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Magnetic instabilities in superconducting composites

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Abstract. The stability of electric currents in multifilamentary superconducting composites against flux jumps is discussed in detail. The normal procedure for such a quantitative stability investigation is carried out. In some particular cases the stability criteria are found and analysed.

1. Introduction

The flux-jumping phenomenon in hard superconductors imposes certain limits for their stable performance as electric conductors. The theories describing the initial stages of a flux jump have made it possible to derive criteria for the limits of stability of the superconductor (Wipf 1967, Hancox 1965, Hart 1968, Swartz and Bean 1968, Wilson *et al* 1970, Kremlev 1973, 1974, Mints and Rakhmanov 1975). These results are at least in qualitative agreement with experiments.

It is well known that an instability of essentially the same nature may occur in so-called superconducting composites (see, for example, Wilson *et al* 1970). These composites generally consist of a high number (up to a few thousand) of fine superconducting wires embedded by some method or other in a matrix of normal metals or their alloys. By reducing the size of the superconducting wires and by choosing an appropriate metal for the matrix it is possible to ensure the stabilization of the individual superconducting filaments (it is worth noting that any eventual breaks of the superconducting wires are also shunted by the matrix).

Instability can grow in these composites as a collective effect since individual filaments are coupled to each other in one way or another. The complex nature of the composite makes it difficult to carry out an exact stability analysis. However, for a sufficiently large number of superconducting filaments N and with some definite assumptions about their behaviour it is possible to treat the composite as a quasi-continuous medium with some effective parameters. This approach to the problem was outlined in a preliminary report (Kremlev *et al* 1976). In this paper we present a more thorough derivation of the basic equations for a wider set of different conditions. The stability criteria are found for some particular cases.

An analogous approach has been developed by Duchateau and Turk (1975a, b), however without consideration of the applicability of the method.

2. Basic equations

To describe the flux jumping in a composite conductor quantitatively one has to derive the equation for a small perturbation averaged over the volume including a large number of filaments $n \gg 1$, but with $n \ll N$. Such an equation is evidently adequate if

(1) the total number of filaments N in the composite is sufficiently high ($N \gg 1$);
(2) the rise-time of a flux jump is greater than the thermal and electromagnetic relaxation times of an individual element of a conductor;
(3) all our variables (temperature, currents, fields, etc.) do not change too much over the distances comparable with the dimensions of the filaments. In particular, this requires that most of the filaments are saturated with current, i.e. the transport current in each filament is equal to its critical value. This is the case for a non-twisted composite and also for a twisted one carrying the transport current. However, for sufficiently large variations of external field the necessary distributions can be realized in a twisted and even in a transposed conductor.

If all the above-mentioned conditions are satisfied one can write the thermal and Maxwell equations for the small perturbations of temperature θ and electric field E. In the approximation linear with respect to small θ and E they are

$$\nu\dot{\theta} = \kappa \nabla^2 \theta + \boldsymbol{j}_s . \boldsymbol{E}$$
$$\nabla^2 \boldsymbol{E} = \frac{4\pi}{c^2} \frac{\partial \boldsymbol{j}}{\partial t} \tag{1}$$

and the relation between current density j and E:

$$j = j_s + \sigma E. \tag{2}$$

In accordance with the general idea of the proposed method all our variables (θ, E, j) are mean values taken over the volumes including some finite number of filaments.

The parameters ν, j_s, κ, σ correspondingly represent the mean values of specific heat, current density, thermal and electric conductivities. If x_s and x_n are the fractions of the superconductor and of the normal metal in the composite ($x_s + x_n = 1$), one obtains

$$\nu = x_s \nu_s + \nu_n x_n \qquad j_s = x_s j_c \qquad \sigma = x_s \sigma_f + x_n \sigma_n$$

where ν_s and ν_n are the corresponding values of specific heat, $j_c = j_c(T)$ is the critical current density of the superconductor (T is the temperature), σ_n is the electric conductivity of the normal metal and σ_f that of the superconductor in the flux–flow regime (the small nonlinear part of the volt–ampere curve is not of interest here). The mean value of the thermal conductivity κ (transverse with respect to the filaments) is determined by the internal structure of the composite. It can be easily shown that as a rather good approximation one can take $\kappa = (1 - x_s^{1/2})\kappa_n$, where κ_n is the normal metal conductivity.

