

2004/5
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מזכ" 2 - פיתרון 11

1. פתרון

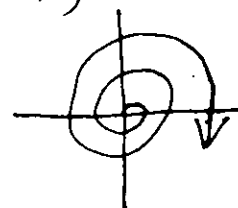
דמיון באופן איכותי את המיון הפני.

פיתרון

$$1. \begin{cases} \dot{x} = y \\ \dot{y} = -2x + y \end{cases} \quad \text{נק' קריטי} \quad (0,0) \quad |A-\lambda I| = \begin{vmatrix} -\lambda & 1 \\ -2 & 1-\lambda \end{vmatrix} = \lambda^2 - \lambda + 2$$

$$\lambda_{1,2} = \frac{1 \pm i\sqrt{7}}{2}$$

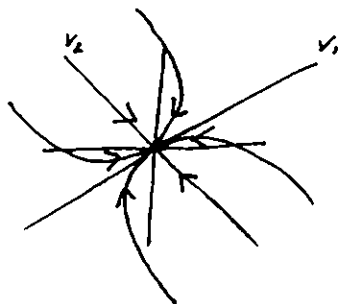
$$A \cdot (0) = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$



$$2. \begin{cases} \dot{x} = -3x + 2y \\ \dot{y} = x - 4y \end{cases} \quad \text{נק' קריטי} \quad (0,0) \quad |A-\lambda I| = \begin{vmatrix} -3-\lambda & 2 \\ 1 & -4-\lambda \end{vmatrix} = \lambda^2 + 7\lambda + 10$$

$$\lambda_1 = -2 \quad \lambda_2 = -5$$

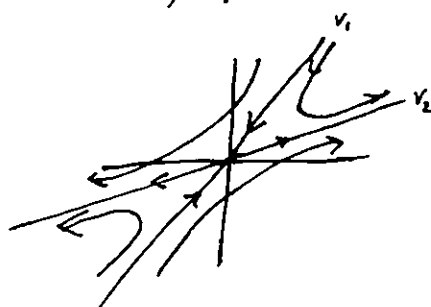
$$V_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad V_2 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$



$$3. \begin{cases} \dot{x} = 3x - 4y \\ \dot{y} = x - 2y \end{cases} \quad \text{נק' קריטי} \quad (0,0) \quad |A-\lambda I| = \begin{vmatrix} 3-\lambda & -4 \\ 1 & -2-\lambda \end{vmatrix} = \lambda^2 - \lambda - 2$$

$$\lambda_1 = -1 \quad \lambda_2 = 2$$

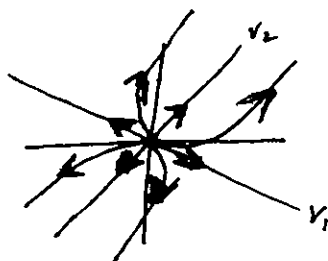
$$V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad V_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$



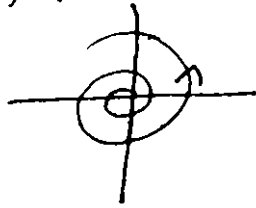
$$4. \begin{cases} \dot{x} = 2x + 3y \\ \dot{y} = x + 4y \end{cases} \quad \text{נק' קריטי} \quad (0,0) \quad |A-\lambda I| = \begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = \lambda^2 - 6\lambda + 5$$

$$\lambda_1 = 1 \quad \lambda_2 = 5$$

$$V_1 = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

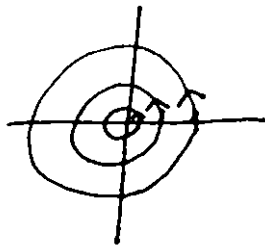


5. $\begin{cases} \dot{x} = 3x - 2y \\ \dot{y} = 4x - y \end{cases}$ מרכז, $\dot{y} = (0)$ $|A - \lambda I| = \begin{vmatrix} 3-\lambda & -2 \\ 4 & -1-\lambda \end{vmatrix} = \lambda^2 - 2\lambda + 5$
 $\lambda_{1,2} = 1 \pm 2i$



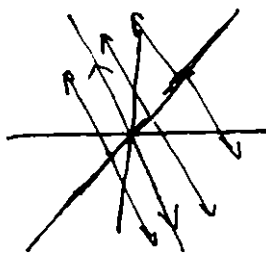
$$A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

6. $\begin{cases} \dot{x} = x - y \\ \dot{y} = 2x - y \end{cases}$ מרכז, $\dot{y} = (0)$ $|A - \lambda I| = \begin{vmatrix} 1-\lambda & -1 \\ 2 & -1-\lambda \end{vmatrix} = \lambda^2 + 1$
 $\lambda = \pm i$

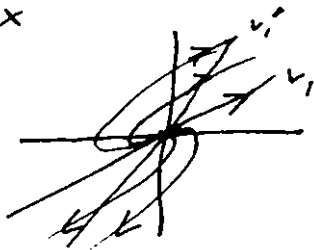


$$A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

7. $\begin{cases} \dot{x} = 3x - 2y \\ \dot{y} = 4y - 6x \end{cases}$ מרכז, $\dot{y} = (0)$ $|A - \lambda I| = \begin{vmatrix} 3-\lambda & -2 \\ -6 & 4-\lambda \end{vmatrix} = \lambda^2 - 7\lambda$
 $\lambda_1 = 0$ $\lambda_2 = 7$
 $v_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $v_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

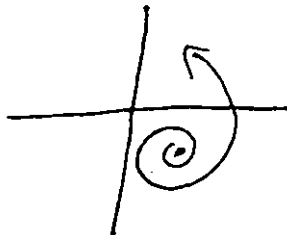


8. $\begin{cases} \dot{x} = x + 2y \\ \dot{y} = 5y - 2x \end{cases}$ מרכז, $\dot{y} = (0)$ $|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 \\ -2 & 5-\lambda \end{vmatrix} = (\lambda - 3)^2$
 $\lambda_{1,2} = 3$



$$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad v_2' = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

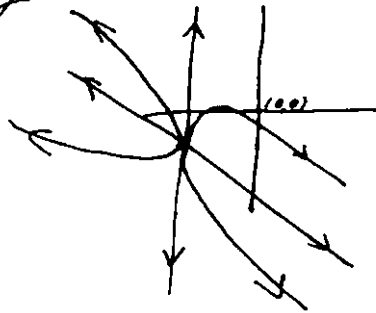
9. $\begin{cases} \dot{x} = x - 2y - 5 \\ \dot{y} = 2x + y \end{cases}$ מרכז, $\dot{y} = (-2)$ $|A - \lambda I| = \begin{vmatrix} 1-\lambda & -2 \\ 2 & 1-\lambda \end{vmatrix} = \lambda^2 - 2\lambda + 5$
 $\lambda_{1,2} = 1 \pm 2i$



$$A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

10. $\begin{cases} \dot{x} = 3x + 6 \\ \dot{y} = -x + 2y \end{cases}$ נק' קריטית $-\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ $|A - \lambda I| = \begin{vmatrix} 3-\lambda & 0 \\ -1 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda)$

$\lambda_1 = 2$ $\lambda_2 = 3$
 $v_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$



11. $\begin{cases} \dot{x} = x - y - 2 \\ \dot{y} = x - y - 2 \end{cases}$ נק' מיוחדת $\begin{pmatrix} x \\ x-2 \end{pmatrix}$ נק' קריטית יחיד

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & -1 \\ 1 & -1-\lambda \end{vmatrix} = \lambda^2$ ניקח למשל את הנק' $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\lambda_{1,2} = 0$
 $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $v_1' = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

