J. Phys. D: Appl. Phys., 13 (1980) 847–51. Printed in Great Britain

# Flux jumps and training in superconducting composites

R G Mints
Institute for High Temperatures, Moscow 127412, USSR

Received 3 July 1979, in final form 24 October 1979

**Abstract.** The critical state stability in a composite superconductor under an external stress, causing plastic strain yield, is considered. The obtained criterion of the stability against thermomagnetic–mechanical perturbations allows the explanation of the training phenomenon as a process of consequent strain hardening of the superconductor.

The training of superconducting materials is a well known phenomenon in superconducting magnetic systems. Recently the existence of training has been observed in short superconducting samples under significant mechanical stress (resulting from either external or ponderomotive forces) (Anashkin *et al* 1975, 1977, Schmidt 1976, Schmidt and Pasztor 1977). The results obtained in this work confirmed the connection between the training phenomenon and the mechanical properties of superconducting materials.

In the present paper we found the stability criteria of the current, magnetic field and mechanical stress distributions in the critical state in the composite superconductor, the stress being supposed to cause the plastic yield of the material. The thermomagnetic–mechanical instability (i.e. both flux jump and serrated yielding development) was investigated in the linear approximation for small perturbations. The plastic strain rate nonuniformity affecting the critical state stability of multifilamentary superconductors was also treated.

Being interested in the critical state stability in the whole sample, we shall regard the composite superconductor as a uniform anisotropic superconducting medium. The physical properties of such a medium are defined by superconducting filaments and normal conducting matrix characteristics, averaged over the cross-section of the composite (see Mints and Rakhmanov 1977 and references therein). This approach is evidently valid provided that the scale of perturbations of interest is large compared to the characteristic dimensions of the composite superconductor structure and that the rise time of the perturbation is larger than the relaxation times of the individual elements of the medium. After averaging we obtain the equations for the temperature ($T$) perturbation $\theta=T-T_0$ ($T_0$ is the initial temperature) and the electric field $E$ in the linear $\theta$ and $E$ approximation:

$$\nu\dot{\theta}=\kappa\nabla^2\theta+\boldsymbol{j}_0\boldsymbol{E}+\sigma(\partial\dot{\epsilon}/\partial T)\theta \tag{1}$$

$$\text{curl curl } \boldsymbol{E}=-(4\pi/c^2)(\partial\boldsymbol{j}/\partial t) \tag{2}$$

and the connection between the current density and the electric field

$$\boldsymbol{j}=\boldsymbol{j}_0+\frac{1}{\rho}\boldsymbol{E}. \tag{3}$$

 

The quantities in (1)–(3) are averaged appropriately: $\nu$—the heat capacity, $\kappa$—the heat conductivity, $j_0$—the critical current density (the currents are assumed to flow in the same direction in the region of averaging), $\sigma$—the applied stress, $\dot{\epsilon}$—the plastic strain rate, and $\rho$—the resistivity. The last term in equation (1) describes the variation of the plastic deformation heating $\sigma\dot{\epsilon}$ due to the temperature perturbation $\theta$, in the linear in $\theta$ approximation. In the case when only a part of the work of plastic deformation $[\sigma(\partial\dot{\epsilon}/\partial T)\theta]$ is released in the form of heat, the necessary modification can be introduced to the final result by a correction factor. Note, that the heating connected with the electric field induced by the superconductor motion in the magnetic field was omitted in equation (1). This term was estimated to be small (as far as $H^2/4\pi\sigma < 1$, $H$ being the characteristic magnitude of the magnetic field) in comparison with that directly describing deformation losses $\sigma(\partial\dot{\epsilon}/\partial T)\theta$.

The set of equations (1), (2) enables one to investigate the critical state stability against thermomagnetic–mechanical perturbations for the given critical current density and plastic strain rate dependence on the temperature, magnetic field, strain and applied stress. Using $j_0=0$, (the normal state), equation (1) was used to determine the stability criteria of plastic yields of metals (Petukhov and Estrin 1975, Petukhov 1977).

Let the dependence $\theta(t)$, $E(t)$ be $\theta(t)$, $E(t)\sim\exp(\Gamma t)$, where $\Gamma$ is the increment of instability (Mints and Rakhmanov 1977). To define the stability criterion of interest the heat and electrodynamics boundary conditions should be added to the system of equations (1) and (2). The solution of equations (1) and (2) exists for some definite (eigenvalues) values of $\Gamma$. The instability evidently appears for $\Gamma \geqslant 0$.

In this paper we consider the simplest case: $j_0=j_0(T)$ (Bean's model), $\dot{\epsilon}=\dot{\epsilon}(\sigma, T)$ in the usual situation for composite superconductors, when the ratio of the coefficients of thermal diffusion, $D_t$ is $\kappa/\nu$ and of magnetic diffusion, $D_m$ is $c^2\rho/4\pi$ and $\tau=D_t/D_m \gg 1$. Then in general for the $\tau\gg 1$ approximation $\partial j/\partial t=0$ (Mints and Rakhmanov 1977), and the relation between $\theta$ and $E$ follows:

$$\boldsymbol{E} = -\rho(\mathrm{d}\boldsymbol{j}_0/\mathrm{d}T)\,\theta. \tag{4}$$

The condition $\partial j/\partial t=0$ used means that in the main for the $\tau\gg 1$ approximation the instability in the composite superconductor develops, the magnetic field being frozen. It is obvious physically since $D_m \ll D_t$.