For the following it is necessary to choose a definite model of the critical state. Here we shall consider only Bean's (1964) model, i.e. we assume that $\partial j_c / \partial H = \partial j_s / \partial H = 0$.

Excluding E and j from the system (1–2) one can obtain the equation for the perturbation θ only. The time dependence $\theta(t)$ may be taken in the form $\theta \sim \exp[\lambda(\kappa/\nu b^2)t]$ where b is some characteristic dimension of the sample (e.g. its half-width). For the geometrical configuration of figure 1 (the one-dimensional case) we can write, using non-dimensional variables (Kremlev *et al* 1976)

$$\theta^{\mathrm{iv}} - \lambda(1+\tau)\,\theta'' + \lambda(\lambda\tau - \beta)\,\theta = 0 \tag{3}$$

where τ is the ratio of temperature and magnetic diffusivities:

$$\tau = \frac{D_t}{D_m} = \frac{4\pi}{c^2} \frac{\kappa\sigma}{\nu}$$

and

$$\beta = -\frac{4\pi b^2}{c^2} \frac{j_s}{\nu} \frac{\partial j_s}{\partial T}. \tag{4}$$

The space derivatives in (3) are taken with respect to the variable x/b.

Non-trivial solutions of (3) with appropriate boundary conditions exist only for some values of $\lambda = \lambda(\beta, \tau, \ldots)$. If, for the given values of the parameters, there exists some $\lambda > 0$ for which the equations have a solution, instability occurs.

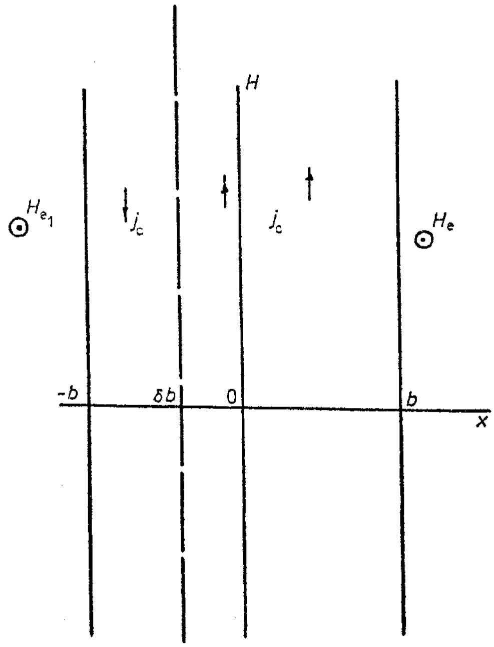

Figure 1. The flat sample.

For further analysis it is necessary to specify the thermal and electromagnetic boundary conditions for (3). The heat transfer on the boundary of a composite can be represented by

$$\left.\left(\kappa \frac{d\theta}{dx} \pm h_0\theta\right)\right|_{x=\pm b} = 0$$

or in a dimensionless form:

$$\theta'(\pm 1) \pm h\theta(\pm 1) = 0; \quad h = h_0 b/\kappa \tag{5}$$

where h_0 is the coefficient of heat transfer from the surface.

The electromagnetic boundary conditions in the general case can be of the form

$$\frac{\partial H(\pm b)}{\partial t} = f(t) \tag{6}$$

where $f(t)$ is a known function. However, it can be shown that the stability criterion (in the linear theory) is independent on $f(t)$ as long as $\dot{H}_e$ does not lead to a large spatial variation of the internal magnetic field gradient. This requires that the skin depth $\delta_{sk}=\delta_{sk}(\dot{H}_e)\sim c/(2\pi\sigma\dot{H}_e/H_e)^{1/2}$ should be greater than b, i.e. that

$$b\ll\delta_{sk}(\dot{H}_e)\sim\frac{c}{(2\pi\sigma\dot{H}_e/H_e)^{1/2}}$$

or

$$\dot{H}_e\ll\frac{c^2H_e}{2\pi\sigma b^2}.$$

In order to investigate the stability when this inequality is satisfied, the condition (6) may be replaced by

$$\frac{\partial H(\pm b)}{\partial t}=0. \tag{7}$$

Supposing that $H_e=10^4$ G, $\sigma=10^{19}$ cgse and $b=10^{-1}$ cm, one has $\dot{H}_e\ll 10^7$ G s^{-1} for the applicability of (7).

