Integral High-Order Sliding Modes

Arie Levant, Lela Alelishvili

School of Mathematical Sciences, Tel-Aviv University, Ramat-Aviv, 69978 Tel-Aviv, Israel

E-mail: levant@post.tau.ac.il, lela@post.tau.ac.il . Tel.: 972-3-6408812, *Fax:* 972-3-6407543

Abstract - Integral sliding mode approach is extended to high-order sliding modes, and allows choosing transient dynamics, or assigning a transient-time function of initial conditions. The resulting controller is robust and capable of controlling outputs of uncertain smooth SISO systems of a known permanent relative degree. The control smoothness can be deliberately increased.

Index Terms - high-order sliding mode, robustness, output feedback control, finite-time stability

I. Introduction

Sliding mode control copes with system uncertainty keeping a properly chosen constraint by means of high-frequency control switching. Featuring robustness and high accuracy [6, 23], the standard (first order) sliding-mode usage is however restricted due to the chattering effect [4, 9, 10, 22] caused by the control switching, and the equality of the constraint relative degree to 1 [22, 24].

High-order sliding mode (HOSM) approach [2 - 4, 8, 13 - 21] suggests to treat the chattering effect using a time derivative of control as a new control, thus integrating the switching. Let $\sigma = 0$ be the constraint to be kept, where σ is the output of an uncertain single-input-single-output (SISO) dynamic system. HOSMs are applicable with any relative degree r, i.e. when the control u explicitly appears for the first time in the rth total time derivative $\sigma^{(r)}$, and $\frac{\partial}{\partial u}\sigma^{(r)}$ is separated from zero. The corresponding finite-time-convergent controllers (r-sliding controllers) [15 - 17] produce control being a bounded discontinuous function of σ and its derivatives $\dot{\sigma}$, $\ddot{\sigma}$, ..., $\sigma^{(r-1)}$. The controllers are predefined for any r, and provide in finite time for keeping the equalities $\sigma = \dot{\sigma} = ... = \sigma^{(r-1)} = 0$ (the r-sliding mode [14, 15]). The missing derivatives can be estimated by the robust exact finite-time-convergent differentiators [11, 15, 20] generating output-feedback controllers [3, 15 - 17]. The approach provides also for higher accuracy with discrete sampling [14, 15 - 17].

Some realization problems of r-sliding modes are caused by the complicated structure of the transient process, which is difficult to monitor with $r > 2$ [8, 13, 17]. Another specific problem concerns the above-mentioned control-smoothing procedure, when $u^{(l)}$ is treated as a new control. Due to the interaction of u and its derivatives during the convergence to the $(r + l)$-sliding mode $\sigma = \dot{\sigma} = ... = \sigma^{(r+l-1)} = 0$, any $(r + l)$-sliding controller is only locally effective in some vicinity of the. mode. The global convergence is so far assured only for $r = l = 1$ [14].

The above issues can be resolved eliminating the transient process. The corresponding technique has a lot of applications [23, 24] with the traditional 1-sliding mode, and is known as integral sliding mode (ISM). The traditional ISM manifold has the codimension 1 and changes in time, containing the current state t, $x(t)$, so as to coincide with the goal manifold $\sigma = 0$ at the moment when $\sigma(t,x(t))$ vanishes. The r-sliding ISM is of the codimension r and is better described by a trajectory in the space $\sigma, \dot{\sigma}, ..., \sigma^{(r-1)}$ terminating at the origin. We show that a transient dynamics with a predefined settling time function of initial conditions can be prescribed to HOSM in such a way, assuring global ISM stability and, respectively, global convergence to the r-sliding mode.

The semi-global convergence is provided by this procedure, when the control smoothness is raised, artificially increasing the relative degree. A drawback of the approach is some complexity of the HOSM implementation, which may be redundant in many practical cases, when the natural local convergence to HOSM is already sufficient [3]. Simulation confirms the approach feasibility.

