



FIG. 2. Distribution of  $n_k$  at infinite excess above threshold ( $\gamma_L = 0$ ) for successive instants of time ( $t = t_1, t_2, t_3, t_4$ ) in arbitrary units. The point  $k = z$  corresponds to the maximum of the increment.

In the inertial region, neglecting the linear damping and the noise, we have

$$\frac{\partial f_n}{\partial t} = f_n(f_{n+1} - f_{n-1}). \quad (4)$$

Equation (4) has an exact solution  $f_n(t) = f(t - n/s - \tau_0)$ , where

$$f(\xi) = f_0 \left( 1 + \frac{a}{1 - b + b \operatorname{ch} \gamma \xi} \right). \quad (5)$$

Here  $f_0$ ,  $a$ , and  $\tau_0$  are arbitrary parameters, while  $s$ ,  $b$ , and  $\gamma$  are functions of  $a$  and  $f_0$ ; when  $a \gg 1$  we have

$$\gamma = 2f_0 a; \quad b^2 = \frac{1}{a}; \quad \frac{\gamma}{s} = \ln a.$$

The solution (5) is a soliton propagating in  $k$ -space along a chain of peaks. The nonstationary process observed in the numerical experiment at  $\Gamma/\gamma_L > k^*/k_{diff}$  can be visualized as a process of successive "detachment" of the solitons from the instability zone  $k^*$ , their propagation in the inertial region, and their "annihilation" in the region of small wave numbers. The soliton propagation velocity is  $v \sim 2k_{diff}\Gamma/\ln \ln(n_e/n_0 \Delta k)$ . A similar qualitative character is possessed by the initial stage of the process of establishment of the stationary state at  $\Gamma/\gamma_L < k^*/k_{diff}$ ; in this case, however, the solitons are damped and stop before reaching the region of small  $k$ .

We note that the nonstationary character of the spectrum of the Langmuir waves should lead, in experiments on parametric excitation, to oscillations of the absorption of the high-frequency field energy in the plasma. The time-averaged energy flux into the plasma coincides then with the value obtained in [4] within the framework of the diffusion approximation, and the results of the present paper can be regarded as an investigation of the fine structure of spectra of the "jet" type.

It can be shown that the difference system (4) is fully integrable, and that the soliton (5) is a stable formation.

<sup>1</sup>The possible existence of spectra of this type was first suggested in a paper by Kingsep and Rudakov.<sup>[7]</sup>

<sup>1</sup>N. E. Andreev, A. Yu. Kirii, and V. P. Silin, Zh. Eksp. Teor. Fiz. 57, 1024 (1969) [Sov. Phys.-JETP 30, 559 (1970)].

<sup>2</sup>E. Valeo, C. Oberman, and F. W. Perkins, Phys. Rev. Lett. 28, 340 (1972).

<sup>3</sup>A. A. Galeev and R. Z. Sagdeev, Nuclear Fusion 13, 603 (1973).

<sup>4</sup>B. N. Breizman, V. E. Zakharov, and S. L. Musher, Zh. Eksp. Teor. Fiz. 64, 1297 (1973) [Sov. Phys.-JETP 37, No. 4 (1973)].

<sup>5</sup>C. Oberman, Paper at School of Plasma Physics, Tbilisi, 1972.

<sup>6</sup>V. E. Zakharov, Zh. Eksp. Teor. Fiz. 62, 1745 (1972) [Sov. Phys.-JETP 36, 908 (1972)].

<sup>7</sup>A. S. Kingsep and L. I. Rudakov, *ibid.* 58, 582 (1970) [31, 313 (1970)].

## Spin resonance and Knight shift in superconductors

R. G. Mints

Institute of High Temperatures, USSR Academy of Sciences

(Submitted January 9, 1974)

ZhETF Pis. Red. 19, 253-255 (March 5, 1974)

Spin resonance in bulky superconductors of type I and II, including under conditions when surface superconductivity exists, is considered. The Knight shift in thin superconducting films, which appears when no allowance is made for the spin-orbit interaction, is also considered.

As mentioned repeatedly in the literature, excitations with energy  $\epsilon < \Delta$ , where  $\Delta$  is the superconducting gap, exist in pure superconductors. In type-I superconductors (bulky sample or plate), these excitations are localized on magnetic surface levels,<sup>[1,2]</sup> and in type-II superconductors they are localized on levels in the core of the vortex,<sup>[3]</sup> or else on magnetic surface levels.<sup>[4]</sup> Starting with a certain value  $H = H_2$  of the external mag-

netic field<sup>[4]</sup> (and at all fields in the core of the vortex), the minimal energy of such levels vanishes. As a result, the state density  $\nu(\epsilon_F)$  remains finite at  $H > H_2$  for superconductors of type I and at  $H < H_{C1}$  for superconductors of type II.

The vanishing of the minimal excitation energy is connected with the magnetic field. Allowance for the

spin leads in this situation, obviously, to a corresponding splitting of the levels:  $\Delta E = 2\mu_0 \bar{H}$ , where  $\mu_0$  is the Bohr magneton and  $\bar{H}$  is the average magnetic field acting on the excitation. The presence of spin splitting simultaneously with  $\nu(\epsilon_F) \neq 0$  leads to the appearance, even at  $T=0$ , of a paramagnetic moment. The susceptibility  $\chi_s$  is then determined, as usual, by the state density  $\nu(\epsilon_F)$ . It is easy to show that in all the listed cases the value of  $\nu(\epsilon_F)$  per unit volume in which the corresponding levels exist is of the order of the density of states in the normal metal; by the same token,  $\chi_s$  turns out to be of the order of  $\chi_n$ . Let us examine the results of these singularities of the excitation spectra in superconductors.