The solutions of (3) can be found independently for two regions $x<\delta b$ and $x>\delta b$ (figure 1). For $x=\delta b$ we impose the usual conjugation conditions (Mints and Rakhmanov 1975)

$$\theta(\delta+0)=\theta(\delta-0) \qquad \theta'(\delta+0)=\theta'(\delta-0) \qquad E(\delta)=0. \tag{8}$$

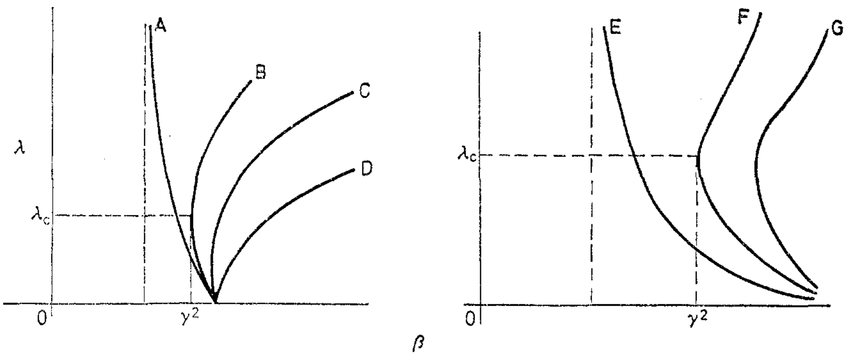

Figure 2. The functions $\lambda(\beta)$ for different h and τ. (*a*) $h=0$: (A) $\tau=0$, (B) $0<\tau<\tau_c$, (C) $\tau=\tau_c$, (D) $\tau>\tau_c$. (b) $h>0$: (E) $\tau=0$, (F) $\tau=\tau_1>0$, (G) $\tau>\tau_1$.

The function $\lambda(\beta, \tau)$ obtained by the use of equation (3) with the boundary conditions (5)–(8) for different heat transfer conditions is depicted schematically in figure 2. The stability criterion can evidently be written in the form

$$\beta<\gamma^2(\tau, h, \delta, \dots) \tag{9}$$

where γ is some definite number to be determined for the given problem.

Let us now investigate in more detail the limits of applicability of the proposed treatment. The shortest space scale for the variation of the solution of (3) in the case $\tau\gg 1$ is equal approximately to $b/(\lambda_c\tau)^{1/2}$ for the dangerous perturbation with the eigenvalue λ_c. This scale must be greater than the structure scale $b/N^{1/2}$ or

$$N\gg\lambda_c\tau. \tag{10}$$

The same condition arises if we demand that the magnetic relaxation time inside the normal elements be smaller than the rise-time of the perturbation t_j.

Another 'slow' local process is the thermal diffusion inside the superconducting filament. The corresponding relaxation time $t_{\kappa s}$ is of the order of $t_{\kappa s} \sim \nu_s x_s b^2/\kappa_s N$ and it should again be smaller than t_j; or

$$N \gg \lambda_c \frac{\kappa}{\kappa_s} \frac{\nu_s x_s}{\nu}. \tag{11}$$

In the case of a twisted conductor for 'fast' variation of the external field H_e, such that

$$\dot{H}_e \gg \frac{c^2 H_e}{2\pi\sigma L^2}$$

(where L is the twist pitch) the field distribution in the composite is largely analogous to the case of an untwisted sample (Wilson *et al* 1970). Therefore, our treatment can be applied when the rate of change of the external field satisfies the inequalities

$$\frac{c^2 H_e}{2\pi\sigma L^2} \ll \dot{H}_e \ll \frac{c^2 H_e}{2\pi\sigma b^2}, \tag{12}$$

if $N \gg 1$ and conditions (10) and (11) are fulfilled.