Substituting (4) into (1) we obtain the equation determining the temperature perturbation $\theta$ in the region of the superconductor where the current is present:

$$\kappa\nabla^2\theta + \left(\rho j_0 \left|\frac{\mathrm{d}j_0}{\mathrm{d}T}\right| + \sigma\frac{\partial\dot{\epsilon}}{\partial T} - \Gamma\nu\right)\theta = 0. \tag{5}$$

In the region without current the equation for $\theta$ has the form:

$$\kappa\nabla^2\theta + [\sigma(\partial\dot{\epsilon}/\partial T) - \Gamma\nu]\,\theta = 0. \tag{6}$$

The second-order equation system (5) and (6) should be completed by the thermal boundary conditions and the temperature and heat flux continuity conditions on the boundary of the region with the current. The corresponding solution can be easily found for the samples with various geometries of magnetic field and current distributions, the existence of the solution with $\Gamma\geqslant 0$ meaning the loss of stability.

Consider for example the critical state stability in a cylindrical composite superconductor with radius $R$ and current $I \leqslant I_c = \pi R^2 j_0$. The cooling conditions at the surface are as usual

$$\kappa\,(\mathrm{d}\theta/\mathrm{d}r) = -W_0\theta \tag{7}$$

where $W_0$ is the heat transfer coefficient to the refrigerant. Under the characteristic conditions for composite superconductors $W = W_0 R/\kappa < 1$ (in particular, for liquid helium as refrigerant $W \sim 0{\cdot}1$). In this case the temperature of the sample is practically uniform. This allows one to calculate the increment $\Gamma$ for any current distribution in the composite by integrating equations (5) and (6) over the sample cross-section, taking into account the cooling condition expressed in equation (7). As a result we find

$$\Gamma = \frac{1}{\nu}\left[\rho j_0 \,|\mathrm{d}j_0/\mathrm{d}T|\,(I/I_c) + \sigma(\partial\dot{\epsilon}/\partial T) - (2W_0/R)\right].$$

From the condition $\Gamma < 0$ we obtain the critical state stability criterion against the codeveloping magnetic flux jump and plastic deformation jerk:

$$\rho j_0 \,|\mathrm{d}j_0/\mathrm{d}T|\,(I/I_c) + \sigma(\partial\dot{\epsilon}/\partial T) < 2W_0/R \tag{8}$$

or

$$\alpha + (I/I_c)\tilde{\beta} < 1$$

where

$$\alpha = \sigma\frac{\partial\dot{\epsilon}}{\partial T}\frac{R}{2W_0} \qquad \tilde{\beta} = \frac{\rho R j_0}{2W_0}\left|\frac{\mathrm{d}j_0}{\mathrm{d}T}\right|.$$

The physical sense of the stability criterion of equation (8) is the following: the power released in the process of the flux jump and plastic deformation jerk should not exceed the power that is transported to the refrigerant.

It should be mentioned that the stability criterion of the stationary regime of plastic yield in the absence of current in the sample has the form (Petukhov and Estrin 1975)

$$\alpha < 1 \tag{9}$$

and the stability condition for the critical state in the absence of mechanical stress (Mints and Rakhmanov 1977)

$$(I/I_c)\tilde{\beta} < 1. \tag{10}$$

Combining the obtained criterion from equation (8) with equations (9) and (10) we see that in the critical state the flux jumps and serrations strongly interact, stimulating each other. Physically this is due to the slow character of both instability developments compared to the thermal diffusion time. Note also, that from the stability condition (8) it follows that the smaller the current, the larger the stress (strain) producing the instability. This fact has been found experimentally (Schmidt and Pasztor 1977). Thus, the quenching current in the sample essentially depends on its mechanical properties ($\partial\dot{\epsilon}/\partial T$) and the applied stress magnitude, which in its turn can be determined by the ponderomotive forces, i.e. by the current. The criterion of stability against thermomagnetic–mechanical perturbations (8) allows one to understand the training phenomenon as a consequent process of strain hardening of the superconductor. In practice, if the current increases so that condition (8) is not satisfied, then as a result of instability development the flux jump occurs. This process is accompanied by intense heating of the sample and by a plastic deformation jerk and consequently strain hardening. The magnitude of $\partial\dot{\epsilon}/\partial T$ (for given stress applied) will decrease, and this, in accordance with equation (8), results in the increase of the quenching current in the next cycle. Provided the stress applied is not too large, the quenching current magnitude can be achieved, which can be determined from equation (10).

