

אוניברסיטת תל-אביב

הפקולטה למדעים מדויקים

בחינה במבוא מתמטי לגיאופיזיקאים 1,

0341-1204-01, סמ' א', מועד א', תשס"ד

06.02.2004

המורה: דר' א. לבנט

משך הבחינה: 3 שעות

חומר מותר לשימוש: שני דפים A4 אישיים הממלאים בכתב יד משני הצדדים.

יש לענות על 4 שאלות בלבד ולציין את מספרי השאלות במחברת.

כל שאלה שווה ל-25 נקודות.

1.

א. (10 נק') למצוא $\alpha \in \mathbf{R}$ שבשבילו הזווית ביין הווקטורים $(0, 1, \alpha)$, $(2, 2, \alpha)$ שווה ל- $\pi/4$.

ב. (15 נק') לחשב את האינטגרלים $\int_0^1 \sin^2 x e^{-x} dx$, $\int_0^{\pi/4} \cos^2 x \sin^4 x dx$.

2.

א. (13 נק') להעריך $\cos(1.1\pi)$ ואת השגיאה. הדיוק חייב להיות טוב מ-0.002. לנמק.

ב. (12 נק') לחשב את הגבול $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos x)}{x^2}$.

3.

א. (15 נק') לחקור את הפונקציה $y = \frac{\ln x}{x}$ ולבנות את הגרף.

ב. (10 נק') לחשב את $z + \bar{z}^6$ כאשר $z = 1 - i\sqrt{3}$.

4.

א. (13 נק') ידוע שהפונקציה $z = f(x, y)$ מקיימת את השוויון $x^2 + y - \sin(xz) = 1$, ו- $f(1, 0) = 0$. למצוא את הדיפרנציאל הראשון של $f(x, y)$ בנקודה $(1, 0)$ ולהעריך בקירוב ליניארי את $f(1.001, 0.002)$.

ב. (12 נק') לחשב את האינטגרל $\iint_{\Omega} x dx dy$ כאשר התחום Ω חסום על-ידי הקווים $y = x^3$, $x + y = 2$, $x = 0$.

5.

א. (13 נק') למצוא x, y המקיימים את $x^2 + y^2 = 4$ שעבורם שטח המשולש עם הקדקודים $(0, 0, 1)$, $(0, 1, 0)$, $(x, y, 0)$ מקבל את הערך המקסימלי והמינימלי.

ב. (12 נק') למצוא ערכים עצמיים ווקטורים עצמיים של המטריצה $\begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$.

בהצלחה!

הפתרונות

1, $\alpha \in \mathbb{R}$

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$$\cos \angle \vec{a} \vec{b} = \frac{(\vec{a}, \vec{b})}{|\vec{a}| |\vec{b}|}$$

$$\cos \frac{\pi}{4} = \frac{((0, 1, \alpha), (2, 2, \alpha))}{\sqrt{1 + \alpha^2} \sqrt{8 + \alpha^2}} = \frac{2 + \alpha^2}{\sqrt{1 + \alpha^2} \sqrt{8 + \alpha^2}}$$

$$(2 + \alpha^2) = \frac{\sqrt{2}}{2} \sqrt{1 + \alpha^2} \sqrt{8 + \alpha^2}$$

$$\alpha^2 = t, \quad t \geq 0$$

$$(2 + t)^2 = \frac{1}{2} (t^2 + 9t + 8)$$

$$2t^2 + 8t + 8 = t^2 + 9t + 8$$

$$t^2 - t = 0 \Rightarrow t = 0, t = 1$$

$$\alpha = 0, \alpha = \pm 1 \quad \text{: } \lambda / \mu / \nu / \dots$$

$$\int \sin^2 x e^{-x} dx = \int \sin^2 x d(-e^{-x}) = \quad 1, 2 \text{ } \delta \kappa e$$