3. The solution of the basic equations

The solution of (3) can be written in the following form:

$$\theta = c_1 \exp\left(\frac{k_1 x}{b}\right) + c_2 \exp\left(-\frac{k_1 x}{b}\right) + c_3 \exp\left(\frac{ik_2 x}{b}\right) + c_4 \exp\left(-\frac{ik_2 x}{b}\right) \tag{13}$$

where

$$k_{1,2} = \left[\left(\frac{\lambda^2(1-\tau)^2}{4} + \lambda\beta\right)^{1/2} \pm \frac{\lambda(1-\tau)}{2}\right]^{1/2}.$$

Substituting (13) into the corresponding boundary conditions produces a system of linear equations for the constants c_i. A non-trivial solution of this system exists if its determinant $\Delta(\lambda, \beta, \tau, \ldots)$ is equal to zero. This condition can be used to determine the desired functions $\lambda = \lambda(\beta, \tau, \ldots)$.

For the sake of simplicity we will consider in this section only the case $I=0$ ($I = 2j_c\delta b = 0$, cf. figure 1). The generalization for $I \neq 0$ can be easily produced.

3.1. Isothermal cooling

For the very intense cooling of the composite surface ($h \to \infty$) the corresponding condition for the determination of $\lambda(\beta, \tau)$ is written

$$\Delta_\infty = 16i\{2k_1k_2(\lambda + k_2^2)(\lambda - k_1^2) - k_1k_2[(\lambda + k_2^2)^2 + (\lambda - k_1^2)^2] \cosh k_1 \cos k_2 + (k_1^2 - k_2^2)(\lambda - k_1^2)(\lambda + k_2^2) \sinh k_1 \sin k_2\} = 0 \tag{14}$$

The results of numerical computations of γ^2 as a function of τ for this equation are presented in figure 3.

For $\tau \ll 1$ the value of λ_c is high, $\lambda_c \gg 1$ and $\gamma^2(\tau)$ can be found explicitly. To the first approximation

$$\lambda_c = \left(\frac{\pi}{2}\right)^{4/3} \tau^{-2/3} \simeq 1{\cdot}82\, \tau^{-2/3} \tag{15}$$

$$\gamma^2(\tau) = \frac{\pi^2}{4}\left[1 + 3\left(\frac{2}{\pi}\right)^{2/3} \tau^{1/3}\right] = 2{\cdot}47\,(1 + 2{\cdot}2\, \tau^{1/3}). \tag{16}$$

It follows from (15)

$$t_j \sim \frac{t_\kappa}{\lambda_c} \sim t_\kappa \tau^{2/3} \ll t_\kappa.$$

Let us consider in more detail another limiting case, $\tau \gg 1$, which is of the most interest for composite study. It can be shown that for $\tau \gg 1$ the following relation holds:

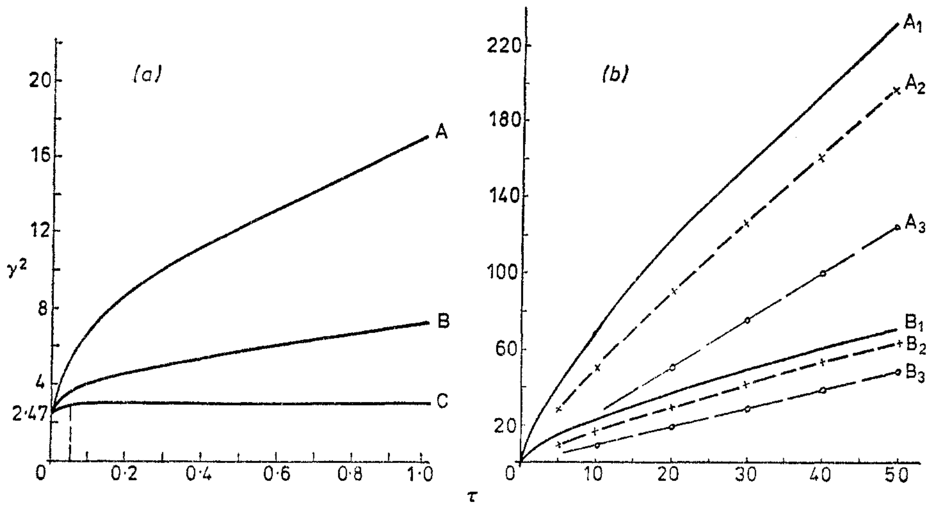

Figure 3. The dependences $\gamma^2(\tau)$ for different h. (a) $\tau \ll 1$: (A) $h \gg 1$, (B) $h=1$, (C) $h=0$; (b) $0<\tau<50$. Curves A_i correspond to $h=\infty$ and B_i correspond to $h=1$. Computational results are shown by solid lines A_1 and B_1. Curves A_2 and B_2 are calculated by means of formulae (20) and (27). Straight lines A_3 and B_3 are obtained by means of Hart (1969) stability criterion.

