ANOMALOUS QUANTUM OSCILLATIONS OF SURFACE IMPEDANCE

R. G. Mints
L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
Submitted 30 October 1969; resubmitted 18 December 1969
ZhETF Pis. Red. 11, No. 2, 128 - 132 (20 January 1970)

It is known that the condition of thermodynamic stability of the homogeneously magnetized state, $(\partial H/\partial B)_T > 0$, is violated periodically (with periodicity in H), owing to the de Haas - van Alphen effect, at sufficiently low temperatures ($\pi^2 T < \hbar\Omega$, where $\Omega = eB/mc$ is the cyclotron frequency) [1]. In analogy sith the vapor-liquid system, it can be shown that there exist critical points $T_0^{(i)}$ and $H_0^{(i)}$, at which $\frac{\partial H}{\partial B}(T_0^{(i)}, H_0^{(i)}) = 0$, $\frac{\partial^2 H}{\partial B^2}(T_0^{(i)}, H_0^{(i)}) = 0$, and $\frac{\partial^3 H}{\partial B^3}(T_0^{(i)}, H_0^{(i)}) > 0$. Depending on the boundary conditions, there occurs at $T < T_0$ either a stratification into phases (a domain structure) with different values of the induction B_1 and B_2 (if $\vec{H}$ is parallel to $\vec{n}$, where $\vec{n}$ is the normal to the surface), or else a jumpwise change of the induction from B_1 to B_2 (if $\vec{H} \perp \vec{n}$) [1]. Such a singularity should affect the propagation in the metal of electromagnetic waves at a frequency ω such that the system has time to "adjust itself" to the thermodynamics, i.e., under the condition $\omega\tau \ll 1$ (τ - free path time).

Let us assume, for simplicity, that the dispersion is isotropic, and that the depth of penetration δ, the free path time τ, and the external magnetic field are such that $\delta > \ell >$ (R is the radius of the electron orbit), and $T > T_0$. In this case the connection between the current and the electric field is local ($\vec{j} = \sigma\vec{E}$), and all that is left to solve the problem of the penetration of the electromagnetic wave in the metal is to specify the connection between the alternating components of the magnetic field $\vec{h}$ and the magnetic induction $\vec{b}$. In the linear approximation (the estimate is presented below) this connection is given by $h = (\frac{\partial H}{\partial B})_T b$ (if $\vec{h} \parallel \vec{H}$) and $h = (1 - 4\pi M/B)b$ (if $\vec{h} \perp \vec{H}$). We see therefore that near the critical point we have $\mu = b/h = (\partial H/\partial B)_T^{-1} \to \infty$ if $\vec{h} \parallel \vec{H}$ and μ has no singularities if $\vec{h} \perp \vec{H}$. In the case considered by us, that of the normal skin effect, the penetration depth is $\delta = c(2\pi\sigma\omega\mu)^{-1/2}$, and if $\vec{h} \parallel \vec{H}$, then $\delta \sim \sqrt{(\partial H/\partial B)_T} \to 0$. Thus, at sufficiently low temperatures, anomalous quantum oscillations occur in the surface impedance $Z \sim \mu\delta \sim \sqrt{\mu}$; these oscillations are essentially anisotropic with respect to the mutual orientation of the vectors $\vec{h}$ and $\vec{H}$.

Deferring the detailed calculations to a separate communication, we shall point out now some of the most essential features of the anomalous quantum oscillations of surface impedance. We note first that the depth of penetration can never decrease below the radius R of the conduction-electron orbit. Physically this is connected with the fact that the magnetic moment is produced by the self-consistent induction field B at distances on the order of R. Thus, the magnetic moment senses relatively small changes of the induction ($b \ll B$) at distances $d \gtrsim R$, and in the opposite case ($d < R$) the magnetic susceptibility decreases in a ratio $(d/R)^2$. This leads to a depth of penetration $\delta \gtrsim R$ and to saturation of the surface impedance.

Let us estimate now the amplitude Z_{max} and the width of the oscillation peaks $(\Delta H)_{res}$ of the surface impedance, and also the maximum value of the derivative of the surface impedance with respect to the external magnetic field. In the region $\delta > \ell$ under consideration we have $Z = Z_0(\partial H/\partial B)^{-1/2}$. Near the critical point we have

$$\left(\frac{\partial H}{\partial B}\right)_T = \alpha\,\frac{\Delta T}{T_0} + \beta\left(\frac{\Delta B}{\delta B}\right)^2 \qquad (1)$$

where α, $\beta \sim 1$, $\delta B \sim B_0(\hbar\Omega/\varepsilon_0)$ is the period of the de Haas oscillations, $\Delta T = T - T_0$, and $\Delta B = B - B_0$. The magnetic-field and temperature regions in which (1) is valid are limited by the condition that the induction fluctuations be small compared with the distance ΔB to the critical point.