II. The problem statement and the integral *r*-sliding mode concept

Consider a SISO dynamic system with the state $x \in \mathbf{R}^n$, input $u \in \mathbf{R}$, and output σ of the form

$$\dot{x} = a(t,x) + b(t,x)u, \qquad \sigma = \sigma(t, x) \in \mathbf{R}, \quad u \in \mathbf{R}, \tag{1}$$

where a, b and σ are unknown smooth functions, n can be also uncertain. The task is to get $\sigma \equiv 0$.

All differential equations are understood in the Filippov sense [7], which allows discontinuous dynamics. The system relative degree r is assumed to be constant and known, which causes [12] that

$$\sigma^{(r)} = h(t,x) + g(t,x)u, \tag{2}$$

where $h(t,x) = \sigma^{(r)}|_{u=0}$, $g(t,x) = \frac{\partial}{\partial u}\sigma^{(r)} \neq 0$ are some unknown functions. It is supposed that

$$0 < K_{\mathrm{m}} \leq \frac{\partial}{\partial u}\sigma^{(r)} \leq K_{\mathrm{M}}, \qquad |\,\sigma^{(r)}|_{u=0}\,| \leq C \tag{3}$$

for some K_{m}, K_{M}, $C > 0$. Trajectories of (2) are assumed infinitely extendible in time for any Lebesgue-measurable bounded control $u(t, x)$. Though formally not needed, it is probably required in practice that the system feature bounded-input-bounded-state property.

The above problem statement is standard and is solved by known r-sliding controllers [15 - 17]

$$u = \alpha\Psi_r(\sigma, \dot{\sigma}, ..., \sigma^{(r-1)}), \tag{4}$$

which actually solve the problem for the differential inclusion $\sigma^{(r)} \in [-C, C] + [K_{\mathrm{m}}, K_{\mathrm{M}}]u$ instead of (2), (3). Here Ψ_r is a bounded discontinuous function. Only the control gain $\alpha > 0$ needs to be adjusted for the concrete values of C, K_{m}, K_{M}, providing for the finite-time convergence of the inclusion trajectories to zero.

Suppose that it is needed to avoid the uncertainty of the transient process, and/or some transient time restrictions are present, transient-trajectory coordinates are to be bounded, etc. Let these requirements be fulfilled by a transient trajectory $\sigma(t, x(t)) = \varphi(t)$, $t_0 \leq t \leq t_f$, so that

$$\varphi(t_0) = \sigma(t_0), \quad \dot{\varphi}(t_0) = \dot{\sigma}(t_0), \quad ..., \quad \varphi^{(r-1)}(t_0) = \sigma^{(r-1)}(t_0), \qquad \varphi(t) \equiv 0 \quad \text{with } t \geq t_f, \tag{5}$$

where t_0 and $t_f > t_0$ are respectively the initial and the final times. Alternatively, some dynamics

$$\sigma^{(i)} = z^{(i)}, \quad z^{(r)} = V(t, x, z, \dot{z}, ..., z^{(r-1)}), \quad z^{(i)}(t_0) = \sigma^{(i)}(t_0), \quad i = 0, ..., r - 1, \tag{6}$$

can be required. Here and further, for the sake of brevity, $\sigma(t)$ is written instead of $\sigma(t, x(t))$ whenever the ambiguity is avoided.

Standard integral sliding mode. A stable polynomial $\mu^{r-1} + d_1\mu^{r-2} + ... + d_{r-1}$ is chosen, and the auxiliary function $\Sigma = \sigma^{(r-1)} - \varphi^{(r-1)} + d_1(\sigma^{(r-2)} - \varphi^{(r-2)}) + ... + d_{r-1}(\sigma - \varphi)$ is introduced. Alternatively, $\Sigma = \sigma^{(r-1)} - z^{(r-1)} + d_1(\sigma^{(r-2)} - z^{(r-2)}) + ... + d_{r-1}(\sigma - z)$, if (6) is required. The problem is solved by means of the 1-sliding-mode control of the form $u = -k(t, x)\,\mathrm{sign}\,\Sigma$. Theoretically, the equalities $\Sigma = 0$ and $\sigma^{(i)} = \varphi^{(i)}$ (or $\sigma^{(i)} = z^{(i)}$) are kept from the very beginning and forever. In practice, nevertheless, any initial measurement error leads to asymptotic convergence only. The control magnitude depends on all derivatives of σ and is necessarily unbounded in spite of restrictions (3). The final accuracy is proportional to the sampling time interval [22, 14].