$t_\kappa \ll t_j \ll t_m = b^2/D_m$, i.e. $\lambda_c \ll 1$ and $\tau\lambda_c \gg 1$. For the 'wavenumbers', k_1 and k_2 in the first approximation one can write:

$$k_1 = (\lambda\tau)^{1/2}; \quad k_2 = \left(\frac{\beta}{\tau}\right)^{1/2}\left(1 - \frac{\lambda\tau}{2\beta}\right) \tag{17}$$

Substituting these into (14) we get

$$(\lambda\tau)^{1/2} = \left(\frac{\beta}{\tau}\right)^{1/2} \tan k_2 \tag{18}$$

As $(\lambda_c\tau)^{1/2} \gg 1$, we can search for k_2 in the form $k_2 = \pi/2 + \phi$, $0 < \phi \ll 1$. The value of ϕ now can be easily found from (18) along with the following equation for $\lambda(\beta,\tau)$ in the vicinity of $\lambda=\lambda_c$:

$$\phi=\frac{\pi}{2}\frac{1}{(\lambda\tau)^{1/2}} \qquad \lambda^{3/2}-\lambda^{1/2}\pi\left[\left(\frac{\beta}{\tau}\right)^{1/2}-\frac{\pi}{2}\right]+\frac{\pi^2}{2\,\tau^{1/2}}=0. \tag{19}$$

To find λ_c from equation (19) we can make use of the fact that for $\lambda=\lambda_c$, $\partial\lambda/\partial\beta=\infty$.

The solution of this cubic equation then gives

$$\lambda_c=\left(\frac{\pi}{2}\right)^{4/3}\tau^{-1/3}\simeq 1{\cdot}82\,\tau^{-1/3}\ 0$$
$$\gamma^2(\tau)=\frac{\pi^2}{4}\tau\left[1+3\left(\frac{2}{\pi}\right)^{2/3}\tau^{-1/3}\right]\simeq 2{\cdot}47\tau(1+2{\cdot}2\,\tau^{-1/3}). \tag{20}$$

The expression, $\gamma^2=\pi^2\tau/4$, has already been found by Hart (1968). It represents the so-called dynamic stability criterion for the case considered. Note that the next term in the expansion of $\gamma^2(\tau)$ is of the order of $\tau^{-1/3}$, and so it is of significance even for rather high values of τ (see figure 3). For example, at $\tau=10$ the expression (20) diverges from the exact value of γ^2 as found by the direct computation by not more than 20%.

With the results obtained above we can determine the critical number N_c of filaments for which the conductor as the whole becomes unstable, although every filament is just stable. The criterion for stability of the individual filament in the massive normal block can be expressed in the form (Mints and Rakhmanov 1975):

$$r_0{}^2<r_c{}^2=i_0{}^2\,\frac{c^2\nu_s}{4\pi j_c\,|\mathrm{d}j_c/\mathrm{d}T|} \tag{21}$$

where r_0 is the filament radius, r_c is the maximum radius at which the current distribution in the filament is stable, $i_0=2, 4, \ldots$ is the first root of Bessel function J_0. By comparing (21) with (20) we find:

$$N_c\simeq 0{\cdot}4\tau\,\frac{\nu}{\nu_s x_s}\left(\frac{r_c}{r_0}\right)^2. \tag{22}$$

Thus, for $\tau=10^3$, $N_c\simeq 10^2\ (r_c/r_0)^2$. It should be noted, however that the condition $h\gg 1$ can hardly be realized by direct cooling with liquid helium. In fact, $h_0\sim 10^7$ erg cm^{-2} s^{-1} K^{-1}, $\kappa\sim 10^6$–10^7 erg cm^{-2} s^{-1} K^{-1}, $b\sim 0{\cdot}1$ cm, and it follows that $h\lesssim 1$.