$$= -e^{-x} \sin^2 x + \int 2 \sin x \cos x e^{-x} dx = e^{-x} \sin^2 x + I$$

$$I = \int \sin 2x e^{-x} dx \quad 7 \text{ } \kappa >$$

$$I = \int \sin 2x d(e^{-x}) = -\sin 2x e^{-x} + \int 2 \cos 2x e^{-x} dx =$$

$$= -\sin 2x \cdot e^{-x} + 2 \int \cos 2x d(e^{-x}) =$$

$$= -\sin 2x \cdot e^{-x} - 2 \cos 2x \cdot e^{-x} - 4 \int \sin 2x e^{-x} dx$$

$$\Rightarrow I = -\sin 2x e^{-x} - 2 \cos 2x e^{-x} - 4I$$

$$I = -\frac{1}{5} (\sin 2x e^{-x} + 2 \cos 2x e^{-x})$$

$$\int_0^1 \sin^2 x e^{-x} dx = \left[-e^{-x} \sin^2 x + I(x) \right]_0^1 =$$

$$= -e^{-1} \sin^2 1 - \frac{1}{5} (\sin 2 + 2 \cos 2) e^{-1} + \frac{2}{5}$$

$$\int_0^{\frac{\pi}{4}} \cos^2 x \sin^4 x dx = \frac{1}{4} \int_0^{\frac{\pi}{4}} (2 \sin x \cos x)^2 \sin^2 x dx =$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{4}} \sin^2 2x \frac{1 - \cos 2x}{2} dx =$$

$$= \frac{1}{8} \int_0^{\frac{\pi}{4}} \sin^2 2x dx - \frac{1}{16} \int_0^{\frac{\pi}{4}} \sin^2 2x d(\sin 2x) =$$

$$= \frac{1}{16} \int_0^{\frac{\pi}{4}} (1 + \cos 4x) dx - \frac{1}{16} \frac{\sin^3 2x}{3} \Big|_0^{\frac{\pi}{4}} =$$

$$= \frac{1}{16} x \Big|_0^{\frac{\pi}{4}} + \frac{1}{16 \cdot 4} \sin 4x \Big|_0^{\frac{\pi}{4}} - \frac{1}{16} \frac{\sin^3 \frac{\pi}{2}}{3} = \frac{\pi}{64} - \frac{1}{48}$$

$$\cos(1.1\pi) = \cos\pi + \cancel{\left(-\sin\pi\right)} \cdot 0.1\pi + \frac{1}{2} \cancel{\left(-\cos\pi\right)} \cdot (0.1\pi)^2 + \frac{1}{6} \cancel{\sin\pi} \cdot (0.1\pi)^3 + R(x), \quad .K 2, \delta K \epsilon$$

$$\begin{aligned} \cos x' &= -\sin x \\ (\cos x)'' &= -\cos x \\ (\cos x)''' &= \sin x \\ (\cos x)^{IV} &= \cos x \end{aligned}$$

$$R(x) = \frac{\cos c}{24} (0.1\pi)^4 \quad \begin{matrix} \forall c > \\ \pi < c < 1.1\pi \end{matrix}$$

$$\cos(1.1\pi) = -1 + \frac{1}{2}(0.1\pi)^2 + R(x) \quad | \approx \delta$$

$$|R(x)| \leq \frac{|\cos c|}{24} (0.1\pi)^4 \leq \frac{1}{24} \cdot 10^{-4} \cdot 4^4 \leq \frac{256}{24} \cdot 10^{-4} < 11 \cdot 10^{-4} = 0.0011$$

$$\frac{0}{0} \quad \delta \lambda \epsilon \pi \delta \lambda \epsilon$$

$$\lim_{x \rightarrow 0} \frac{\sin(\pi \cos x)}{x^2} \stackrel{\text{L'Hôpital}}{=} \lim_{x \rightarrow 0} \frac{-\cos(\pi \cos x) \pi \sin x}{2x} = \frac{\pi}{2} \quad . \sim$$

$$\cos(\pi \cos x) \rightarrow \cos \pi = -1, \quad \frac{\sin x}{x} \rightarrow 1$$

$\delta \lambda \epsilon \pi \delta \lambda \epsilon \delta \lambda \epsilon$ $\lambda \epsilon$ $\epsilon \lambda \epsilon \delta \lambda \epsilon$ $\delta \lambda \epsilon$ $\delta \lambda \epsilon$

