

Ordinary Differential Equations – 1 (ODE-1)

Exercise 1

Question 1

Solve the following DE for each $a \in \mathbf{N}$, $\begin{cases} y' = y^a \\ y(a) = a - 2 \end{cases}$

Question 2

- a. Prove that the function $y = \frac{2x}{c - x^2}$ satisfies the DE $y' = \frac{y}{x} + y^2$ for any $c \in \mathbf{R}$.
- b. How many solutions has the DE for the initial condition $y(0) = 0$?
- c. How many solutions of the DE correspond to the initial condition $y(1) = 0$?

Question 3

- a. Solve the equation $\begin{cases} y' = 3\sqrt[3]{y^2} \\ y(2) = 0 \end{cases}$;
- b. Find all solutions $y(x)$ of $3y^2 y' + 16x = 2xy^3$ bounded as $x \rightarrow \infty$.

Question 4

- a. Solve $y' = \sqrt{5x + 2y - 3}$
- b. Solve $(2x + y + 1)dx = (4x + 2y - 3)dy$

Question 5

Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be an invertible differentiable and strictly increasing function

- a. Find the general solution of the DE $f'(y)y' = xf(y)$;
- b. Find the solution which satisfies $\lim_{x \rightarrow \infty} |y(x)| < \infty$.