The exact criteria for applicability of our treatment can now be derived. Substituting λ_c (20) into (11) and (12) we get

$$N\gg\tau^{2/3} \qquad N\gg\frac{\kappa}{\kappa_s}\frac{\nu_s x_s}{\nu}\,\tau^{-1/3} \tag{23}$$

3.2. *Adiabatic insulation*

For the fully insulated composite ($h=0$) the condition for the determination of $\gamma^2(\tau)$ is written

$$\Delta_0=-16i\,k_1k_2(k_1{}^2+k_2{}^2)\cos k_2\cosh k_1[k_1(\lambda+k_2{}^2)\tanh k_1-k_2(k_1{}^2-\lambda)\tan k_2]=0. \tag{24}$$

The dependence $\gamma^2(\tau)$ for this equation is plotted in figure 3. For $\tau>\tau_c=1/21$ the value of γ becomes constant ($\gamma^2=3$) while $\lambda_c\to 0$. This is because flux jumping is damped by the normal currents, which are not effective for $\lambda\to 0$ (as $j_n\sim\sigma E\propto\lambda$). It is these 'slow' perturbations that become most dangerous for $\tau>1/21$ (Kremlev 1974).

By comparing (21) with the stability criterion, for the new case ($\beta < 3$) we can find the value of N_c:

$$N_c \simeq 0.5 \frac{\nu}{x_s \nu_s} \left(\frac{r_c}{r_0}\right)^2 \sim 1. \tag{25}$$

Thus, for a fully insulated conductor the stability is not greatly affected by the use of a composite.

3.3. The case of an arbitrary heat transfer

For finite h the determinant $\Delta(\lambda, \beta, \tau, h)$ has the form

$$\Delta = \Delta_0 + h\Delta_\infty. \tag{26}$$

Numerical computations of the equation $\Delta = 0$ allow us to find the dependences $\gamma^2(\tau, h)$. As an example in figure 3, there is plotted such a dependence of $\gamma^2(\tau)$ for $h = 1$.

It should be noted that for $\tau \gg 1$, γ^2 is proportional to τ, the slope of the corresponding lines increasing with the value of h up to the value of $\pi^2/4$ for $h \to \infty$.

For the most interesting case with $\tau \gg 1$ and $h\tau \gg 1$ it can be shown that $\lambda_c\tau \gg 1$. As in the previous sections we can make use of power expansions. If $h \lesssim 1$, one finds

$$\lambda_c = \left(\frac{h}{2}\right)^{2/3} \tau^{-1/3}$$
$$\gamma^2(\tau) = h\tau\left(1 + \frac{1.2}{(h\tau)^{1/3}}\right). \tag{27}$$

Substituting λ_c (27) into (11) and (12) we find the following applicability criteria:

$$N \gg 0.3\,(h\tau)^{2/3} \qquad N \gg \left(\frac{h^2}{\tau}\right)^{1/3} \frac{\kappa}{\kappa_s} \frac{\nu_s x_s}{\nu}.$$

By comparing (27) with the criterion (21) we again find the value of N_c:

$$N_c \simeq 0.2\,\tau h \frac{\nu}{\nu_s x_s} \left(\frac{r_c}{r_0}\right)^2. \tag{28}$$

4. The simplified theory

In this section we shall discuss only the composites with $\tau \gg 1$. The heat transfer will be considered to be not too weak, so that $h\tau \gg 1$. In this case the value of $\lambda_c\tau$ is also high, $\lambda_c\tau \gg 1$ (see §3.3). The existence of a large parameter $\lambda_c\tau$ allows us to reduce the order of the basic equation (3).

By substituting (2) into Maxwell equation (1) we can write

$$\nabla^2 E = -\lambda\tau\left(E + \frac{1}{\sigma}\frac{\mathrm{d}j_s}{\mathrm{d}T}\theta\right).$$

As $\nabla^2 E$ has some finite value, while $\lambda_c\tau \gg 1$, to the first approximation the following equation must hold:

$$E + \frac{1}{\sigma}\frac{\mathrm{d}j_s}{\mathrm{d}T}\theta = 0;$$

this gives the relation between E and θ. Substituted into the heat diffusion equation (1), it results in

$$\nabla^2\theta+(\beta/\tau-\lambda)\theta=0. \quad (29)$$

This equation can be obtained directly by dividing equation (3) by $\lambda\tau\gg1$, with $\tau\gg1$. The above derivation is presented in order to illustrate how the instability for $\tau\gg1$ actually occurs.