פאקט 3.

$$y = \frac{\ln x}{x}$$

א. תחום ההשערה $x > 0$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} \stackrel{\text{ל'יטל}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0 \Rightarrow y = 0 \text{ אסימטוטה}$$

אשר גם הפונקציה גוברת (צמיחה):

$$a = \lim_{x \rightarrow \infty} \frac{\ln x}{x^2} \stackrel{\text{ל'יטל}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{2x} = 0$$

$$b = \lim_{x \rightarrow \infty} \left[\frac{\ln x}{x} - ax \right] = \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0$$

$$y = ax + b = 0 \text{ אסימטוטה אנכית}$$

$$\lim_{\substack{x \rightarrow 0^+ \\ x \rightarrow 0^+}} \frac{\ln x}{x} \xrightarrow{-\infty} -\infty$$

$$x = 0 \text{ אסימטוטה}$$

$$\frac{\ln x}{x} = 0 \Rightarrow x = 1$$

$$y' = \frac{1}{x^2} - \frac{\ln x}{x^2}$$

$$y' = 0 \Rightarrow x = e$$

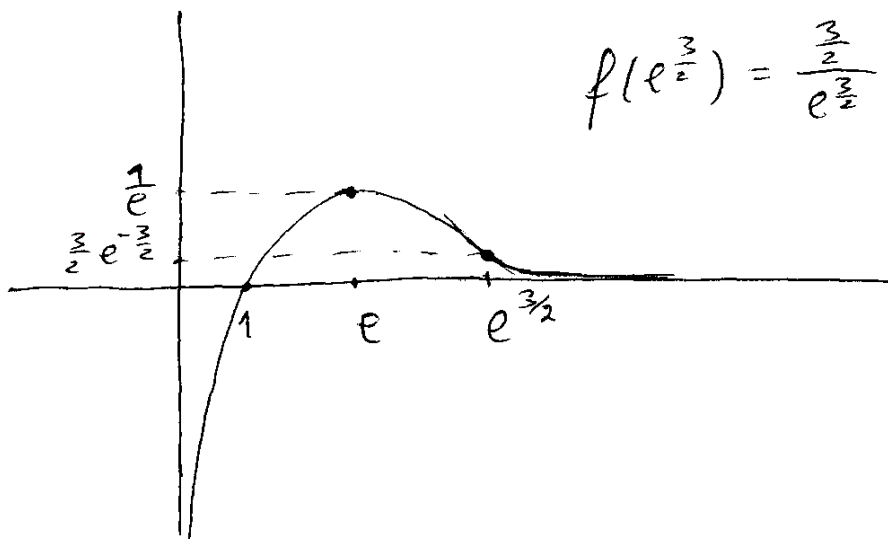
$x > e$ ירידה
 $x < e$ עלייה

$$y'' = -\frac{2}{x^3} - \frac{1}{x^3} + 2 \frac{\ln x}{x^3}, \quad y'' = 0 \Rightarrow \ln x = \frac{3}{2} \\ x = e^{\frac{3}{2}}$$

$$y'' = \frac{2 \ln x - 3}{x^3}$$

נק' כיתה $\Rightarrow$ נק' פנימי ונק' גבול $x = e^{\frac{3}{2}}$

$$f(e^{\frac{3}{2}}) = \frac{\frac{3}{2}}{e^{\frac{3}{2}}} = \frac{3}{2} e^{-\frac{3}{2}}$$



$$z = 1 - i\sqrt{3} \quad z + \bar{z}^6 = ?$$

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$$\bar{z} = 1 + i\sqrt{3} = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$\bar{z}^6 = 2^6\left(\cos\frac{6\pi}{3} + i\sin\frac{6\pi}{3}\right) = 64(\cos(2\pi) + i\sin(2\pi)) = 64$$

$$z + \bar{z}^6 = 65 - i\sqrt{3}$$

10.4.2020

$$F(x, y, z) = x^2 + y - \sin(xz) = 1, \quad z = f(x, y)$$

$$\text{נדרש } f(1, 0) = 0$$

$$F(1, 0, f(1, 0)) = 1 + 0 - \sin(0.1) = 1, \quad F'_z(1, 0, 0) = -\cos(0) \cdot 1 = -1 \neq 0$$