Consider the critical state stability in the case when the plastic yield is nonuniform along the sample axis ($x$ axis). Let the $x$-dependence of the nonperturbed plastic yield rate be that, as shown in figure 1, where $l \ll R$ (strong thin disc-shaped nonuniformity) and $W<1$. The equation for the temperature perturbation $\theta$ dependence on $x$ can be easily obtained by integrating equations (5) and (6) over the cross-section, taking into

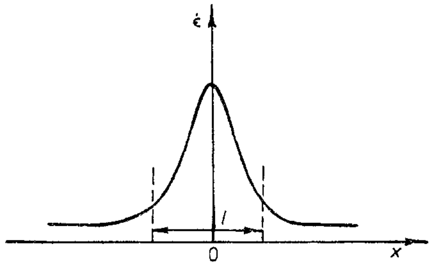


**Figure 1.** Variation of the plastic strain rate $\dot{\epsilon}$ along the $x$ axis of the sample.

account the cooling condition in equation (7), the physical characteristics of the sample being uniform in the cross-section plane. The result is

$$\kappa \frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} - \frac{2W_0}{R}\theta + \rho j_0 \left|\frac{\mathrm{d}j_0}{\mathrm{d}T}\right| \frac{I}{T_c}\theta + \sigma \frac{\partial \dot{\epsilon}}{\partial T}\theta - \Gamma \nu \theta = 0$$

or

$$L^2 \frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} + \left(\frac{I}{I_c}\tilde{\beta} + \alpha - 1 - \gamma\right)\theta = 0 \tag{11}$$

where

$$L = R/\sqrt{(2W)} \qquad \gamma = (\nu R/2W_0)\Gamma.$$

In the case under consideration the instability occurs for the first time near the plane $x=0$, where the plastic deformation power is maximal. Away from this region the temperature perturbation is evidently absent. Thus, the solutions of equation (11), which are of interest, vanish at infinity and the instability criterion can be easily found from these solutions at $\Gamma=0$.

In the case of strong nonuniformity ($l \ll R$) the solution of equation (11) slowly varies on the characteristic scale of nonuniformity $l$. The stability criterion can be easily obtained from solving equation (11) by means of the Fourier transform

$$1 - (I/I_c)\tilde{\beta} > \tfrac{1}{4}\bar{\alpha}^2 \tag{12}$$

where

$$\bar{\alpha} = \frac{1}{L}\int_{-\infty}^{\infty} \mathrm{d}x\, \alpha(x) \exp\{-(|x|/L)\,[1-(I/I_c)\tilde{\beta}]^{1/2}\}.$$

It is seen from criterion (12) that the presence of the 'weak' link can essentially diminish the critical state stability (provided that the magnitude $\partial\dot{\epsilon}/\partial T$ at $x=0$ is fairly large). The instability appearing in the region of strong nonuniformity causes the flux jump, accompanied by intense heating and the plastic deformation jerk, and results in strain hardening in the hot region. In this case the training phenomenon is connected with the consequent process of strain hardening of 'weak' links.

Let us estimate the characteristic magnitude of the external stress, at which the plastic yield of the material becomes unstable. Taking $R \sim 3 \times 10^{-1}$ cm, $\partial\dot{\epsilon}/\partial T \sim 3 \times 10^{-2}$ s$^{-1}$ K$^{-1}$, $W_0 \sim 1$ W s$^{-1}$ cm$^{-2}$ we find from equation (9) that $\sigma \sim 2 \times 10^4$ N cm$^{-2}$. A characteristic value of the parameter $\tilde{\beta}$ is about unity for composite superconductors. In this case (see equation (8)) the presence of current in the sample can essentially decrease the given estimate.

Thus, we see that under the applied external stress, which causes the plastic yield of the sample, the critical state in multifilamentory superconductors can become unstable against the collective thermomagnetic–mechanical perturbation. This instability, which is due to the interaction between the flux jump and the plastic deformation jerk, allows one to understand the training process in superconductors as a process of consequent strain hardening, stimulated by the thermal softening. The nonuniformity of superconductors leads to the training connected with the sequence of local stability breakdown in weak links.

The author would like to thank Dr BV Petukhov for fruitful discussions on the results of the work.

## References

Anashkin O P, Keilin V E and Lyikov V V 1975 *Proc. Conf. Appl. Supercond. Alushta, USSR* BNL-TR-624

Anashkin O P, Varlakhin V A, Keilin V E, Kriviskh A V and Lyikov V V 1977 *IEEE Trans. Magn.* **13** 673–7

Mints R G and Rakhmanov A L 1977 *Sov. Phys.–Usp.* **20** 249–63

Petukhov B V and Estrin Yu Z 1975 *Solid St. Phys.* **17** 2041–4 (New York: Academic Press)

Petukhov B V 1977 *Solid St. Phys.* **19** 2058–63 (New York: Academic Press)

Schmidt C 1976 *Appl. Phys. Lett.* **28** 463–5

Schmidt C and Pasztor R 1977 *IEEE Trans. Magn.* **13** 116–9