Ordinary Differential Equations – 1 (ODE-1)

Exercise 13

Question 1 Draw the phase portraits of the critical points for the following systems. Indicate the type of each point.

$$\begin{cases} \dot{x} = x - y \\ \dot{y} = x + y + 2 \end{cases}, \quad \begin{cases} \dot{x} = 4x - y + 1 \\ \dot{y} = 9x - 2y \end{cases}, \quad \begin{cases} \dot{x} = -5x + 3y \\ \dot{y} = -9x + 7y \end{cases}, \quad \begin{cases} \dot{x} = 3x - 7y \\ \dot{y} = 2x - 6y \end{cases},$$

$$\begin{cases} \dot{x} = 3x - 8y + 5 \\ \dot{y} = 2x - 5y + 3 \end{cases}, \quad \begin{cases} \dot{x} = 2x - 3y \\ \dot{y} = 4x - 5y \end{cases}, \quad \begin{cases} \dot{x} = 3x - 2y \\ \dot{y} = 2x - 2y + 2 \end{cases}, \quad \begin{cases} \dot{x} = -3x + y - 2 \\ \dot{y} = -5x + y \end{cases}.$$

Question 2

Find the solution for the Cauchy problem

$$\ddot{y} = 2\arctan\, y + \tfrac{3}{2}\dot{y}\cos\varepsilon - \varepsilon(e^{2t}y + \dot{y}^2) - \tfrac{1}{2}\pi, \quad y(0) = \cos\varepsilon, \dot{y}(0) = \varepsilon$$

in the linear approximation with respect to the small parameter $\varepsilon \approx 0$ whereas t belongs to some bounded closed time interval around 0.

Question 3 Find the general solutions for the following systems of DEs.

Find the coefficients $c_0, c_1, c_2, c_3, c_4 \in \mathbb{R}$ of the Taylor-expansion for the solution $x(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + o(t^4)$ of the Cauchy problem

$$e^t \ddot{x} - t\dot{x} + x = \ln(1+t), \quad x(0) = 1, \dot{x}(0) = 0 \ .$$