Ordinary Differential Equations – 1 (*ODE-1*)

Exercise 9

Question 1 Solve the following DEs over $\mathbb{R}$

a. $\dot{y} = \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix} \vec{x}$, $\dot{y} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} y$;

b. $\dot{y} = \begin{pmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ -2 & -1 & 0 \end{pmatrix} y$, one eigenvalue is known to be -2;

c. $\dot{y} = \begin{pmatrix} 4 & 9 & 9 \\ -6 & -11 & -6 \\ 3 & 3 & -2 \end{pmatrix} y$, one eigenvalue is known to be 1;

d. $\dot{y} = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} y$;

e. $\dot{y} = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 5 & 3 \end{pmatrix} y$.

Question 2

Check which of the following functions are quasi-polynomials (real or complex). Indicate the order and the eigenvalue for each quasi-polynomial.

a. 2,

b. $\cos t$,

c. $\sin t \cos t + e^t$,

d. $e^{5t} \cos t - te^{5t} \sin t$,

e. $t^{-2}e^{-t}$,

f. $e^{it}(\cos t - t \sin t)$,

g. $e^{(1-i)t}t^3$,

h. $e^t \tan t$,

i. $t^7 + 5t$,

j. $\sin^2 t - \sin t$.

a. $y' = \dfrac{y}{3x - y^2}$