# *Ordinary Differential Equations – 1 (ODE-1)*

## *Exercise 10*

## Question 1

Motion of a stretched spring on a smooth table is described by the DE $m\ddot{x}+kx=0$ where $m>0$ is the spring mass, $k>0$ is the elasticity coefficient. Prove that the sum of the kinetic energy $m\frac{\dot{x}^2}{2}$ and the potential energy $k\frac{x^2}{2}$ is kept constant. How can it be used to decrease the equation order?

## Question 2 Solve the following linear DEs

a. $y''-2y'+y=0$,

b. $y''+6y'+13y=0$,

c. $y''-6y'+8y=0$,

d. $y^{(4)}-8y'=0$,

e. $y^{(4)}-16y=0$,

f. $y^{(4)}+y''-2y=0$.

## Question 3

Consider the linear DE with constant real coefficients

$$y^{(n)}+a_{n-1}y^{(n-1)}+\ldots+a_1y'+a_0y=0 \qquad (*)$$

a. Prove that if $x^k\sin(x)$ is a solution of (*) then $n\ge 2k+2$.

b. Let $\sum_{k=0}^{n-1}a_k=-1$. Find a solution of (*) which satisfies $y(0)=y'(0)=\ldots y^{(n-1)}(0)=1$.

c. Is the Cauchy-problem solution found in b. unique? Explain the answer.

## Question 4 Which of the following DEs have resonances? What are the multiplicities of the resonance roots? Solve DEs a, b, c.

a. $\ddot{y}-y=e^t$,

b. $\ddot{y}+4y=1+\sin 2t+e^t$,

c. $\dddot{y}-y=t+1-\sin t$,

d. $\ddot{y}-2\dot{y}+y=t^2\cos t+1$,

e. $y^{(5)}-y^{(4)}=t+t^3e^t$,

f. $y^{(4)}-16y=t+\cos 2t-\tan t$,

g. $\ddot{y}-8\dot{y}+20y=e^{2t}\sin 2t+t^2e^{4t}\cos 2t-e^{4t}\sin 2t$,

h. $\ddot{y}-6\dot{y}+10y=\frac{1}{t}e^{3t}\sin t+e^{3t}-t\sin t$.