ELECTRODYNAMICS OF SURFACE SUPERCONDUCTIVITY

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The energy spectrum of the excitations and the surface of impedance of a metal are investigated under conditions when surface superconductivity is present.

We consider the energy spectrum of the excitations of a semi-infinite superconductor in an external magnetic field H parallel to the surface (H_{c_2}, $H_c \leq H \leq H_{c_3}$, where H_c, H_{c_2}, and H_{c_3} are respectively the critical and second and third critical fields). It is known [1, 2] that under these conditions the gap Δ is different from zero only near the surface, at a distance on the order of ξ (ξ is the characteristic scale of variation of Δ). If we choose the coordinate axes as in Fig. 1, then the magnetic field and $|\Delta|$ depend only on the y coordinate. We choose also a gauge in which the gap Δ is real, and the vector potential $\vec{A} = (A_x, 0, 0)$ also depends only on y; this is possible because the phase of Δ is a function of x and z. The problem of determining the energy spectrum and the eigenfunctions of the excitations (the eigenfunctions of the Gor'kov equations [3]) reduces thus to a one-dimensional problem, and it can be solved by using a quasi-classical approximation [4, 5].

Near H_{c_3}, in the region of applicability of the Ginzburg-Landau equations, the magnetic field can be regarded as homogeneous, and we obtain for A_x the expression

$$A_x = -H_y + \text{const} = -H(y - y_0), \quad y_0 = 0{,}59\,\xi\,\frac{H_{c_3}}{H}. \tag{1}$$

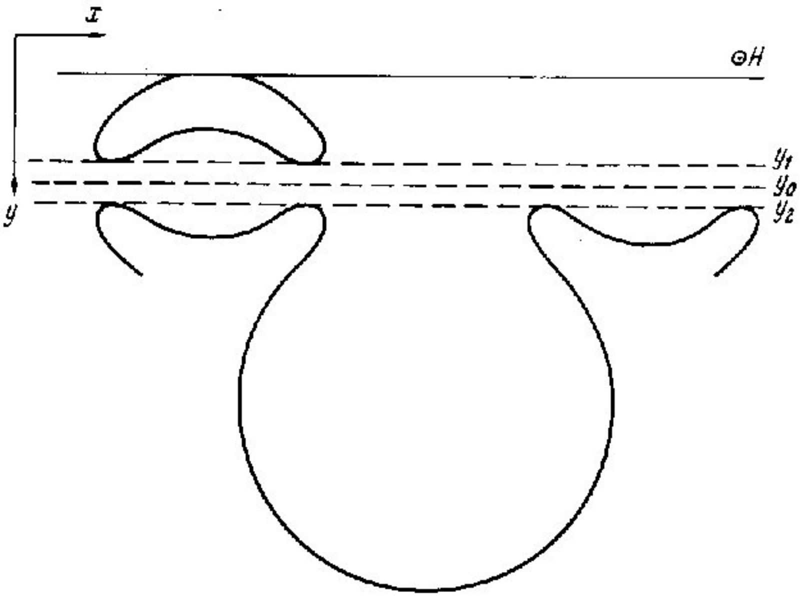

Fig. 1

We note that the fector potential vanishes at the point y_0 (approximately the midpoint of the layer); this is natural, since the current is determined in the chosen gauge by the London expression and can be of alternating sign, since the total current is equal to zero. This property of the vector potential with our choice of gauge remains in force in the entire region of surface conductivity; all that changes is the expression for y_0.

The energy spectrum of the excitation is determined in the quasiclassical approximation from the Bohr rules

$$\oint p_y(y)\,dy = 2\pi n\hbar \tag{2}$$

where p_y is the classical momentum of the excitation along the y axis, and n is an integer. The criterion for the applicability of (2), as usual, is $n \gg 1$ [6], and is automatically confirmed. To determine the $p_y(y)$ dependence we can use the Bogolyubov-deGennes equations [1], where it is necessary, unlike in [4, 5], to retain also the terms quadratic in the magnetic field, since the magnetic field does not attenuate in the interior of the metal. It is easily found that

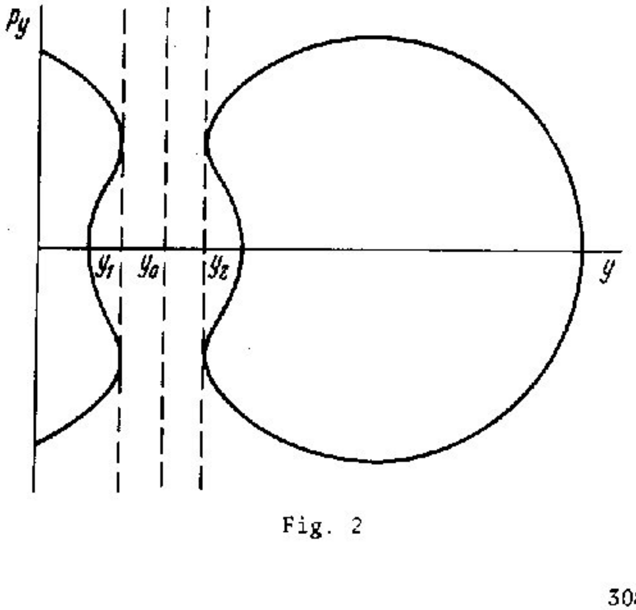

Fig. 2

$$p_y = \pm\sqrt{2m\left[\mu_1 - \frac{1}{2m}\left(\frac{eA_x}{c}\right)^2 \pm \sqrt{(\epsilon - \rho)^2 - \Delta^2}\right]},$$

$$\mu_1 = \epsilon_F - \frac{1}{2m}(P_x^2 + P_z^2), \quad \rho = \frac{e}{mc}P_x A_x. \tag{3}$$