We note first that in thin films, in which the Knight shift is usually measured, we have  $\chi_s \sim \chi_n$  at  $H > H_2$  (even if  $T=0$ ). In weaker fields ( $H < H_2$ ),  $\chi_s$  vanishes (at  $T=0$ ). Consequently, in a strong magnetic field parallel to the film surface the Knight shift remains finite even without allowance for the spin-orbit interaction. A similar behavior of the Knight shift in spheres and cylinders of small radius was obtained in [5].

It should further be noted that in bulky samples there should be observed a spin resonance at the frequency of the corresponding splitting energy, namely  $\Omega = 2\mu_0 \bar{H}/\hbar$ . This frequency, however, depends on the quasiparticle-motion parameters. As always, the resonance frequency is the extremal or limiting frequency, and this decreases the amplitude of the resonance and determines the line shape. For magnetic surface levels,  $\bar{H}$  is several times smaller than the external magnetic field and depends on  $P_x$  and  $P_z$  (spherical Fermi surface) or on  $P_x$  (cylindrical Fermi surface); the axes  $x$  and  $z$  lie here in a plane parallel to the sample surface, and  $P_x$  and  $P_z$  are the momenta of the excitation along the axes  $x$  and  $z$ , respectively. The dependence of the resonance frequency on the parameters of the motion also causes the spin relaxation time to become equal to the free-path time, since the particle leaves the resonant group as a result of any scattering act. We note that here, just as in the case of any resonance with a selected group of carriers, one can use the relaxation-time approximation.<sup>[6]</sup> A distinguishing feature of spin resonance in a superconductor is that excitations with  $\epsilon=0$  cannot leave the skin layer. This leads to enhancement of the resonance relative to the situation in the normal metal,<sup>[7]</sup> but the dependence of  $\Omega$  on the momenta, as shown, decreases the amplitude of the resonance. As a result, resonance in a superconductor differs in amplitude from resonance in a normal metal<sup>[7]</sup> by a factor

$$\frac{\tau}{T_s} \frac{\delta_{\text{eff}}}{\delta} \sqrt{\Omega \tau} \gg 1$$

where  $\tau$  and  $T_s$  are the momentum and spin relaxation times, respectively,  $\delta$  is the depth of the skin layer, and  $\delta_{\text{eff}}$  is the effective depth of damping of the magnetic moment.<sup>[7]</sup> The frequency difference (by a factor of several times) makes it possible to separate the absorption on the surface from the spin-resonance proper.

The levels in the core of the vortex are localized at distances on the order of  $\xi$  (the pair dimension), i.e., they are in a practically homogeneous magnetic field, and all the excitations take part in the resonance; the relaxation time is then the spin-flip time. The resonance frequency does not depend here on the external field and makes it possible to measure the field at the core of the vortex, while the amplitude of the resonance is proportional to the external magnetic field (to the number of vortices). In the case of an external field parallel to the surface of the sample, the excitations with  $\epsilon=0$  do not leave the skin layer, and in comparison with the situation in the normal metal<sup>[7]</sup> the resonance is enhanced by a factor

$$\frac{H}{H_{c2}} \frac{\delta_{\text{eff}}}{\delta} \Omega T_s \gg 1,$$

while the line shape becomes Lorentzian. If the external magnetic field is perpendicular to the surface of the sample, then it is necessary to take into account the diffusion of the spins, and all the processes proceed as in a normal metal. We note also that, owing to the character of the motion of the excitations, the resonance is not sensitive here to the value of the spin relaxation on the surface, a quantity concerning which little can be said under ordinary circumstances.

In the region of existence of surface superconductivity, as shown in [8], excitations with  $\epsilon=0$  (the analog of magnetic surface levels) occur near the surface, and the excitations with  $\epsilon=0$  that move from the interior of the metal experience reflections from the superconducting layer, so that this is equivalent to strictly specular conditions. The spin resonance takes place here both with the surface and with the volume excitations. If the spin scattering by the surface is immaterial, and the magnetic field is practically homogeneous, then the situation is analogous to the already-discussed resonance in type-II superconductors. In the case of strong scattering of spins by the surface, the absorption is determined by the volume excitations, which are not affected by this scattering. Thus, the character of spin relaxation on the surface can be ascertained in this case from the shape and amplitude of the line.

<sup>1</sup>R.A. Pincus, Phys. Rev. 158, 346 (1969).

<sup>2</sup>M. Ya. Azbel', Zh. Eksp. Teor. Fiz. 59, 295 (1970) [Sov. Phys.-JETP 32, 159 (1971)].

<sup>3</sup>C. Caroli, P.G. DeGennes, and J. Matricon, Phys. Lett. 9, 307 (1964).

<sup>4</sup>M. Ya. Azbel', L.B. Dubovskii, and R.G. Mints, Zh. Eksp. Teor. Fiz. 60, 1895 (1971) [Sov. Phys.-JETP 33, 1024 (1971)].

<sup>5</sup>A.I. Larkin, ibid. 48, 232 (1965) [21, 153 (1965)].

<sup>6</sup>I.M. Lifshitz, M. Ya. Azbel', and M.I. Kaganov, Elektron-naya teoriya metallov (Electron Theory of Metals), M., 1971.

<sup>7</sup>M. Ya. Azbel', V.I. Gerasimenko, and I.M. Lifshitz, Zh. Eksp. Teor. Fiz. 35, 691 (1958) [Sov. Phys.-JETP 8, 480 (1959)].

<sup>8</sup>R.G. Mints, ZhETF Pis. Red. 18, 523 (1973) [JETP Lett. 18, 307 (1973)].