Ordinary Differential Equations – 1 (ODE-1)

Exercise 11

Question 1 What is the form of a particular solution to be searched for according to the method of undetermined coefficients for the following DEs?

a. $\dddot{y} - 3\ddot{y} + 3\dot{y} - y = t^2 + t\sin t - te^t$,

b. $\ddot{y} + 6\dot{y} + 13y = 1 - e^{-3t} + te^{-3t}\sin t$,

c. $y^{(4)} - 8\dot{y} = t^2 - te^{-t}\sin\sqrt{3}t + 1 + \cos t$,

d. $y^{(4)} + 8\ddot{y} + 16y = t\sin 2t - e^t$.

Question 2 What is the form of a particular solution to be searched for according to the method of undetermined coefficients for the following systems of DEs?

a. $\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 1+\cos 2t \\ t^2 + te^t \end{pmatrix}$,

b. $\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} te^{2t}\cos 3t \\ t^2 + e^t \end{pmatrix}$.

Question 3 Find the general solutions for the following systems of DEs.

a. $\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} \cos t \\ t \end{pmatrix}$,

b. $\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 0 \\ t + e^{-t} \end{pmatrix}$.

Question 4 What are the values of $a \in \mathbb{R}$ for which all solutions of the system $\dot{y} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} y + \begin{pmatrix} \sin at \\ 0 \end{pmatrix}$ are bounded? Answer without solving the DE.

Question 5

a. Which pair of functions (t, e^t) or $(1, e^t)$ can constitute a pair of fundamental solutions for a linear homogeneous second-order DE with functional coefficients continuous for $t \in \mathbb{R}$? What is the DE?

b. Which pair of vector functions $\begin{pmatrix} \cos t \\ -1 \end{pmatrix}, \begin{pmatrix} t^2+1 \\ \cos t \end{pmatrix}$ or $\begin{pmatrix} t \\ -t \end{pmatrix}, \begin{pmatrix} t^2+1 \\ t \end{pmatrix}$ can constitute a pair of fundamental solutions for a linear homogeneous system of first-order DEs with functional coefficients continuous for $t \in \mathbb{R}$? What is the system?