Integral r-sliding mode. Let $\varphi^{(r-1)}(t)$ be a Lipschitz function, then almost everywhere it has a globally bounded derivative $\varphi^{(r)}(t)$, and the new output $\Sigma(t, x) = \sigma(t, x) - \varphi(t)$ satisfies conditions (2), (3) with some changed constants K_{m}, K_{M}, $C > 0$. Alternatively, $\Sigma = \sigma - z$ is taken, if (6) is to be maintained with some globally bounded function V. Hence, $\Sigma \equiv 0$ can be kept in r-sliding mode by the bounded control $u = \alpha\Psi_r(\Sigma, \dot{\Sigma}, ..., \Sigma^{(r-1)})$. Due to the finite-time convergence, any small initial measurement error is practically immediately compensated. The accuracy $|\sigma^{(i)} - \varphi^{(i)}| \leq \gamma_i\, \tau^{r-i}$ is maintained, τ being the sampling time interval, $i = 0, ..., r - 1$ [14, 15 - 17].

The missing derivatives of σ can be calculated on-line by means of the robust exact differentiators [15] with finite-time convergence. Since the only condition is the boundedness of $\sigma^{(r)}$, the differentiator convergence is global with the bounded r-sliding control, providing for the global convergence to the r-sliding mode $\sigma \equiv 0$. Only local differentiator convergence takes place in the case of the standard integral-sliding-mode approach.

III. Application of high-order integral sliding modes

Transient time assignment for *r*-sliding mode. Introduce few notions. Denote

$$\vec{\sigma} = (\sigma, \dot{\sigma}, ..., \sigma^{(r-1)}), \quad d_\kappa\vec{\sigma} = (\kappa^r\sigma, \kappa^{r-1}\dot{\sigma}, ..., \kappa\sigma^{(r-1)}).$$

The linear transformation d_κ: $\mathbf{R}^r \to \mathbf{R}^r$ is called *the homogeneity dilation* [1]. A function $f(\vec{\sigma})$ is called *r-sliding homogeneous* [16] *with the homogeneity degree* (*weight*) m if the identity $f(d_\kappa\vec{\sigma}) = \kappa^m f(\vec{\sigma})$ holds for any $\vec{\sigma}$ and any positive κ. The controller (4) is *r-sliding homogeneous*, if the identity $\Psi_r(\vec{\sigma}) = \Psi_r(d_\kappa\vec{\sigma})$ holds for all $\vec{\sigma}$ and $\kappa > 0$, i.e. $\Psi_r(\vec{\sigma})$ in (4) is a homogeneous function of the weight 0 [16].

Let the $(r$-1)-smooth function $\varphi(t)$ satisfying (5) have the form

$$\varphi = (t - t_f)^r (c_0 + c_1(t - t_0) + ... + c_{r-1}(t - t_0)^{r-1}) \quad \text{with } t_0 \le t \le t_f, \quad \varphi = 0 \quad \text{with } t > t_f. \tag{7}$$

Parameters c_i are now to be found from the conditions (5) after t_f is assigned. Obviously, any constant value of the transient time $t_f - t_0$ requires unacceptably large control values in order to steer the trajectory to the r-sliding mode $\vec{\sigma} = 0$ from far-distanced initial values, and leads to very low convergence rate if $\vec{\sigma}(t_0)$ is close to zero. Thus, let $t_f - t_0$ be a continuous positive-definite r-sliding homogeneous function of the initial conditions $\vec{\sigma}(t_0)$ of the degree 1, i.e.

$$t_f - t_0 = T(\vec{\sigma}(t_0)), \qquad \forall\kappa > 0 \;\; T(d_\kappa \vec{\sigma}) \equiv \kappa\, T(\vec{\sigma}). \tag{8}$$

For example, the choice $t_f - t_0 = T(\vec{\sigma}(t_0)) = \lambda\, (|\sigma(t_0)|^{p/r} + |\dot{\sigma}(t_0)|^{p/(r-1)} + ... + |\sigma^{(r-1)}(t_0)|^{p}\,)^{1/p}$ is valid, where $p, \lambda > 0$. As a result, the function φ turns out to be a function of $t - t_0$ and initial conditions $\vec{\sigma}(t_0)$.

