Ordinary Differential Equations – 1 (ODE-1)

Exercise 4

Question 1 Consider the DE

$$y^2 dx + (xy + \tan xy)dy = 0$$

a. Solve the DE by means of the substitution $xy = z$

b. Utilize the identity $d(xy) = ydx + xdy$ in the solution

Question 2 Consider the DE

$$y' = \tfrac{1}{2}(-x + \sqrt{x^2 + 4y})$$

a. Find the x-segments over which both functions $y_1 = 1 - x, y_2 = -\tfrac{1}{4}x^2$ are solutions of the DE.

b. Explain why the existence of two solutions does not contradict the theorem of the existence and the uniqueness of solutions.

Question 3 Consider the Cauchy problem

$$y' = x - y^2, y(0) = 0 .$$

a. Perform 3 Picard iterations and find the consecutive approximations y_1, y_2, y_3 of the solution y .

b. Estimate the approximation error $\varphi_3(\tfrac{1}{2}) = | y(\tfrac{1}{2}) - y_3(\tfrac{1}{2}) | .$

Question 4 Consider the Cauchy problem

$$y' = y^2, y(0) = 1.$$

a. Calculate 3 consecutive Picard approximations y_1, y_2, y_3 of the solution y

b. Assuming that the solution is unique, do the Picard approximations converge to the solution for all $x \in \mathbb{R}$?

Question 5 Consider the DEs

$$\begin{cases} \dot{x} = x^2 - | x - y |^{1/2} \\ \dot{y} = \ln t + t^{1/3} x^{1/5} \end{cases} , (y-1)\ddot{y} - | \dot{y} + x | \tan x = 0 \ , \ x\ddot{y} - | \dot{y} + x |^{1/2} = 0$$

where the differentiation is with respect to t . What are the initial conditions for which there exists a unique solution?