$$\text{לכן } (1, 0) \text{ איננו נקודה קריטית של } f(x, y) \Leftarrow$$

$$x^2 + y - \sin(xf(x, y)) = 1$$

$$2x - \cos(xf)(f + xf'_x) = 0 \quad \text{x נגזר}$$

$$2 - f'_x(1, 0) = 0 \quad \Leftrightarrow \begin{cases} (x, y) = (1, 0) \\ f(x, y) = 0 \end{cases} \text{ נציב}$$

$$f'_x(1, 0) = 2$$

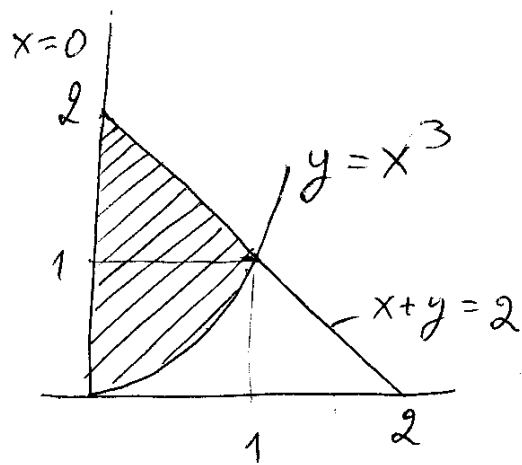
$$1 - \cos(xf) f'_y = 0 \quad \text{y נגזר}$$

$$1 - f'_y(1, 0) = 0 \quad \Leftrightarrow \begin{cases} (x, y) = (1, 0) \\ f(x, y) = 0 \end{cases} \text{ נציב}$$

$$f'_y(1, 0) = 1$$

$$df(1, 0) = f'_x(1, 0)\Delta x + f'_y(1, 0)\Delta y$$

$$\begin{aligned} f(1.001, 0.002) &\approx f(1, 0) + f'_x(1, 0)\Delta x + f'_y(1, 0)\Delta y = \\ &= 0 + 2 \cdot 0.001 + 1 \cdot 0.002 = 0.004 \end{aligned}$$



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$$\begin{aligned}
 \iint_{\Omega} x \, dx \, dy &= \int_0^1 dx \int_{x^3}^{2-x} x \, dy = \\
 &= \int_0^1 x(2-x-x^3) \, dx = \int_0^1 (2x - x^2 - x^4) \, dx = x^2 \Big|_0^1 - \frac{x^3}{3} \Big|_0^1 - \frac{x^5}{5} \Big|_0^1 = \\
 &= 1 - \frac{1}{3} - \frac{1}{5} = \frac{7}{15}
 \end{aligned}$$