Theorem 1. *The function* $\varphi(t - t_0, \vec{\sigma}(t_0))$ *is uniquely determined by* (5), (7), (8). *Let* (4) *be any of finite-time convergent r-sliding homogeneous controllers* [15 - 17], *then with any sufficiently large* α, *independently of the initial conditions* $\vec{\sigma}(t_0)$, *the controller*

$$u = \alpha\Psi_r(\Sigma, \dot{\Sigma}, ..., \Sigma^{(r-1)}), \quad \Sigma(t, x) = \begin{cases} \sigma(t,x) - \varphi(t - t_0, \vec{\sigma}(t_0)), & t_0 \le t \le t_0 + T(\vec{\sigma}(t_0)) \\ \sigma(t,x), & t \ge t_0 + T(\vec{\sigma}(t_0)) \end{cases} \tag{9}$$

establishes the finite-time-stable r-sliding mode $\sigma \equiv 0$ *with the transient time* (8). *The equality* $\sigma(t, x(t)) = \varphi(t - t_0, \vec{\sigma}(t_0))$ *is kept during the transient process.*

Here and further all proofs are omitted due to the lack of place and can be found in [18] and at the internet address http://www.tau.ac.il/~levant. It can be shown that the function $\varphi(\tau, \vec{\sigma})$ is homogeneous in the sense $\varphi(\kappa\tau, d_\kappa\vec{\sigma}) \equiv \kappa^r \varphi(\tau, \vec{\sigma})$, $\forall\kappa > 0$. The homogeneity and continuity result in the global boundedness of $\frac{\partial^r}{\partial t^r}\varphi(t - t_0, \vec{\sigma}(t_0))$, $0 \le t - t_0 \le T(\vec{\sigma}(t_0))$, which implies the Theorem.

Analytic calculations of the function φ and its successive time derivatives are easily performed in real time by computer at the moment t_0, and used afterwards. The corresponding formulas are listed in the simulation Section for $r = 4$.

Suppose that sampling noises are present, being bounded Lebesgue-measurable functions of time of any nature, and sampling is carried out with some sampling intervals. Due to (2), (3) the boundedness of the control (9) provides for the global convergence of $(r - l - 1)$th-order differentiator [15] applied to $\sigma^{(l)}$. The following robustness Theorem follows easily from [16].

Theorem 2. *Under the conditions of Theorem* 1 *let the sampling noise magnitudes of* $\sigma, \dot{\sigma}, ..., \sigma^{(l)}$ *not exceed* $\beta_0\varepsilon, \beta_1\varepsilon^{(r-1)/r}, \dots, \beta_k\varepsilon^{(r-l)/r}$ *respectively with some* $\beta_0, ..., \beta_l > 0$, *and the rest of derivatives be estimated by means of an* $(r - l - 1)$*th order differentiator* [15] *with a proper parameter set. Let also sampling intervals not exceed* $\tau = \varepsilon^{1/r} > 0$. *Then the inequalities* $|\Sigma| \le \gamma_0\tau^r = \gamma_0\varepsilon$, $|\dot{\Sigma}| \le \gamma_1\tau^{r-1} = \gamma_1\varepsilon^{(r-1)/r}$, ..., $|\Sigma^{(r-1)}| \le \gamma_{r-1}\tau = \gamma_{r-1}\varepsilon^{1/r}$ *are established with some constants* $\gamma_0, ..., \gamma_{r-1}$ *independent of* ε.

The case $\varepsilon = 0$ corresponds to the case of exact continuous measurements. That asymptotic accuracy cannot be improved with a constant sampling interval $\tau > 0$ and $|\sigma^{(r)}|$ separated from 0 [14]. Note that the differentiator convergence can be made arbitrarily fast [15].

