Ordinary Differential Equations – 1 (ODE-1)

Exercise 2

Question 1 Solve the following DEs

a. $y' + 2y = x + e^{-x}$

b. $y' - \frac{1}{x} y = x$

c. $x \ln(x) y' + my = \ln^{-m}(x),\ m \in \mathbf{N}$

d. $y' = \frac{y}{3x - y^2}$

Question 2

Prove that if $x(t) \le B + \int_{t_0}^{t} a(s)x(s)ds$ holds for any $t \in [t_0, t_1]$ for some functions continuous in $[t_0, t_1]$, then also $x(t) \le Be^{\int_{t_0}^{t} a(s)ds}$ holds for all $t \in [t_0, t_1]$.

Question 3 Solve the DE $y' = y^4 \cos(x) + y \tan(x)$

Question 4

Let $|f(x)| \le M$, $x \in \mathbf{R}$, $M > 0$, be a bounded function. Consider the DE $y' + ay = f(x)$ for some $a > 0$. Prove that each its solution is bounded over $[0, \infty)$.

Question 5 Solve the following DE

Let y_1, y_2, y_3 be different particular solutions of the linear DE $y' + a(x)y = b(x), x \in \mathbf{R}$, with continuous coefficients. Prove that the function $\frac{y_2 - y_3}{y_3 - y_1}$ is a constant.