The development of the perturbation according to (29) proceeds at fixed current density ($\tau=D_t/D_m\gg1$) while the dissipation grows as a result of lowering of the critical current density. An equivalent equation has been proposed by Hart (1968) from qualitative considerations.

Only thermal boundary conditions need be supplied to the equation (29). The occurrence of $\lambda>0$, as earlier, signifies instability.

In the case of a flat slab with the boundary conditions (5) one can derive the following relation to determine $\gamma(h, \tau)$

$$\frac{\gamma}{\tau^{1/2}}+\tan^{-1}\left(\frac{\gamma}{h\,\tau^{1/2}}\right)=\frac{\pi}{2}. \quad (30)$$

The corresponding dependence of $\gamma(h)/\tau^{1/2}$ is plotted in figure 4. It can be seen that the isothermal regime ($\gamma\simeq\pi\,\tau^{1/2}/2$) is reached at about $h\simeq10$.

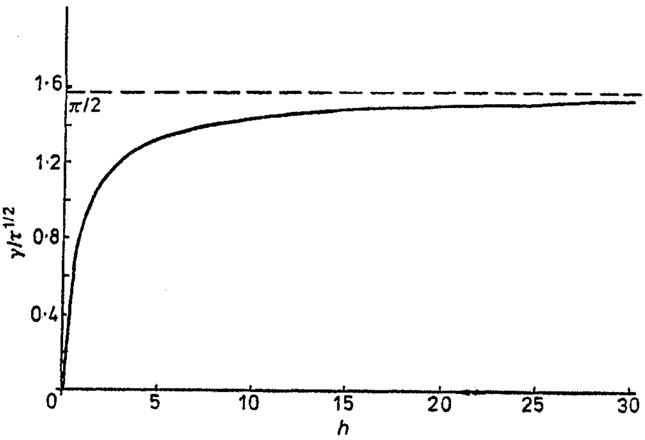

Figure 4. The value $\gamma(h)/\tau^{1/2}$ for $\tau\gg1$ and $h\tau\gg1$.

It follows from (29) that for $\tau\gg1$ the stability is determined only by the total volume occupied by currents and does not depend on their direction. In fact, the electric field occurs neither in the equation (29) nor in the boundary conditions. The perturbation develops simultaneously for both portions $x>\delta b$ and $x<\delta b$ of conductor. This is in sharp contrast to the case of $\tau\ll1$, where the development of the perturbations proceeds quite independently in these two regions.

With the aid of (29) one can investigate the stability of samples with different configurations. Thus, for a circular conductor with given transport current (figure 5) the equation (29) takes the form ($r>\delta$; $\lambda=0$):

$$\theta''+\frac{1}{r}\theta'+\frac{\beta}{\tau}\theta=0. \quad (31)$$

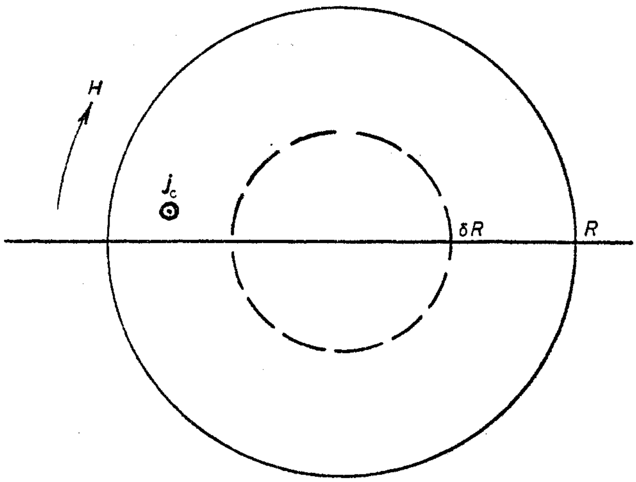

Figure 5. The wire with a fixed transport current.

where

$$\frac{\beta}{\tau}=\left(\frac{R}{R_0}\right)^2 \qquad R_0=\left(\frac{\kappa\sigma}{j_s}\left|\frac{dj_s}{dT}\right|^{-1}\right)^{1/2}$$

R being the wire radius.