Raising the control smoothness degree. Choose some integer $k > r$ and consider $u^{(k-r)}$ as a new control. The new relative degree is k. The function $\varphi(t - t_0, \vec{\sigma}(t_0))$, $\vec{\sigma} = (\sigma, \dot{\sigma}, ..., \sigma^{(k-1)})$, satisfies

$$\varphi = \sigma(t_0), ..., \frac{\partial^{k-1}}{\partial t^{k-1}}\varphi = \sigma^{(k-1)}(t_0) \text{ at } t = t_0, \; \varphi = (t - t_f)^k (c_0 + c_1(t - t_0) + ...+ c_{k-1} (t - t_0)^{k-1}),$$

$$t_f - t_0 = T(\sigma(t_0), \dot{\sigma}(t_0), ..., \sigma^{(k-1)}(t_0)), \quad T(\sigma, \dot{\sigma}, ..., \sigma^{(k-1)}) \equiv T(\kappa^r\sigma, \kappa^{r-1}\dot{\sigma}, ..., \kappa\sigma^{(k-1)}),$$

where $\kappa > 0$ and T is continuous and positive definite. Let the bounded feedback control be

$$u^{(k-r)} = \alpha\Psi_k(\Sigma, \dot{\Sigma}, ..., \Sigma^{(k-1)}), \qquad \Sigma = \begin{cases} \sigma(t,x) - \varphi(t - t_0, \vec{\sigma}(t_0)), & t_0 \le t \le t_0 + T(\vec{\sigma}(t_0)) \\ \sigma(t,x), & t \ge t_0 + T(\vec{\sigma}(t_0)) \end{cases} \tag{10}$$

with arbitrary initial values $u(t_0), ..., u^{(k-r-1)}(t_0)$, where $\alpha\Psi_k$ is one of the finite-time convergent r-sliding homogeneous controllers from [15 - 17].

Theorem 3. *Let the initial conditions* $t_0, x(t_0), u(t_0), ..., u^{(k-r-1)}(t_0)$ *belong to some compact set in* $\mathbf{R}^{n+k-r+1}$. *Then with sufficiently large* α *controller* (10) *establishes the k-sliding mode* $\sigma \equiv 0$ *with the transient time* $T(\vec{\sigma}(t_0))$. *The equality* $\sigma(t, x(t)) = \varphi(t)$ *is kept during the transient process.*

The direct application of any k-sliding controller required the dominance of $u^{(k-r)}g(t, x)$ in $\sigma^{(k)}(t, x, u, ..., u^{(k-r)})$, containing also terms with lower derivatives of u. This interaction is now removed. Indeed, define the smooth function $u_{eq}(t, x) = - h(t, x)/g(t, x)$ from (2) and the condition $\sigma^{(r)} = 0$. Functions $u(t)$, ..., $u^{(k-r-1)}(t)$ track now the smooth function $\tilde{u}_{eq} = u_{eq}(t, x(t)) + \frac{\partial^r}{\partial t^r}\varphi(t - t_0, \sigma(t_0),..., \sigma^{(k-1)}(t_0))/g(t,x(t))$ and its total derivatives $\dot{\tilde{u}}_{eq}$, ..., $\tilde{u}_{eq}^{(k-r-1)}$ calculated with respect to (1) modified by the substitution $u = u_{eq}(t, x)$ (the corresponding zero-dynamics).

IV. Simulation example

Consider a variable-length pendulum control problem. All motions are restricted to some vertical plane, friction is absent. A load of a known mass m moves along the pendulum rod (Fig. 1). Its distance from O equals $R(t)$ and is not measured. An engine transmits a torque w, which is considered as control. The engine dynamics is neglected. The task is to track some function x_c given in real time by the angular coordinate x of the rod.