$$x^2 + y^2 = 4$$

כ 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 143, 149, 151, 157, 163, 167, 173, 179, 181, 187, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 247, 251, 257, 263, 269, 271, 277, 281, 283, 287, 293, 299, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 437, 439, 443, 449, 457, 461, 463, 467, 473, 479, 487, 491, 493, 497, 503, 509, 517, 521, 523, 527, 533, 539, 541, 547, 551, 557, 563, 569, 571, 577, 581, 583, 587, 593, 599, 607, 611, 613, 617, 631, 637, 647, 649, 653, 659, 667, 673, 679, 683, 687, 691, 693, 697, 701, 709, 713, 717, 727, 731, 733, 737, 743, 749, 751, 757, 761, 763, 767, 773, 779, 781, 787, 791, 793, 797, 803, 809, 811, 813, 817, 821, 823, 827, 833, 839, 841, 847, 851, 857, 859, 863, 869, 871, 877, 881, 883, 887, 893, 899, 907, 911, 913, 917, 923, 929, 931, 937, 941, 943, 947, 953, 959, 961, 967, 971, 973, 977, 983, 989, 991, 993, 997, 1003, 1009, 1013, 1017, 1021, 1023, 1027, 1033, 1039, 1041, 1047, 1051, 1057, 1063, 1069, 1071, 1077, 1081, 1083, 1087, 1093, 1099, 1103, 1107, 1113, 1117, 1121, 1123, 1127, 1133, 1139, 1141, 1147, 1151, 1157, 1163, 1169, 1171, 1177, 1181, 1183, 1187, 1193, 1199, 1201, 1203, 1207, 1211, 1213, 1217, 1221, 1223, 1227, 1231, 1233, 1237, 1243, 1249, 1251, 1257, 1261, 1263, 1267, 1271, 1273, 1277, 1283, 1289, 1291, 1297, 1301, 1303, 1307, 1311, 1313, 1317, 1321, 1323, 1327, 1331, 1333, 1337, 1343, 1349, 1351, 1357, 1361, 1363, 1367, 1371, 1373, 1377, 1381, 1383, 1387, 1393, 1399, 1401, 1403, 1407, 1411, 1413, 1417, 1421, 1423, 1427, 1431, 1433, 1437, 1443, 1449, 1451, 1457, 1461, 1463, 1467, 1471, 1473, 1477, 1481, 1483, 1487, 1493, 1499, 1501, 1503, 1507, 1511, 1513, 1517, 1521, 1523, 1527, 1531, 1533, 1537, 1543, 1549, 1551, 1557, 1561, 1563, 1567, 1571, 1573, 1577, 1581, 1583, 1587, 1593, 1599, 1601, 1603, 1607, 1611, 1613, 1617, 1621, 1623, 1627, 1631, 1633, 1637, 1643, 1649, 1651, 1657, 1661, 1663, 1667, 1671, 1673, 1677, 1681, 1683, 1687, 1693, 1699, 1701, 1703, 1707, 1711, 1713, 1717, 1721, 1723, 1727, 1731, 1733, 1737, 1743, 1749, 1751, 1757, 1761, 1763, 1767, 1771, 1773, 1777, 1781, 1783, 1787, 1793, 1799, 1801, 1803, 1807, 1811, 1813, 1817, 1821, 1823, 1827, 1831, 1833, 1837, 1843, 1849, 1851, 1857, 1861, 1863, 1867, 1871, 1873, 1877, 1881, 1883, 1887, 1893, 1899, 1901, 1903, 1907, 1911, 1913, 1917, 1921, 1923, 1927, 1931, 1933, 1937, 1943, 1949, 1951, 1957, 1961, 1963, 1967, 1971, 1973, 1977, 1981, 1983, 1987, 1993, 1999, 2001, 2003, 2007, 2011, 2013, 2017, 2021, 2023, 2027, 2031, 2033, 2037, 2043, 2049, 2051, 2057, 2061, 2063, 2067, 2071, 2073, 2077, 2081, 2083, 2087, 2093, 2099, 2101, 2103, 2107, 2111, 2113, 2117, 2121, 2123, 2127, 2131, 2133, 2137, 2143, 2149, 2151, 2157, 2161, 2163, 2167, 2171, 2173, 2177, 2181, 2183, 2187, 