# Topics in non-linear control

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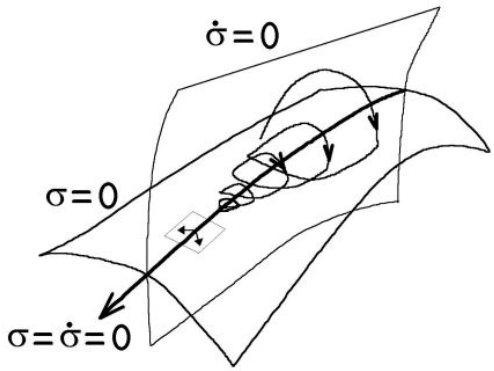
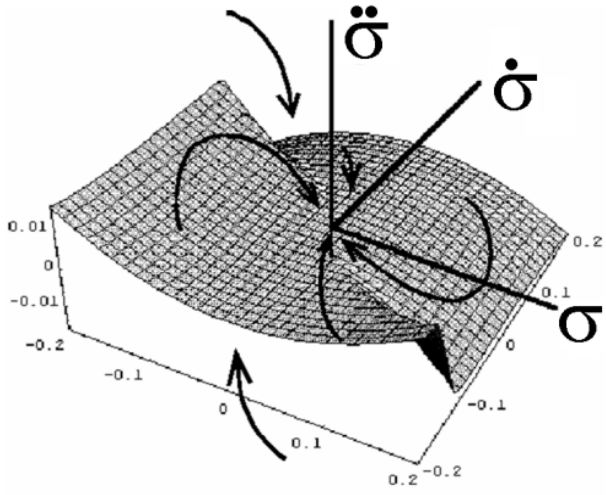
The goal of the course is to introduce the student to the latest effective and easily applicable methods of control under heavy uncertainty conditions. The most advanced results are reached by means of a simple intuitively clear and analytically simple approach. The developed technique is based on the most modern results of sliding-mode control theory. At the end of the course the student will be able to design effective non-linear controllers to control general-case Single-Input-Single-Output (SISO) uncertain systems with known relative degree, differentiate noisy signals and to solve a large number of practical problems. The approach has proved to be especially effective in robotics, avionics, power plant industry, electric motor design, any kinds of targeting and tracking problems, signal and image processing.

Standard basic courses of ordinary differential equations and infinitesimal calculus are sufficient for understanding the course.




Second-order sliding mode (on the line)




Third-order sliding mode (at the origin only)

# Syllabus:

1. Control theory and its problems. Dynamic system. Stability notions and Lyapunov theory overview. Controllability and observability. Normal form of controllable linear autonomous system.
2. Lie derivative. Frobenius theorem. Controllability. Feedback linearization. Relative degree of non-linear control system. Scalar and vector relative degree. Zero dynamics. Minimum-phase feature. Uncertainty conditions and robustness problem.
3. Introduction to Sliding-Mode Control. Equivalent control method. Differential equations with discontinuous right-hand sides: the Filippov theory. Theorems on solution existence, continuous dependence on the right-hand side and initial conditions.
4. Invariance of the sliding mode motions and matching conditions. Mathematical equivalence between classical sliding mode control and high-gain feedback. Chattering effect and sliding mode regularization.
5. Higher-order sliding mode: connection with zero dynamics and vector relative degree. Existence theorem. Discrete sampling sliding-mode control. Sliding order and sliding accuracy.
6. Infinite-time and finite-time convergence. Higher-order sliding modes in systems with actuators. The Anosov theorem. Dynamical sliding modes. Arbitrary-order sliding problem and its importance.
7. The homogeneity notion and the homogeneity approach to sliding-mode design. 2-sliding technique: main algorithms. Practical example: aircraft pitch-control contour design. Arbitrary-order sliding controllers with finite-time convergence. Quasi-continuous sliding-mode control.
8. Real-time differentiation problem. Intrinsic accuracy restrictions. Asymptotically-optimal differentiation. Arbitrary-order optimal robust exact differentiator with finite-time convergence. Signal and image processing applications.
9. Universal output-feedback controller for uncertain Single-Input-Single-Output system with known relative degree. Homogeneity approach and the accuracy asymptotics. Some results in the Multi-Input-Multi-Output case.

**References**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4. Isidori, A., 1995, *Nonlinear Control Systems*, (New York: Springer Verlag).
5. Edvards C. and Spurgeon S.K., 1998, *Sliding mode control: theory and applications*, Taylor & Francis, London.
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9. Levant A., 2006, Homogeneous quasi-continuous sliding-mode control. In Edwards C., Fossas E., Fridman L. (Eds.), *Advances in Variable Structure and Sliding Mode Control*, *Lecture Notes in Control and Information Science*, Springer Verlag, 334, 143--168
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