The system is described by the equations $\ddot{x} = -2\frac{\dot{R}}{R}\dot{x} - g\frac{1}{R}\sin x + \frac{1}{mR^2}w, \qquad \ddot{w} = u,$

where $g = 9.81$ is the gravitational constant, $m = 1$. Let R, $\dot{R}$, $\ddot{R}$, $\dot{x}_c$ and $\ddot{x}_c$ be bounded, R be separated from 0, $\sigma = x - x_c$, $\dot{\sigma} = \dot{x} - \dot{x}_c$ be available. The initial conditions are $x(0) = \dot{x}(0) = 0$. The natural relative degree of the system is 2, but is artificially raised to 4, using $\ddot{w} = u$ as a new control, in order to smooth the control and to avoid unacceptable torque switching, which cannot be performed by the engine.

Let $w(0) = \dot{w}(0) = 0$. Since $\sigma^{(4)}|_{u=0}$ linearly depends on $\dot{x}$, it is not uniformly bounded. Nevertheless, all assumptions are satisfied in any bounded region of the system coordinates, which provides for the semi-global application of the method. Following are the functions R and x_c considered in the simulation:

$$R = 1 + 0.25 \sin 4t + 0.5 \cos t, \qquad x_c = 0.5 \sin 0.5t + 0.5 \cos t .$$

The parameters of the controller were tuned during simulation, avoiding complicated calculations and crude estimations leading to excessively large gains. In any case the controlled class still allows significant disturbances of considered functions R and x_c.

The transient dynamics is chosen according to (5), (7), (8) as follows:

$$\varphi(t) = (t - t_f)^4(c_0 + c_1(t - t_0) + c_2(t - t_0)^2 + c_3(t - t_0)^3),\ T = t_f - t_0= \lambda(|s_0(t_0)|^3+s_1(t_0)^4+| s_2(t_0)|^6+s_3(t_0)^{12})^{1/12},$$

$$c_0 = s(t_0)\,T^{-4},\qquad c_1 = s_1(t_0)\,T^{-4}+ 4\, s(t_0)\,T^{-5},\qquad c_2 = [s_2(t_0)\,T^{-4}+ 8\, s_1(t_0)\,T^{-5} + 20\, s(t_0)\,T^{-6}]\,/2,$$

$$c_3 = [s_3(t_0)T^{-4}+12\, s_2(t_0)T^{-5} + 60\, s_1(t_0)\,T^{-6}+ 120\, s(t_0)T^{-7}]\,/6.$$

Here s_0, s_1 are some noisy measured values of σ and $\dot\sigma$, while s_2, s_3 are the outputs of the 2nd-order differentiator estimating $\ddot\sigma$, $\dddot\sigma$ respectively. The transient time parameter λ takes on values 2 and 6. Parameters of the function φ are calculated at the moment $t_0 = 1$ providing sufficient time for the differentiator convergence. Function φ and its derivatives are calculated further analytically. The output-feedback controller takes now the form

$$\xi(t) = \begin{cases} 0 & \text{with } t \notin [t_0, t_f] \\ \varphi(t) & \text{with } t \in [t_0, t_f] \end{cases};\quad u = \begin{cases} 0 & \text{with } t < t_0 \\ 70\Psi_4(s_0 - \xi, s_1 - \dot\xi, s_2 - \ddot\xi, s_3 - \dddot\xi) & \text{with } t \ge t_0 \end{cases},$$

where the function Ψ_4 is defined a few lines below. The second-order differentiator [16]

$$\dot y = v_1,\qquad v_1 = - 2\, L^{1/3}\, | y - s_1|^{2/3}\, \mathrm{sign}(y - s_1) + s_2,$$

$$\dot s_2 = v_2,\qquad v_2 = - 1.5\, L^{1/2}\, | s_2 - v_1|^{1/2}\, \mathrm{sign}(s_2 - v_1) + s_3,$$

$$\dot s_3 = - 1.1\, L\, \mathrm{sign}(s_3 - v_2).$$

supplies estimates of $\ddot\sigma$, $\dddot\sigma$. Here $L = 300$ ($L > \sup|\sigma^{(4)}|$ is required), y is an additional auxiliary variable approximating $\dot\sigma$, $y(0) = s_2(0) = s_3(0) = 0$.

