

Ordinary Differential Equations – 1 (ODE-1)

Exercise 12

Question 1 Find general solutions for the following DEs by means of the method of coefficients' variation.

- a. $\dot{y} = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} y + \begin{pmatrix} t^{-3} \\ -t^2 \end{pmatrix},$
- b. $\ddot{y} + \dot{y} = \frac{1}{\sin t}$ for $0 < t < \pi$.

Question 2 Solve the following DEs.

- a. $t^2 \ddot{y} + 3t\dot{y} - 3y = 0,$
- b. $t^2 \ddot{y} + 3t\dot{y} + 5y = 0,$
- c. $t^2 \ddot{y} - 4t\dot{y} + 6y = t^{-2} + \sin(\ln t) + t^3 e^t$ for $t > 0$. Remark: it is impossible to calculate the integral $\int \frac{e^x}{x} dx$ in elementary functions, leave it as it is.

Question 3 Find the general solutions for the following systems of DEs.

- a. Solve the DE $t\ddot{y} + 2\dot{y} + ty = 0$, $t \neq 0$, given its particular solution $\frac{\sin t}{t}$.
- b. Solve the system of DEs $t\dot{y} = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix} y$, $t \neq 0$, given its particular solution $\begin{pmatrix} t \\ t \end{pmatrix}$.