The coordinate r is normalized here by the conductor radius R. The value of the parameter δ is determined by the transport current I:

$$\delta=\left(1-\frac{I}{I_c}\right)^{1/2} \qquad I_c=\pi R^2 j_s.$$

The solution of (31) with the corresponding boundary conditions is readily performed. The maximum permissible transport current I_m can be found from the following

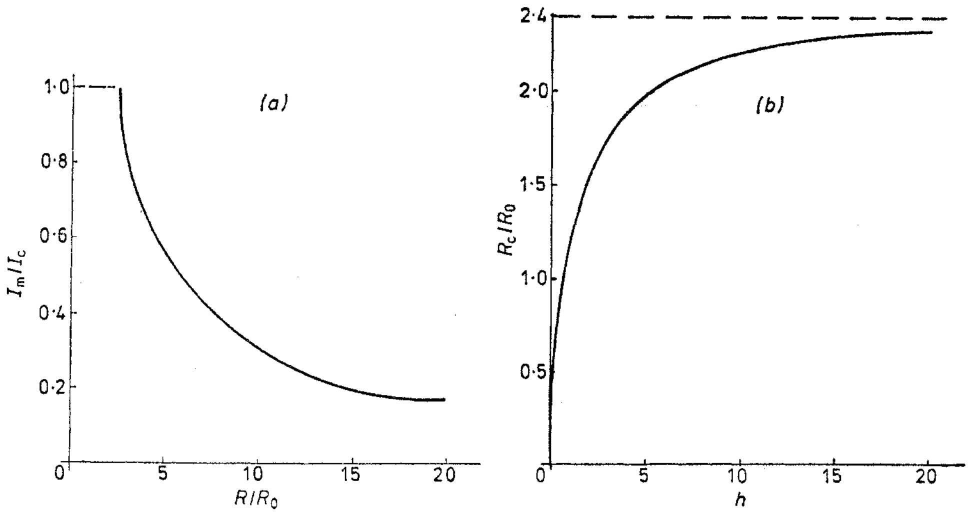

Figure 6. (*a*) The function $I_m(R)/I_c$ for the wire: $h \gg 1$, $\tau \gg 1$. (*b*) The value $R_c(h)/R_0$ for the wire: $\tau \gg 1$, $h\tau \gg 1$.

relation:

$$N_1\left(\frac{R\delta}{R_0}\right)\left[hJ_0\left(\frac{R}{R_0}\right)-\frac{R}{R_0}J_1\left(\frac{R}{R_0}\right)\right]=J_1\left(\frac{R\delta}{R_0}\right)\left[hN_0\left(\frac{R}{R_0}\right)-\frac{R}{R_0}N_1\left(\frac{R}{R_0}\right)\right] \quad (32)$$

where $\delta=\delta(I_m)$ and J_0, J_1, N_0, N_1 are the corresponding Bessel and Neumann functions. The dependence of I_m/I_c on R/R_0 for $h \gg 1$ is plotted in figure 6(*a*).

For $R<R_c$ (figure 6*a*) stability is not violated even for $I=I_c$ ($\delta=0$, i.e. the whole cross-section is filled with current). To determine R_c we can find from (31) and from the boundary conditions

$$hJ_0\left(\frac{R_c}{R_0}\right)-\frac{R_c}{R_0}J_1\left(\frac{R_c}{R_0}\right)=0.$$

The dependence of R_c on h is plotted in figure 6(*b*). The value of R_c essentially depends on the heat transfer coefficient h:

$R_c/R_0 \to 2, 4$ for $h\to\infty$, and $R_c\to 0$ for $h\to 0$.

5. Conclusions

Equations have been deduced describing the development of the collective mode instabilities in superconducting composites with a sufficiently high number of filaments, $N \gg 1$. On the basis of these equations, stability criteria have been derived for arbitrary values of external heat transfer coefficient h_0.

The stability of the composite has been shown to depend strongly on the heat exchange intensity and not, in the first approximation, on heat capacity and on the value of transport current. For $h \ll 1$ the stability also does not depend on the composite heat conductivity.

It has been shown that for some $N>N_c$ the composite as the whole can become unstable, even though all its constituent filaments are stable.

In the case of a high value of the ratio of temperature and magnetic diffusivities τ and for $h\tau \gg 1$ the simplified calculation scheme has been derived based on the second-order equation (first proposed by Hart 1968 from the qualitative considerations). Within this simplified scheme the stability of a circular composite has been investigated.

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