It is easy to show that $V\overline{\Delta B^2} \sim T/(\partial H/\partial B)_T$, where $\overline{\Delta B^2}$ is the mean-square fluctuation of the induction in the volume V. It is clear that the fluctuations in the volume $V \gtrsim R^3$ play an essential role.

Thus, the expression for the surface impedance in which formula (1) is substituted has a definite (and not accidental) character in the region where

$$T/R^3\left(\frac{\partial H}{\partial B}\right)_T < (\Delta B)^2 \qquad (2)$$

Substitution of the numerical values shows that for all $\Delta T_0 \gtrsim 10^{-4}$ the relation

$$\beta\left(\frac{\Delta B}{\delta B}\right)^2 \leqslant \alpha\,\frac{\Delta T}{T_0} \qquad (3)$$

is satisfied on the boundary of the region (2). Thus, the first term predominates in $(\partial H/\partial B)_T$ near the "resonance," i.e., Z_{max} is determined by the proximity to the critical temperature. It follows from (1) and (3) that

$$\frac{Z_{max}}{Z_0} \sim 1/\sqrt{\alpha\,\frac{\Delta T}{T_0}} \gg 1$$

and the oscillations are indeed anomalously large. Using (1), we can readily find that the maximum value of the derivative of the surface impedance with respect to the magnetic field is

$$\left(\frac{H}{Z}\,\frac{dZ}{dH}\right)_{max} = \left(\frac{H}{Z}\,\frac{dZ}{dB}\,1/\left(\frac{\partial H}{\partial B}\right)_T\right)_{max} \sim \frac{\varepsilon_0}{\hbar\Omega}\Big/\left(\frac{\alpha\Delta T}{T_0}\right)^{3/2} \gg 1.$$

This maximum is reached when $(\Delta B)_m \sim \Delta B\sqrt{\alpha\Delta T/T_0}$. Knowing the value of $(\Delta B)_m$, we can estimate the width of the "resonance" region $(\Delta H)_{res}$. Integrating (1), we get $\Delta H = \alpha\Delta T/T_0)\Delta B$ (with (3) taken into account). Substituting here the value of $(\Delta B)_m$ we get $(\Delta H)_{res} \sim \delta B(\alpha\Delta T/T_0)^{3/2}$

An important role is usually played near phase transitions by nonlinear effects. An estimate shows that if $b > \delta B\sqrt{\alpha\Delta T/T_0}$, then the problem becomes nonlinear close enough to the critical point ($b > \Delta B$). If

$$\delta B\sqrt{\alpha\Delta T/T_0} < b < \delta B, \qquad (4)$$

then the connection between h and b takes the form $h = (b^3/6)(\partial^3 H/\partial B^3)$, but if $b > \delta B$, then it is necessary to use the exact H(B) relation. The inequality (4) for the magnetic field h takes the form $\delta B(\alpha\Delta T/T_0)^{3/2} < h < \delta B$.[1)] When $T < T_0$, the analysis is carried out in similar fashion. It must be remembered, however, that a phase transition takes place at $H = H_0(T)$, and the value of the induction changes jumpwise from $B_1 = B_0 - \Delta B_1$ to $B_2 = B_0 + \Delta B_2$ [1]. Near the critical point, as is well known, we have $\Delta B_1 = \Delta B_2 = \sqrt{-3\alpha\Delta T/T_0}$ [2]. As a result, when

$$h \geq |H - H_0(T)|, \qquad (5)$$

the magnetic permeability μ is determined in the linear approximation by the quantity $(\partial H/\partial B)_m^{-1}$ and depends only on the temperature. Thus, in that region of the magnetic field H where the inequality (5) is satisfied, the surface impedance is independent of the magnetic field, and consequently the derivative of the surface impedance experiences a jump on the boundary of the region.

The author thanks M. Ya. Azbel' for a useful discussion.

[1] J. H. Condon, Phys. Rev. 145, 526 (1966)
[2] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Nauka, 1964 [Addison-Wesley, 1958].
[3] M. Ya. Azbel' and L. B. Dubovskii, ZhETF Pis. Red. 5, 414 (1967) [JETP Lett. 5, 338 (1967)].

---

1) It is known that nonlinear effects are usually significant under the condition that $h > \delta B$, but in this case the linear approximation remains the fundamental one [3].