2193, 2199, 2201, 2203, 2207, 2211, 2213, 2217, 2221, 2223, 2227, 2231, 2233, 2237, 2243, 2249, 2251, 2257, 2261, 2263, 2267, 2271, 2273, 2277, 2281, 2283, 2287, 2293, 2299, 2301, 2303, 2307, 2311, 2313, 2317, 2321, 2323, 2327, 2331, 2333, 2337, 2343, 2349, 2351, 2357, 2361, 2363, 2367, 2371, 2373, 2377, 2381, 2383, 2387, 2393, 2399, 2401, 2403, 2407, 2411, 2413, 2417, 2421, 2423, 2427, 2431, 2433, 2437, 2443, 2449, 2451, 2457, 2461, 2463, 2467, 2471, 2473, 2477, 2481, 2483, 2487, 2493, 2499, 2501, 2503, 2507, 2511, 2513, 2517, 2521, 2523, 2527, 2531, 2533, 2537, 2543, 2549, 2551, 2557, 2561, 2563, 2567, 2571, 2573, 2577, 2581, 2583, 2587, 2593, 2599, 2601, 2603, 2607, 2611, 2613, 2617, 2621, 2623, 2627, 2631, 2633, 2637, 2643, 2649, 2651, 2657, 2661, 2663, 2667, 2671, 2673, 2677, 2681, 2683, 2687, 2693, 2699, 2701, 2703, 2707, 2711, 2713, 2717, 2721, 2723, 2727, 2731, 2733, 2737, 2743, 2749, 2751, 2757, 2761, 2763, 2767, 2771, 2773, 2777, 2781, 2783, 2787, 2793, 2799, 2801, 2803, 2807, 2811, 2813, 2817, 2821, 2823, 2827, 2831, 2833, 2837, 2843, 2849, 2851, 2857, 2861, 2863, 2867, 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4101, 4103, 4107, 4111, 4113, 4117, 4121, 4123, 4127, 4131, 4133, 4137, 4143, 4149, 4151, 4157, 4161, 4163, 4167, 4171, 4173, 4177, 4181, 4183, 4187, 4193, 4199, 4201, 4203, 4207, 4211, 4213, 4217, 4221, 4223, 4227, 4231, 4233, 4237, 4243, 4249, 4251, 4257, 4261, 4263, 4267, 4271, 4273, 4277, 4281, 4283, 4287, 4293, 4299, 4301, 4303, 4307, 4311, 4313, 4317, 4321, 4323, 4327, 4331, 4333, 4337, 4343, 4349, 4351, 4357, 4361, 4363, 4367, 4371, 4373, 4377, 4381, 4383, 4387, 4393, 4399, 4401, 4403, 4407, 4411, 4413, 4417, 4421, 4423, 4427, 4431, 4433, 4437, 4443, 4449, 4451, 4457, 4461, 4463, 4467, 4471, 4473, 4477, 4481, 4483, 4487, 4493, 4499, 4501, 4503, 4507, 4511, 4513, 4517, 4521, 4523, 4527, 4531, 4533, 4537, 4543, 4549, 4551, 4557, 4561, 4563, 4567, 4571, 4573, 4577, 4581, 4583, 4587, 4593, 4599, 4601, 4603, 4607, 4611, 4613, 4617, 4621, 4623, 4627, 4631, 4633, 4637, 4643, 4649, 4651, 4657, 4661, 4663, 4667, 4671, 4673, 4677, 4681, 4683, 4687, 4693, 4699, 4701, 4703, 4707, 4711, 4713, 4717, 4721, 4723, 4727, 4731, 4733, 4737, 4743, 4749, 4751, 4757, 4761, 4763, 4767, 4771, 4773, 4777, 4781, 4783, 4787, 4793, 4799, 4801, 4803, 4807, 4811, 4813, 4817, 4821, 4823, 4827, 4831, 4833, 4837, 4843, 4849, 4851, 4857, 4861, 4863, 4867, 4871, 4873, 4877, 4881, 4883, 4887, 4893, 4899, 4901, 4903, 4907, 4911, 4913, 4917, 4921, 4923, 4927, 4931, 4933, 4937, 4943, 4949, 4951, 4957, 4961, 4963, 4967, 4971, 4973, 4977, 4981, 4983, 4987, 4993, 4999, 5001, 5003, 5007, 5011, 5013, 5017, 5021, 5023, 5027, 5031, 5033, 5037, 5043, 5049, 5051, 5057, 5061, 5063, 5067, 5071, 5073, 5077, 5081, 5083, 5087, 5093, 