Denote $z_i = s_i - \xi^{(i)}$, $i = 0, 1, 2, 3$. The 4-sliding homogeneous quasi-continuous controller [18]

$$\Psi_4 = - \{z_3 + 3[|z_2| + (|z_1| + 0.5|z_0|^{3/4})^{-1/3}\, |z_1 + 0.5\, |z_0|^{3/4}\mathrm{sign}\, z_0|\,]\, [|z_2| +(| z_1| + 0.5|z_0|^{3/4})^{2/3}]^{-1/2}\}/$$
$$\{| z_3| + 3[|z_2| + (|z_1| + 0.5\, |z_0|^{3/4})^{2/3}]^{1/2}\}.$$

was applied. Ψ_4 is continuous everywhere except the set $z_0 = z_1 = z_2 = z_3 = 0$, $|\Psi_4| \le 1$ [18].

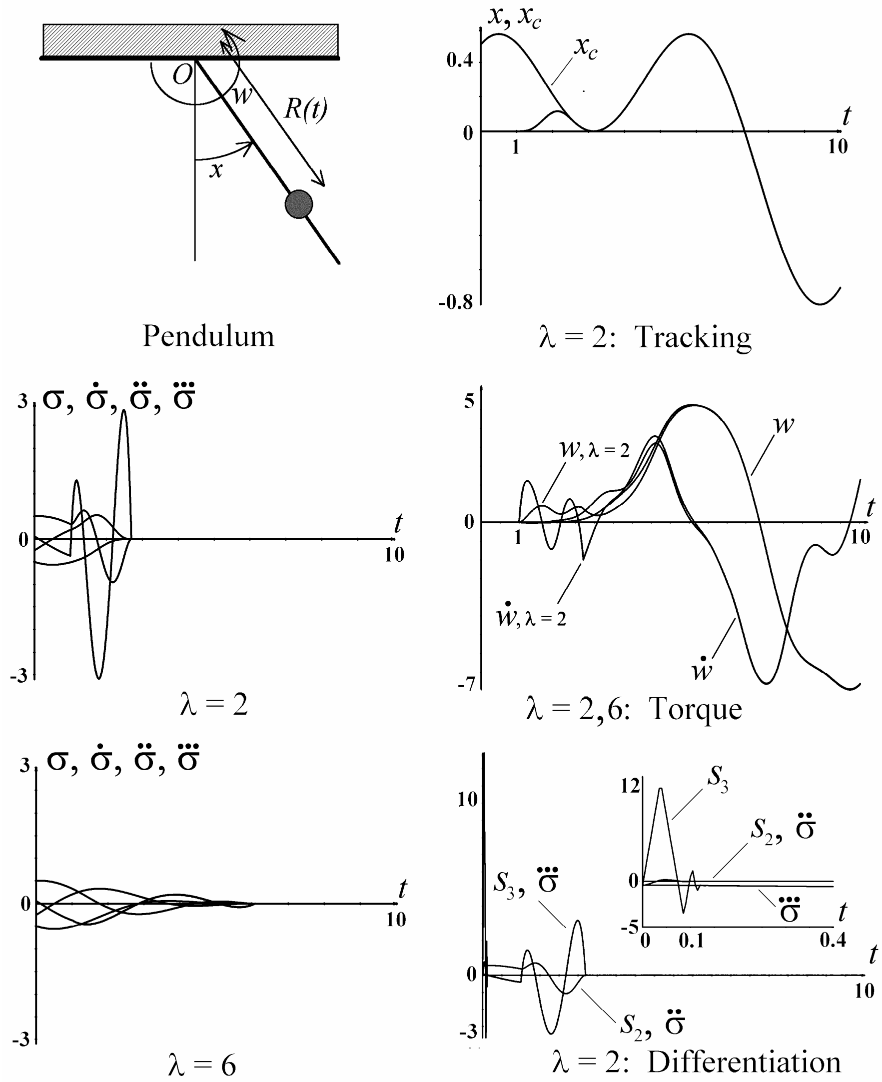

Fig. 1: 4-sliding pendulum control and transient adjustment

The integration was carried out according to the Euler method (the only reliable method with discontinuous dynamics). The 4-sliding deviations for $\lambda = 2, 6$, the tracking performance, the differentiator convergence in the absence of output noises with $\lambda = 2$, and the torques for $\lambda = 2, 6$ are shown in Fig. 1. The accuracies $|\sigma| \le 8.4\cdot 10^{-15}$, $|\dot{\sigma}| \le 2.7\cdot 10^{-11}$, $|\ddot{\sigma}| \le 2.0\cdot 10^{-7}$, $|\dddot{\sigma}| \le 0.0036$ were obtained with the sampling step $\tau = 10^{-5}$. It is seen that the accurate differentiation is ensured already with $t = 0.15$. Larger λ correspond to smaller torque magnitude during the transient.