Here ϵ_F is the Fermi energy, P_x and P_z are the conserved (by virtue of the homogeneity) momenta of the excitations along the x and z axis, and ε is the energy of the excitations. The characteristic trajectories in the coordinate and phase spaces are shown in Figs. 1 and 2, respectively. It is seen from (2) and (3) that there are always excitations with $\varepsilon = 0$, since

$$\rho(0) \gg \Delta(0) \qquad (P_x \sim p_F).$$

Some of the excitations (the surface ones) cannot penetrate into the metal, since there is a turning point y_1 ($y_1 < y_0$) where $\rho(y_1) = \Delta(y_1)$. They move with a frequency

$$\Omega\sqrt{R/\xi} >> \Omega$$

(R and Ω are the cyclotron radius and frequency, respectively), and mainly in analogy with the motion considered in [4, 5]. Nor can excitations moving from the interior of the metal (volume excitations) reach the surface $y = y_0$ at $\varepsilon = 0$, in view of the existence of the turning point y_2 ($y_2 > y_0$) at which $|\rho(y_2)| = \Delta(y_2)$. The frequency of the corresponding motion differs slightly (by $\sqrt{(\xi/R)}\Omega << \Omega$) from the cyclotron frequency. We emphasize that the presence of the turning points y_1 and y_2 is due to the vanishing of the vector potential at the point y_0.

Let us see now what occurs when an electromagnetic wave is incident on the metal. At $T < T_c$, the region considered from now on, the energy absorption occurs mainly in the regions where there are excitations with $\varepsilon = 0$, i.e., in the surface layer ($y < y_1$) and in the volume ($y > y_2$). The dependence of the impedance on the external magnetic field and on the frequency is determined essentially by the relation between the other parameters of the problem and the depth of penetration of the alternating field δ. Deferring the detailed exposition to another article, we stop to discuss here the case when $\delta > \xi$, a case that can be realized if the frequency is not too low or if the magnetic field is sufficiently close to H_{c_3}. In this approximation, the surface impedance Z can obviously be expressed in the form

$$\frac{1}{Z} = \frac{1}{Z_{sur}} + \frac{1}{Z_{vol}},$$

where Z_{sur} is determined by the nondissipative superconducting and dissipative surface currents, and S_V is determined by the current of the volume excitations. The expression for Z_V is analogous, with a relative accuracy on the order of $\sqrt{\xi/R} << 1$, to the corresponding expression for the impedance of a normal metal with strictly specular reflection from the surface. In particular, there should also be observed cyclotron resonance at a slightly modified (to the extent that $\sqrt{\xi/R}$ is small) cyclotron frequency. The value of Z_{sur} is determined by the effective conductivity of the surface layer, and if the scattering by the surface is diffuse, then the effective free-path time is of the order of $\sqrt{Ry_0}/v_F$, and

$$\sigma_{eff} \sim -i\frac{ne^2}{m\omega} + \sigma_0\frac{\sqrt{Ry_0}}{\ell}\frac{y_0}{\xi}, \quad Z_{sur} \sim \frac{1}{\sigma_{eff}\,\xi}.$$

We see that at frequencies that are not too low, or in a field sufficiently close to H_{c_3}, a situation is possible wherein

$$Z_{sur} \sim \frac{\ell}{\sqrt{Ry_0}\,y_0\sigma_0} \propto H^2.$$

Let us stop also to discuss the obvious anisotropy of the impedance Z_{sur}, and hence of the entire surface impedance Z, with respect to the angle between the constant and alternating magnetic fields (currents). In our approximation $\delta > \xi$, when $H_1 \perp H$ (H_1 is the amplitude of the alternating magnetic field), the gap Δ does not change in the linear approximation in H_1, but at $H_1 \parallel H$ the change of Δ can obviously be expressed in the form

$$\Delta_1 = \frac{\partial\Delta}{\partial H}H_1.$$

The effect of the change in the gap Δ can be easily estimated from the excitation dispersion law (2). The relative contribution made to the absorption by the change of the gap is of the order of

$$\frac{\xi}{\delta}\frac{H_{c_3}}{\Delta}\frac{\partial\Delta}{\partial H}$$

and can be appreciable, in spite of the small factor ξ/δ in this approximation, because $\partial\Delta/\partial H$ is large. Such an anisotropy was observed experimentally in [7].

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