5099, 5101, 5103, 5107, 5111, 5113, 5117, 5121, 5123, 5127, 5131, 5133, 5137, 5143, 5149, 5151, 5157, 5161, 5163, 5167, 5171, 5173, 5177, 5181, 5183, 5187, 5193, 5199, 5201, 5203, 5207, 5211, 5213, 5217, 5221, 5223, 5227, 5231, 5233, 5237, 5243, 5249, 5251, 5257, 5261, 5263, 5267, 5271, 5273, 5277, 5281, 5283, 5287, 5293, 5299, 5301, 5303, 5307, 5311, 5313, 5317, 5321, 5323, 5327, 5331, 5333, 5337, 5343, 5349, 5351, 5357, 5361, 5363, 5367, 5371, 5373, 5377, 5381, 5383, 5387, 5393, 5399, 5401, 5403, 5407, 5411, 5413, 5417, 5421, 5423, 5427, 5431, 5433, 5437, 5443, 5449, 5451, 5457, 5461, 5463, 5467, 5471, 5473, 5477, 5481, 5483, 5487, 5493, 5499, 5501, 5503, 5507, 5511, 5513, 5517, 5521, 5523, 5527, 5531, 5533, 5537, 5543, 5549, 5551, 5557, 5561, 5563, 5567, 5571, 5573, 5577, 5581, 5583, 5587, 5593, 5599, 5601, 5603, 5607, 5611, 5613, 5617, 5621, 5623, 5627, 5631, 5633, 5637, 5643, 5649, 5651, 5657, 5661, 5663, 5667, 5671, 5673, 5677, 5681, 5683, 5687, 5693, 5699, 5701, 5703, 5707, 5711, 5713, 5717, 5721, 5723, 5727, 5731, 5733, 5737, 5743, 5749, 5751, 5757, 5761, 5763, 5767, 5771, 5773, 5777, 5781, 5783, 5787, 5793, 5799, 5801, 5803, 5807, 5811, 5813, 5817, 5821, 5823, 5827, 5831, 5833, 5837, 5843, 5849, 5851, 5857, 5861, 5863, 5867, 5871, 5873, 5877, 5881, 5883, 5887, 5893, 5899, 5901, 5903, 5907, 5911, 5913, 5917, 5921, 5923, 5927, 5931, 5933, 5937, 5943, 5949, 5951, 5957, 5961, 5963, 5967, 5971, 5973, 5977, 5981, 5983, 5987, 5993, 5999, 6001, 6003, 6007, 6011, 6013, 6017, 6021, 6023, 6027, 6031, 6033, 6037, 6043, 6049, 6051, 6057, 6061, 6063, 6067, 6071, 6073, 6077, 6081, 6083, 6087, 6093, 6099, 6101, 6103, 6107, 6111, 6113, 6117, 6121, 6123, 6127, 6131, 6133, 6137, 6143, 6149, 6151, 6157, 6161, 6163, 6167, 6171, 6173, 6177, 6181, 6183, 6187, 6193, 6199, 6201, 6203, 6207, 6211, 6213, 6217, 6221, 6223, 6227, 6231, 6233, 6237, 6243, 6249, 6251, 6257, 6261, 6263, 6267, 6271, 6273, 6277, 6281, 6283, 6287, 6293, 6299, 6301, 6303, 6307, 6311, 6313, 6317, 6321, 6323, 6327, 6331, 6333, 6337, 6343, 6349, 6351, 6357, 6361, 6363, 6367, 6371, 6373, 6377, 6381, 6383, 6387, 6393, 6399, 6401, 6403, 6407, 6411, 6413, 6417, 6421, 6423, 6427, 6431, 6433, 6437, 6443, 6449, 6451, 6457, 6461, 6463, 6467, 6471, 6473, 6477, 6481, 6483, 6487, 6493, 6499, 6501, 6503, 6507, 6511, 6513, 6517, 6521, 6523, 6527, 6531, 6533, 6537, 6543, 6549, 6551, 6557, 6561, 6563, 6567, 6571, 6573, 6577, 6581, 6583, 6587, 6593, 6599, 6601, 6603, 6607, 6611, 6613, 6617, 6621, 6623, 6627, 6631, 6633, 6637, 6643, 6649, 6651, 6657, 6661, 6663, 6667, 6671, 6673, 6677, 6681, 6683, 6687, 6693, 6699, 6701, 6703, 6707, 6711, 6713, 6717, 6721, 6723, 6727, 6731, 6733, 6737, 6743, 6749, 6751, 6757, 6761, 6763, 6767, 6771, 6773, 6777, 6781, 6783, 6787, 6793, 6799, 6801, 