Check now the robustness of the method. The accuracies changed to $|\sigma| \le 4.4\cdot 10^{-4}$, $|\dot{\sigma}| \le 2.8\cdot 10^{-3}$, $|\ddot{\sigma}| \le 0.054$, $|\dddot{\sigma}| \le 1.7$ with $\tau = 0.005$ (Fig. 2). The tracking accuracy $|\sigma| \le 0.013$ was obtained with $\tau = 10^{-5}$ in the presence of non-smooth non-centered noises with the same magnitude $\varepsilon = 0.01$, the accuracy |σ| ≤ 0.052 was obtained with τ = 0.005, ε = 0.01 (Fig. 2). The graph of the torque $w(t)$ shows that, while noises and discrete sampling lead to the loss of accuracy, no significant chattering is observed. The results practically do not depend on the noise frequencies.

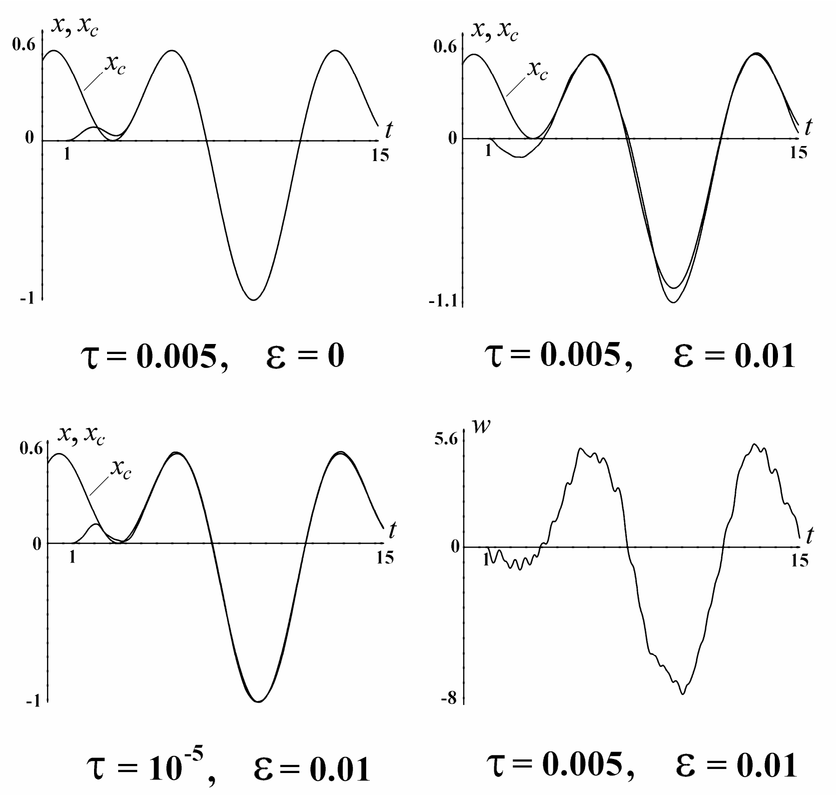

Fig. 2: Performance with various values of the noise amplitude ε and sampling step τ

V. Conclusions

The integral sliding mode approach allows prescribing any needed dynamics to high-order sliding-mode transient. In particular, any continuous positive-definite *r*-sliding-homogeneous function of the weight 1 of initial values of the output and its derivatives can be realized as a transient-time function. A robust global output-feedback controller is obtained, when combined with robust exact differentiator [15]. The same approach solves the long-lasting problem of the control interaction during the procedure of raising the control smoothness degree.

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