig. 3. The stability criterion may be written as (see Fig. 3):

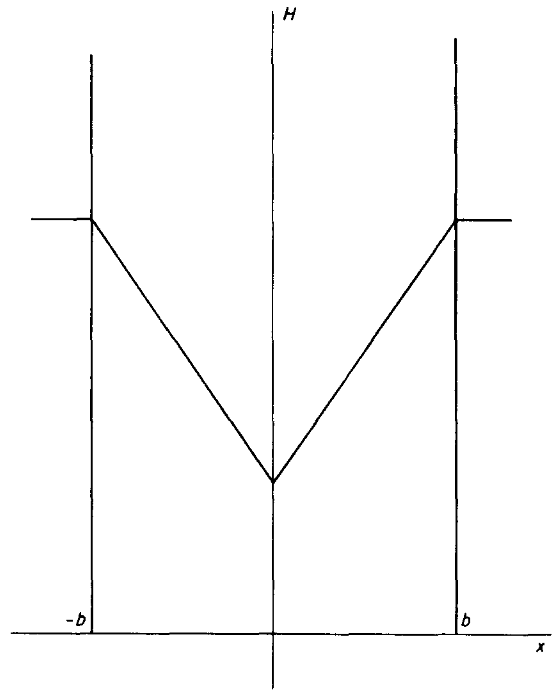

Fig. 2 The distribution of magnetic field in the sample assuming full flux penetration

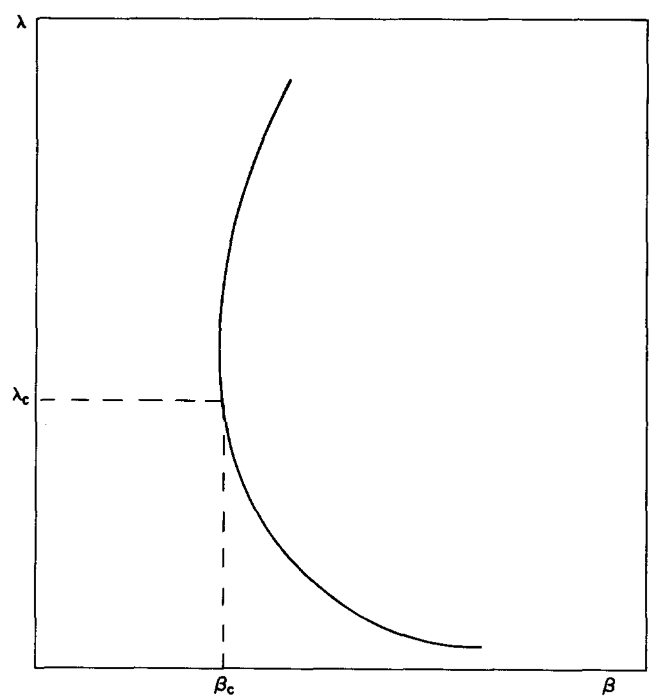

Fig. 3 The curve $\lambda = \lambda\ (\beta)$

$$\beta < \beta_c = \beta(\lambda_c) \quad (6)$$

where λ_c is the instability increment.

If W_f is not too small, $\lambda_c\tau >> 1$. In this limit we have ($\tau^{-1} << W_f << \tau^2$):

$$\frac{\beta_c}{\tau} = W_f \left(1 + \frac{3}{(4W_f\tau)^{1/3}}\right) \simeq W_f$$

$$\lambda_c = \left(\frac{W_f^2}{4\tau}\right)^{1/3}$$

Similarly in the case of finite surface cooling: a – if $W\tau >> 1$ and $W < 1$ ($W = W_0 b/\kappa_\perp$) then $\lambda_c\tau >> 1$ and $\beta_c/\tau \simeq W + W_f$; b – $W\tau >> L$ and $W >> 1$ then $\lambda_c\tau >> 1$, $\beta_c/\tau = \pi^2/4 + W_f$.

Since for the composites cooled by liquid helium the condition $\lambda_c\tau >> 1$ is usually valid; we may reduce the fourth order equations (1), (2) to a second order ones. Following the method already published[2,5] we have for θ:

$$\sum_{i,k} \frac{b^2\kappa_{ik}}{\kappa_\perp} \frac{\partial^2\theta}{\partial x_i \partial x_k} + (\beta/\tau - W_f)\,\theta = 0 \quad (7)$$

The thermal boundary conditions must be imposed on (7). The parameter β_c in the criterion (6) is found from the condition of existence of the nontrivial solution of the (7) with the appropriate boundary conditions. Just as in the case of $W_f = 0$ at $\tau >> 1$ the stability does not depend in the first approximation of the magnetic flux distribution in the sample, but only on the volume occupied by the supercurrents.

For example, in the case of the geometry shown in Fig. 1, and $w < 1$, we have:

$$\beta/\tau < \beta_c/\tau = W_f + W(1 + b/L) \quad (8)$$

If $W_f = 0$, $b/L = 0$ then (7) coincides with the dynamic stability criterion[1]. Note, that in the approximation which corresponds to (7), $\lambda_c = 0$ and the results obtained by means of it must be equivalent to the cryostatic stability criterion by Steckly[3].

Using (8) one easily finds:

$$j_s < j_1 = \frac{\sigma_{11} W_0}{b\,|\,dj_s/dT|} \left(1 + \frac{b}{L} + 2\eta_H \frac{b}{r_0}\right) \quad (9)$$

The value j_1 has a maximum at some $\eta_H = \eta_{opt}$ as may be seen from (9). For example, one can assume that $j_s = x_s j_c$, $j_c = j_0\,(1 - T/T_c)$, $\sigma_{11} = \eta_n\sigma_n$, where η_s and η_n are the concentrations of the superconducting and normal metals, respectively ($\eta_n + \eta_s + \eta_H = 1$), σ_n is the normal metal conductivity. As the values r_0/b, r_0/L $W_0 T_c \sigma_n / r_0 j_c^2$ are usually small, one readily finds from (9): $\eta_{opt} \simeq 0.5$ and $j_{max} \sim \sigma_n W_0 (T_c - T_0)/r_0$.

In the case of an arbitrary W we have for β_c:

$$\frac{\beta_c}{\tau} - W_f = k^2 + q^2$$

$$K \tan K = W; \; q \tan\left(q\frac{L}{b}\right) = W$$

In particular, for $W >> 1$.

$$\frac{\beta_c}{\tau} = \frac{\pi^2}{4}\left(1 + \frac{b^2}{L^2}\right) + W_f$$

This result shows that the above limitation on j_s cannot be substantially changed by enhancing the heat removal from the outer surface.

Let $W_0 = 10^6$ erg cm^{-2}. s^{-1}K^{-1}, $T_c = 10$ K, $\sigma_n = 10^{20}$ s^{-1}, $r_0 = 10^{-1}$ cm. Then $j_{max} \sim 10^4$ Acm^{-2}. The higher current densities may be attained not by a complete stabilization of the conductor, but rather by eliminating the large scale perturbations.

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