6803, 6807, 6811, 6813, 6817, 6821, 6823, 6827, 6831, 6833, 6837, 6843, 6849, 6851, 6857, 6861, 6863, 6867, 6871, 6873, 6877, 6881, 6883, 6887, 6893, 6899, 6901, 6903, 6907, 6911, 6913, 6917, 6921, 6923, 6927, 6931, 6933, 6937, 6943, 6949, 6951, 6957, 6961, 6963, 6967, 6971, 6973, 6977, 6981, 6983, 6987, 6993, 6999, 7001, 7003, 7007, 7011, 7013, 7017, 7021, 7023, 7027, 7031, 7033, 7037, 7043, 7049, 7051, 7057, 7061, 7063, 7067, 7071, 7073, 7077, 7081, 7083, 7087, 7093, 7099, 7101, 7103, 7107, 7111, 7113, 7117, 7121, 7123, 7127, 7131, 7133, 7137, 7143, 7149, 7151, 7157, 7161, 7163, 7167, 7171, 7173, 7177, 7181, 7183, 7187, 7193, 7199, 7201, 7203, 7207, 7211, 7213, 7217, 7221, 7223, 7227, 7231, 7233, 7237, 7243, 7249, 7251, 7257, 7261, 7263, 7267, 7271, 7273, 7277, 7281, 7283, 7287, 7293, 7299, 7301, 7303, 7307, 7311, 7313, 7317, 7321, 7323, 7327, 7331, 7333, 7337, 7343, 7349, 7351, 7357, 7361, 7363, 7367, 7371, 7373, 7377, 7381, 7383, 7387, 7393, 7399, 7401, 7403, 7407, 7411, 7413, 7417, 7421, 7423, 7427, 7431, 7433, 7437, 7443, 7449, 7451, 7457, 7461, 7463, 7467, 7471, 7473, 7477, 7481, 7483, 7487, 7493, 7499, 7501, 7503, 7507, 7511, 7513, 7517, 7521, 7523, 7527, 7531, 7533, 7537, 7543, 7549, 7551, 7557, 7561, 7563, 7567, 7571, 7573, 7577, 7581, 7583, 7587, 7593, 7599, 7601, 7603, 7607, 7611, 7613, 7617, 7621, 7623, 7627, 7631, 7633, 7637, 7643, 7649, 7651, 7657, 7661, 7663, 7667, 7671, 7673, 7677, 7681, 7683, 7687, 7693, 7699, 7701, 7703, 7707, 7711, 7713, 7717, 7721, 7723, 7727, 7731, 7733, 7737, 7743, 7749, 7751, 7757, 7761, 7763, 7767, 7771, 7773, 7777, 7781, 7783, 7787, 7793, 7799, 7801, 7803, 7807, 7811, 7813, 7817, 7821, 7823, 7827, 7831, 7833, 7837, 7843, 7849, 7851, 7857, 7861, 7863, 7867, 7871, 7873, 7877, 7881, 7883, 7887, 7893, 7899, 7901, 7903, 7907, 7911, 7913, 7917, 7921, 7923, 7927, 7931, 7933, 7937, 7943, 7949, 7951, 7957, 7961, 7963, 7967, 7971, 7973, 7977, 7981, 7983, 7987, 7993, 7999, 8001, 800

2.5, 5.8

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$$

$$p(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{vmatrix} =$$

$$= \lambda^2 - 3\lambda + 2 - 6 = \lambda^2 - 3\lambda - 4 = (\lambda - 4)(\lambda + 1)$$

$$\lambda = 4 \quad \text{נצח } \lambda = 4$$

$$(A - \lambda I) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3x + 3y \\ 2x - 2y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x = y \Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{נצח } \lambda = 4 \text{ ו-} \lambda = -1$$

$$\lambda = -1 \quad \text{נצח } \lambda = 4$$

$$\begin{array}{l} 2x + 3y = 0 \\ 2x + 3y = 0 \end{array} \quad \left| \quad \begin{array}{l} 2x + 3y = 0 \\ \Rightarrow \end{array} \right. \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad \text{נצח } \lambda = 4 \text{ ו-} \lambda = -1$$